Homework 4: Work, Energy and Linear Momentum
Due Friday March 2nd

School of Engineering
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1. The 'Morse Potential' is often used to approximate the forces acting between atoms in a diatomic molecule. The work done to separate two interacting atoms from the equilibrium distance between atoms (r_e) to a distance d apart has the following form

$$V(d) = E_0 \left[1 - \exp\{-a(d - r_e)\} \right]^2$$

1.1 Find an expression for the magnitude of the force acting between the atoms, and plot the force as a function of d, for $E_0 = 1, r_e = 1, a = 5$, with 0.9<d<3

1.2 Hence, express the following physical quantities in terms of the parameters in the potential

  (i)   The bond strength (the force required to break the bond)

  (ii) The binding energy (the total energy required to pull the atoms apart from their equilibrium spacing to infinity)

  (iii) The stiffness of the bond (i.e. the slope of the force-separation relation at the equilibrium spacing.

1.3 A hydrogen molecule has bond spacing 0.74A, binding energy 4.72eV (electron-volts), and stiffness 33.72 eV/A^2. Estimate the force required to break the bond. Express your answer in both eV/A.

2. The figure shows the force-v-strain relation for a specimen of spider-silk (from Biophys J. 2007 December 15; 93(12): 4425–4432.). Estimate the total energy per unit length required to stretch the silk to its breaking point (use the 44% relative humidity (RH) curve).
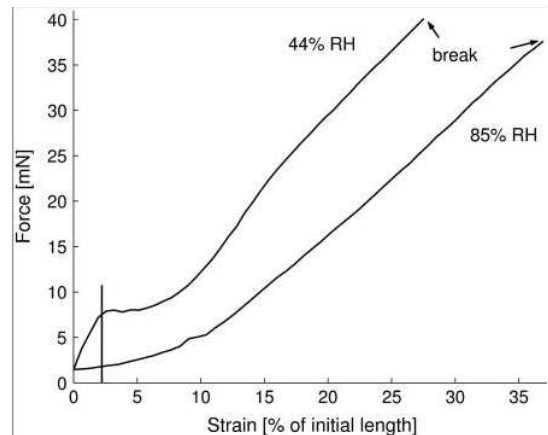
2.2 Consider an idealized spider-web made from three radial threads, as shown in the figure. Take the length of threads to be 15cm. (A detailed study of spider-web dimensions and their correlation with the spider's diet can be found here http://deepblue.lib.umich.edu/bitstream/2027.42/31447/1/0000367.pdf). Suppose that the center of the web is impacted by (a) a house-fly, with mass 12mg and speed 4.5mph; or (b) a horse-fly with mass 12mg and speed 90mph (!! – the reference for these is http://www.speedofanimals.com/animals/housefly, which may not be trustworthy…). Using energy conservation, estimate whether the flies will break the web. Assume that the flies move perpendicular to the plane of the web. Spiral threads (shown dashed in the figure) can be ignored.
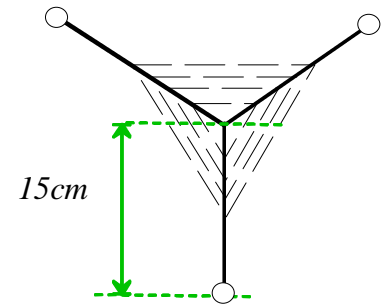
2.3 If the web remains intact, estimate the deflection of the web and the maximum force in the threads. Approximate the force-deflection curve as a straight-line for this calculation.

15cm

3. Reconsider the ski-jump problem analyzed in HW 3. During flight, three forces act on the skier: lift, weight, and drag.
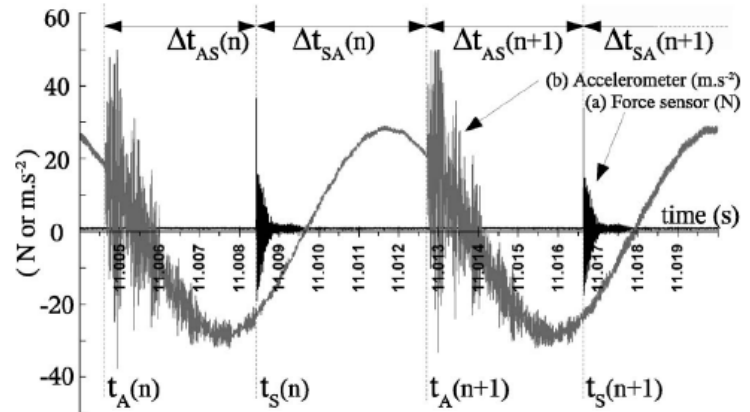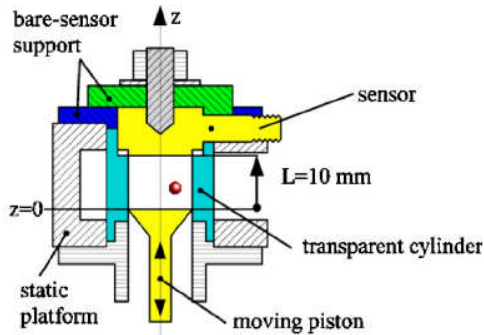
3.1 For each force, state whether the force is conservative, non-conservative, or workless. Explain your reasoning briefly (one or two sentences for each).

3.2 Write down an expression for the rate of work done by each force acting on the skier, as functions of the angle of attack α and the velocity components (or, if appropriate, the velocity magnitude).

3.3 Write down the energy conservation equation relating the skier's kinetic energy to the work done by the forces acting on the skier.

3.4 Write down *two* expressions for the work done on the skier by air resistance (one in terms of a time integral of the force and the skier's velocity, the other in terms of the kinetic and potential energy of the skier at the start and end of the jump).

3.5 Use the MATLAB code you wrote for the ski-jump problem in HW3 (or see the solutions if you weren't able to get the code to work) to calculate the total energy dissipated by air resistance. Use both approaches in step 4, and use parameter values given in Problem 1.5 or HW3. Note that you can use the MATLAB 'Trapz' command to do the time integral. Why are the two answers different? What would you need to do to make them closer together? You do not need to submit a full MATLAB code with your solution – just report the lines of code you added to your solution to HW3 to solve this problem.

Figure labels: bare-sensor support, sensor, L=10 mm, z=0, static platform, transparent cylinder, moving piston

(b) Accelerometer (m.s⁻²): $\Delta t_{AS}(n)$, $\Delta t_{SA}(n)$, $\Delta t_{AS}(n+1)$, $\Delta t_{SA}(n+1)$
(a) Force sensor (N)

$t_A(n)$, $t_S(n)$, $t_A(n+1)$, $t_S(n+1)$

(N or m.s⁻²) vs time (s)

4. The background to this problem can be found in Leconte *et al* Applied Physics Letters, **89**, 243518 (2006). Both figures shown above are from this source. This paper describes an experimental method for measuring restitution coefficient of mm sized particles. The apparatus confines a particle of interest between two flat platens. The upper platen is stationary, while the lower vibrates vertically at a fixed frequency. An accelerometer is mounted to the vibrating platen, and a force sensor is mounted on the fixed platen. The ball bounces between the two surfaces, and the sensors detect the times at which impacts occur. The entire experiment is conducted in zero-g (using an aircraft in parabolic flight). A representative experimental result is shown in the figure above. In the test shown, the ball had a radius of 1mm, and mass 32.7×10^{-6} kg
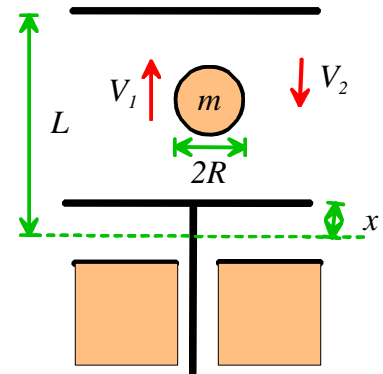
4.1 Note that the lower platen vibrates with simple harmonic motion. Use the experimental data provided in the figure to calculate the period and frequency of the vibration, the amplitude of the velocity V_0 and the amplitude of the displacement X_0 of the platen.

4.2 Use the experimental data provided to estimate the value of x at the time that the particle impacts the lower platen. Also, determine the velocity of the platen at the instant of impact.

4.3 Use the results of 4.2, together with the experimental data, to determine the velocities V_1, V_2 of the particle moving between the platens.

4.4 Find the restitution coefficient of the ball with both the upper and lower platens.

5. The figure (from Kilcast "Solid Food") shows the variation of force with time when tomatoes are dropped onto a flat, stationary surface. Suppose that the variation of impact force with time can be approximated by the following function

$$F(t) = F_0 \left(\frac{t}{t_0} \right) \exp\left(-\left(\frac{t}{t_0} \right)^3 \right)$$

where t_0, F_0 are two numbers that can be adjusted to give the best fit to the experimental data.

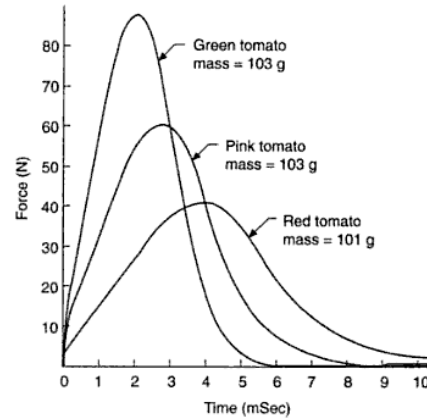5.1 Use Mathematica to plot $F(t)$, for $t_0 = 1, F_0 = 1$.

Fig. 5.12 Impact force response curves for three tomatoes of the same mass dropping 10 cm onto a rigid plate (Lichtensteiger *et al.*, 1988).

5.2 Find a formula for the time at which $F(t)$ is a maximum, in terms of t_0, and determine the corresponding maximum force, in terms of F_0.

5.3 Find a formula for the impulse exerted by the force, in terms of t_0, F_0 (use Mathematica to do the integral – you can use Assuming[assumption,expression] to remove the annoying conditional in the result. Check the Mathematica help for more information the Assuming[] command).

5.4 Determine values for t_0, F_0 that will approximate the three sets of experimental data.

5.5 Hence, estimate the value of the restitution coefficient for red, pink and green tomatoes.

6. The figure shows an apparatus to measure the impulse exerted by a sub-surface explosive device. It consists of a piston with mass m supported by a frame. The system is initially at rest. The explosion then propels the piston vertically, and its maximum height h is measured.   Derive an expression that relates the piston mass m and the height h to the impulse I exerted on the piston by the explosion.   Friction can be neglected.
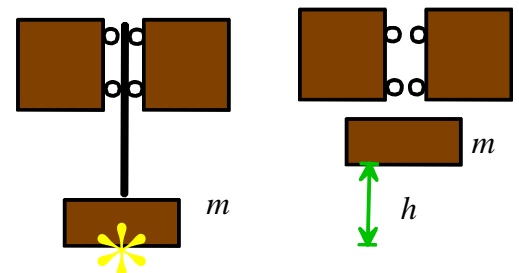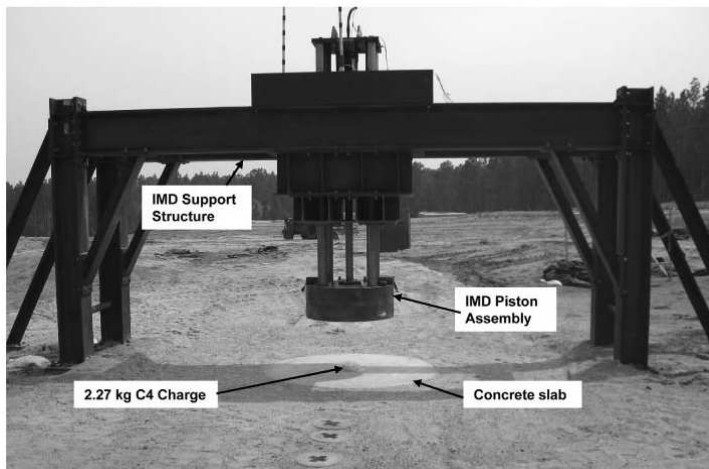
(Figure from Ehrgott, et al *Experimental Techniques,* doi: 10.1111/j.1747-1567.2009.00604.x)

7. The figure shows two masses which roll freely on the surfaces of a wedge. The two masses are tethered by a massless cable that passes over a frictionless pulley. The wedge rolls freely over a horizontal surface. At time *t=0* the system is at rest and mass *A* is a distance *d* from the top of the wedge. The goal of this problem is to calculate the time taken for the mass at *A* to reach the top of the ramp. (Why anyone would care about this is beyond me, but I spent many years solving problems like this as a student… Now I have a wonderful family, many cats, and a substantial job, so you might like to see if it works for you…)
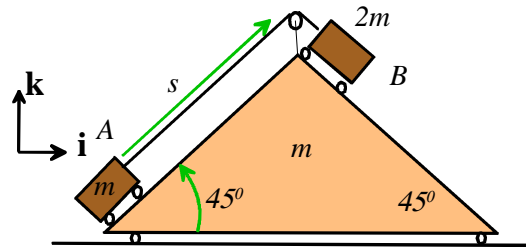

7.1 Suppose that the wedge has horizontal speed *v*. Write down the velocity vectors of each small mass in terms of *v* and the time derivative of the distance *s* of the lighter of the two masses from the top of the wedge

7.2 Hence, write down the total momentum, and energy of the system

7.3 Use momentum and energy conservation to obtain a differential equation for *s*. Your equation should have the form

$$\frac{d^2 s}{dt^2} = nasty \ \ formula$$

7.4 Solve the result of 7.3 to calculate the time. Then go off and start a family, get some kittens, etc.
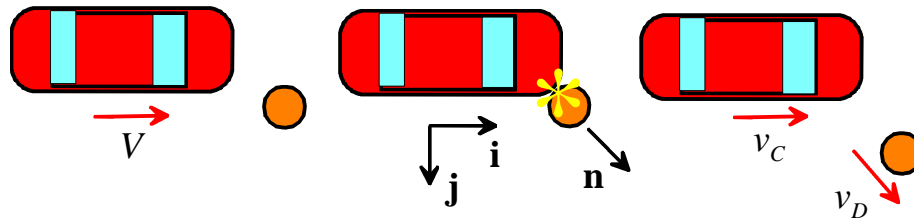
8. The figure shows a plan view of a vehicle with mass *M* and initial speed *V* colliding with one of the sand-filled drums in a 'Fitch Barrier'
http://www.google.com/patents?hl=en&lr=&vid=USPAT3880404&id=3YoyAAAAEBAJ&oi=fnd&dq=fitch+barrier&printsec=abstract#v=onepage&q&f=false. The drum has mass *m*.

The car rolls freely, so the combined momentum of the car and drum is conserved in the **i** direction (parallel to the vehicle's motion) during impact. Friction forces between the car's wheels and the road ensure that the car continues to move in the **i** direction after impact. Friction between the drum and road during the impact may be neglected. After the impact, the car has velocity vector $v_c \mathbf{i}$, while the drum has velocity vector $v_D \mathbf{n}$, where **n** is the normal to the plane of contact.

8.1 Assume that the impact occurs with the normal vector to the contact plane at 45 degrees, and is frictionless. Write down the unit vector **n** as components in the **{i,j}** basis. Hence, determine the

components of velocity in a direction parallel to **n** of the car and drum before and after impact, in terms of V, v_D and v_c

8.2 Assume that the collision has restitution coefficient e. Write down the restitution coefficient formula relating V, v_D and v_c

8.3 Use momentum conservation and 8.2 to determine a formula for the speed of the car v_c after collision, in terms of V, e, and the masses of the car and drum.

8.4 Suppose that the car hits a series of drums with mass $m=M/5$ and $e=1/2$. How many drums are required to reduce the car's speed by a factor of 10?