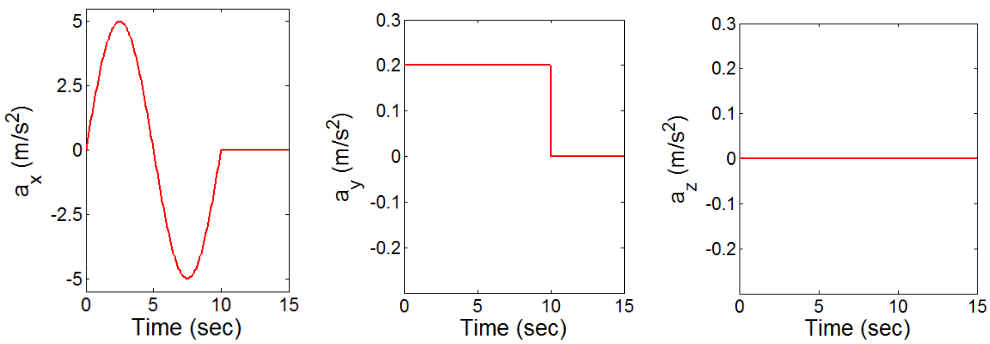
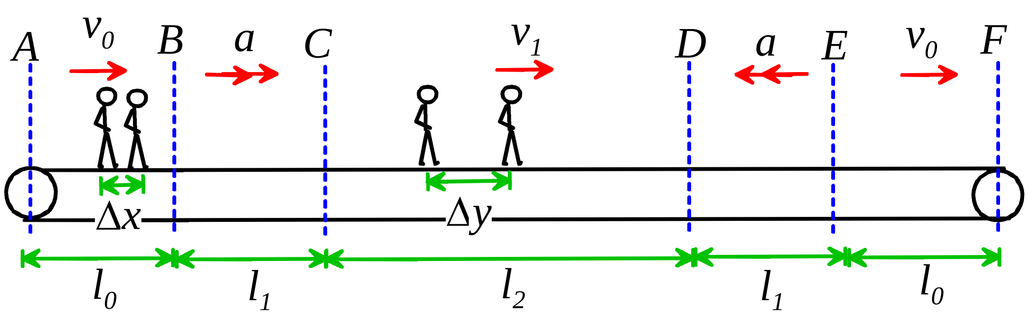
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 2: Dynamics of Particles
Due Friday Feb 10, 2012



1. A three-axis accelerometer mounted on an inertial platform with fixed orientation measures three components of acceleration shown in the figure (the first figure can be approximated as a sinusoidal function). If the platform is initially at rest at the origin, what is its position vector after 15sec?



2. The figure shows a design for a high-speed moving walkway (see http://www.jfe-steel.co.jp/archives/en/nkk_giho/84/pdf/84_10.pdf for a detailed description of this general type of design, or **http://www.youtube.com/watch?v=uwHer1RrYg8** for a movie of such a walkway in action). A passenger standing on the walkway passes through five regions:

(i) between A and B she moves at constant speed v_0 ;

(ii) between B and C she accelerates with constant acceleration a;

(iii) between C and D she moves with constant (high) speed v_1 ; and

(iv) between D and E she decelerates at constant rate $-a$, and

(v) between E and F she travels at speed v_0 again.

In this problem, you will work through a few of the calculations required to design the walkway.

2.1 Consider a passenger who arrives at point B at time $t=0$ and travels between B and C. Write down formulas for her speed $v(t)$ and distance traveled $y(t)$ in terms of v_0 and a while she is in this part of the walkway. (Don't think too hard about this – its just the straight line motion formulas!)

2.2 Noting that the passenger must acceleration from speed v_0, to speed v_1 at constant acceleration between B and C, find an expression for the length l_1 of the walkway between B and C, in terms of v_0, v_1 and a.

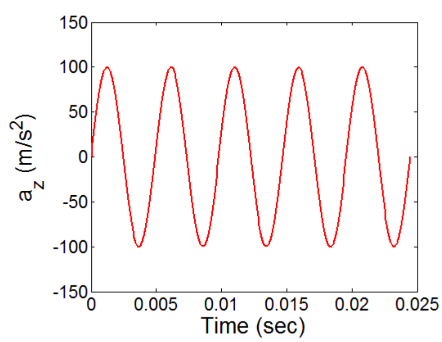
2.3 Find a formula for the total time taken for a passenger to travel from A to F, in terms of l_0, l_2, v_0, v_1, a

2.4 The walkway in Toronto has $v_1 = 400\, ft / \min$ $v_0 = 125\, ft / \min$. Its total length is 912ft (http://www.thyssenkruppelevator.com/downloads/Executive_Manual_Testimonials.pdf). In this design $l_0 = 0$ Use the movie posted online to estimate the acceleration a (you can use the movie to time roughly how long it takes to transition from on velocity to the other), and hence determine l_1. Calculate the total time of travel along the walkway (assume the passenger stands still on the walkway), and calculate how much time is saved per passenger by using a high-speed walkway compared to a low speed (125 ft/min) version.

2.5 **OPTIONAL (hard extra credit problem…)** Suppose that a row of passengers stand on the walkway. They are a small distance Δx apart while traveling from A to B. How far apart are they when traveling between C and D? Express your answer in terms of Δx, v_1 and v_0.

2.6 **OPTIONAL** (also hard, and also extra credit) Suppose that a passenger walks along the walkway at constant speed w *relative to the travelling walkway* between B and C. What is her acceleration? Express your answer as a formula involving her distance y from B, as well as other relevant parameters.

3. In an experiment to measure the stiffness of a thin, extensible wire, an accelerometer with mass 100 grams is suspended from the end of the wire. (Recall that the stiffness of a wire (k) relates the force (F) to the extension x of the wire by $F=kx$). The accelerometer is then tapped lightly to start it vibrating. The figure shows the measured vibration (note the simple harmonic motion).

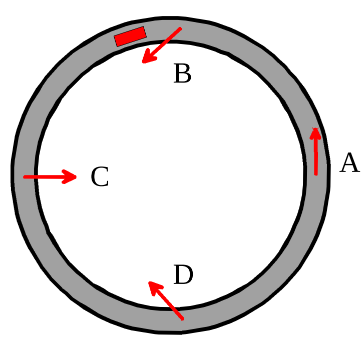


3.1 Determine the period, angular frequency, and frequency of the acceleration

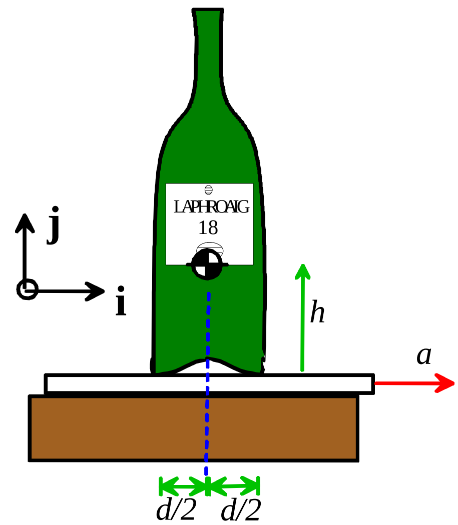
3.2 Hence, determine the amplitude of the displacement of the accelerometer

3.2 By considering the force necessary to cause the measured acceleration, and using 3.2 to determine the corresponding stretch of the wire, determine the stiffness of the wire. Express your answer in *N/m*.

4. The figure shows the direction of the acceleration vector for a vehicle traveling counterclockwise around a circular track, at four different points along the path. The vehicle starts at A. Sketch a graph showing the variation of the car's speed (i.e. the magnitude of the velocity) with distance traveled (label points A,B,C,D on your graph).



5. The figure shows the 'tablecloth' trick demonstrated in class. The bottle has diameter *d* at the base, and its center of mass is a height *h* above the table. The coefficient of friction between cloth and bottle is μ. The cloth is pulled horizontally with an acceleration $a > \mu g$ so the cloth slips under the bottle.

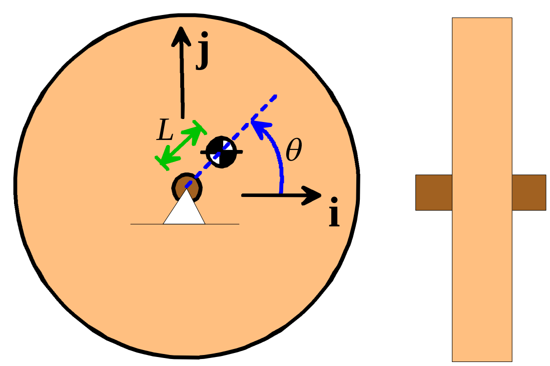


4.1 Draw a free body diagram showing the forces acting on the bottle.

4.2 Assuming that the bottle does not tip, calculate its horizontal acceleration.

4.3 Show that the bottle will tip over if *h/d* exceeds a critical value, and give an expression for the maximum allowable value of *h/d* for the trick to work.

6. An unbalanced rotor that is spun at constant speed by a motor attached to its hub can be idealized as a particle with mass *m* located at the center of mass of the rotor, which is a distance *L* from the hub as shown in the figure.

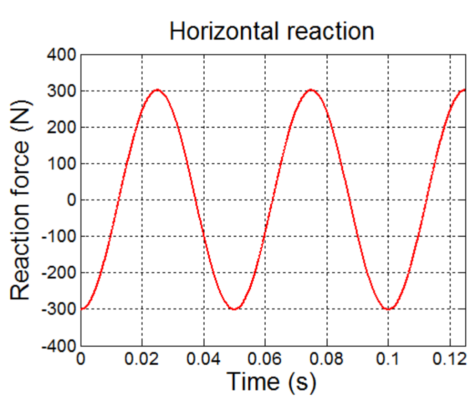


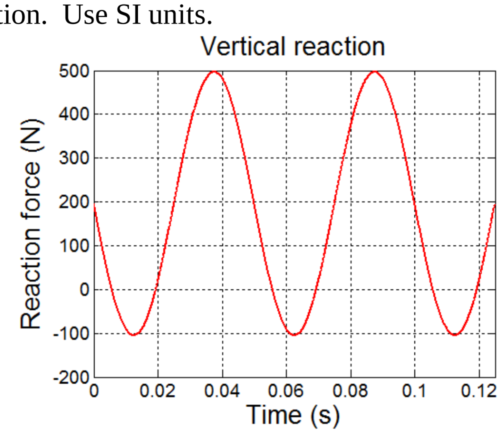
6.1 Write down the position vector of the particle (i.e. center of mass) in terms of *L* and and the angle θ. Hence, derive expressions for its acceleration in terms of θ, $\omega = d\theta / dt$ and *L*. Use the basis shown, and assume that ω is constant.

6.2 Draw the forces and moments acting on the rotor Gravity should be included.

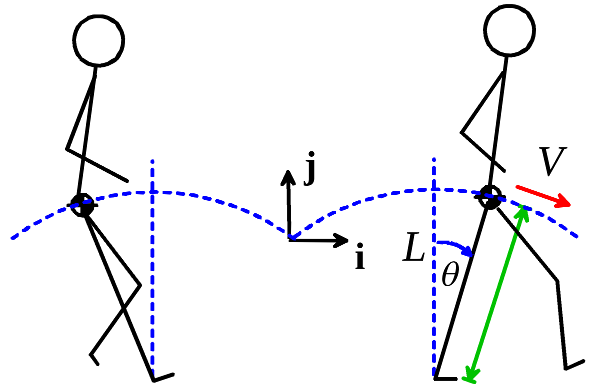
6.3 Hence, calculate expressions for the horizontal and vertical reaction forces acting at the rotor hub as functions of time.

6.4 The graphs show reaction forces measured experimentally. Determine the mass of the rotor, and the distance of the center of mass from the axis of rotation. Use SI units.





7. In this problem you will estimate the maximum possible speed of a human walker. As you probably know, the difference between running and walking is that, when you run, your feet leave the ground, whereas they remain in contact with the ground if you walk. Your goal is to estimate the speed at which your feet lose contact with the ground.



7.1 Suppose that you walk with your load bearing leg straight – your center of mass then travels along a series of circular arcs (with each arc corresponding to single step), as shown in the figure. Assume that your COM moves with constant (absolute) speed V. Find formulas for the horizontal and vertical components of acceleration of the COM as functions of the angle θ as well as V and L.

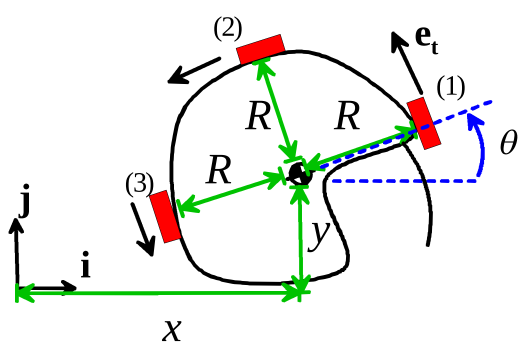
7.2 Draw a free-body diagram for the person walking, and use Newton's laws of motion to calculate formulas for the reaction forces acting on the foot in contact with the ground

7.3 Hence, calculate a formula for the critical speed at which your foot must leave the ground.

7.4 Estimate the length of your legs, and measure your maximum walking speed experimentally. Be sure to walk so that your load-bearing leg remains straight (bent legs are illegal in speed walking – if you practice a special gait with bent legs that allows your COM to move along a straight line, you can easily exceed the theoretical limit). Explain briefly how you did the experiment. Compare your speed with the predictions of 7.3.

8. OPTIONAL (extra credit) Football teams are increasingly using instrumented helmets to detect head impacts that may lead to injury http://www.browndailyherald.com/campus-news/football-helmet-sensors-measure-concussions-1.1672758 , http://www.sportsgrid.com/nfl/nfl-looks-to-technology-to-prevent-head-injuries/

The figure shows a helmet with three accelerometers attached to its surface. Each accelerometer measures the acceleration component in a direction parallel to its axis. It is then necessary to calculate the acceleration of the center of mass of the head a_x, a_y, together with the rotational acceleration $d^2\theta / dt^2$. This data can then be used to deduce the location, direction, and magnitude of the impact force.

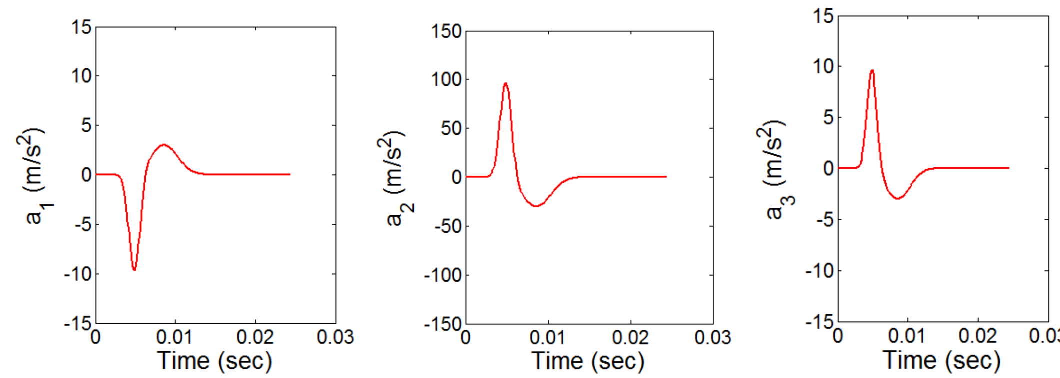


7.1 Suppose that the center of mass of the head has instantaneous position $x(t), y(t)$ as shown, and consider an accelerometer that subtends an angle $\theta(t)$ to the horizontal. Write down the components of the position vector of the accelerometer.

7.2 Write down the components of a unit vector tangent to the accelerometer. Hence, find a formula for the component of acceleration parallel to this direction ($a_t^{(1)}$, in terms of $\theta(t)$ and its time derivatives, as well as time derivatives of $x(t), y(t)$ (Mathematica will make this painless)

7.3 Suppose that three accelerometers are mounted on the helmet, 90 degrees apart, as shown on the figure. Use the result of (7.2) to write down formulas for the tangential accelerations $a_t^{(2)}, a_t^{(3)}$ of accelerometers (2) and (3) in terms of $d^2x/dt^2, d^2y/dt^2, \theta(t), d^2\theta/dt^2$ and R.

7.4 Suppose that $a_t^{(1)}, a_t^{(2)}, a_t^{(3)}$ are measured as functions of time. Derive formulas for $d^2x/dt^2, d^2y/dt^2, d^2\theta/dt^2$ in terms of these three measurements and the angle θ.



7.5 Suppose that a helmet starts at rest and has orientation $\theta = 0$. An impact is then detected with acceleration signals shown in the figure (note that $a_3(t) = -a_1(t)$). If the mass of the head is 4.5kg, what is the peak impact force? What is the angle ϕ of the impact location on the head?