



ENGN0040: Dynamics and Vibrations

Homework 6: Driven vibrations, multiple degrees of freedom

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1. What is the steady-state motion of a forced oscillator with damping if the driving force is of the form

$$F = F_0 \sin \omega t$$

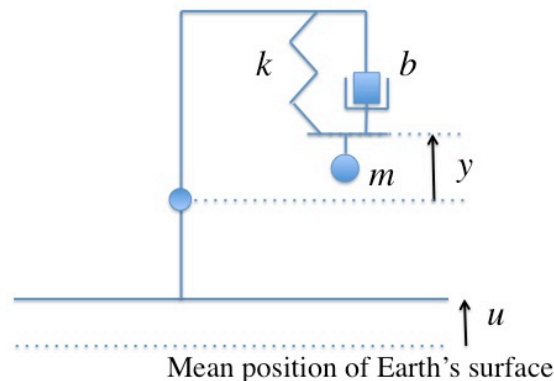
instead of

$$F = F_0 \cos \omega t?$$

2. Consider a simple seismograph consisting of a mass m hung from a spring on a rigid framework attached to the earth, as shown below. The spring force and the damping force depend on the displacement and velocity relative to the earth's surface, but the significant acceleration is the acceleration of the mass relative to the fixed stars.

2.1 Use y to denote the displacement of the mass relative to a fixed point on the supporting arm, which coincides with the static position of the mass when the earth is not shaking. Use u to denote the displacement of the earth's surface relative to its mean position. Show that the equation of motion is

$$\frac{d^2 y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y = -\frac{d^2 u}{dt^2}$$

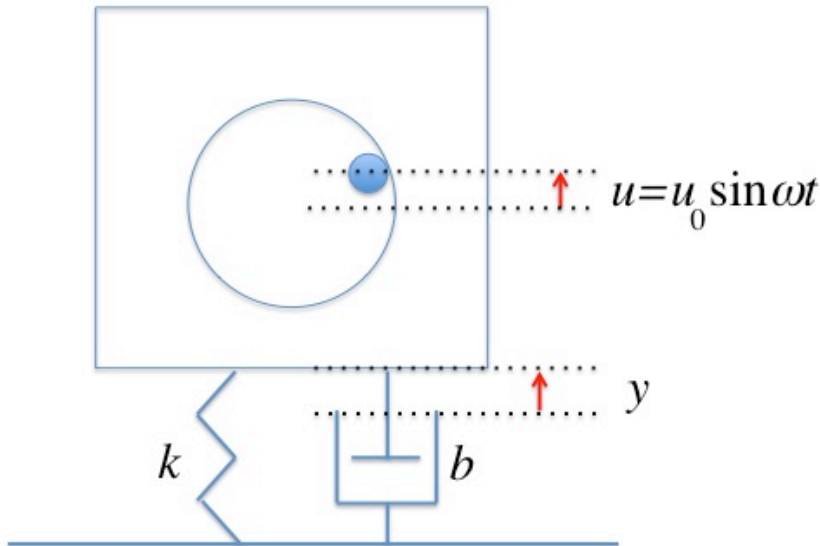


2.2 Solve for y in steady-state vibration if $u = u_0 \cos \omega t$.

2.3 Plot the amplitude A of the displacement y as a function of ω , assuming u_0 is independent of frequency. Suppose $\zeta = 1/4$, and use appropriate dimensionless variables.

2.4 A typical long-period seismometer has a period of about 30 s and a Q of about 2. Suppose that during an earthquake, the surface oscillates with a frequency of about 10 Hz and with an amplitude such that the maximum acceleration is about 10^{-2} m/s^2 , which is the lower limit of acceleration perceptible to people. What value of A will be recorded in the seismograph? (Your answer will be a small number---this type of seismometer is more effective at low frequencies, and modern seismometers use electronic sensors.)

3. The clothes in a front-loading washing machine are all wadded up in a ball. In the spin cycle, the ball rotates about the center of the drum with constant rate ω . The radius of the orbit of the ball about the center of the drum is u_0 (see figure below). Suppose the washing machine sits on a support with effective spring constant k and effective damping constant b .



3.1 Draw free-body diagrams for the drum-ball system and the rest of the washing machine. Disregard gravity.

3.2 What are the forces acting on each system?

3.3 Write the equation of motion for y . Suppose the mass of the ball is m_1 and the mass of the washing machine is m_2 .

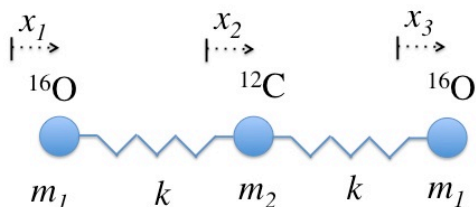
4. An object of mass 2 kg hangs from a spring of negligible mass. The spring is extended by 2.5 cm when the object is attached. The top end of the spring is oscillated up and down in simple harmonic motion with an amplitude of 1 mm. The Q of the system is 15.

4.1 What is the natural frequency for this system?

4.2 What is the amplitude of forced oscillation at the natural frequency?

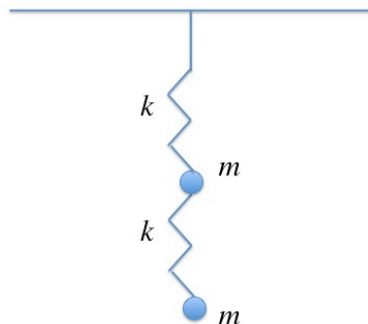
4.3 What is the mean power input required to maintain the forced oscillation at a frequency 2% greater than the natural frequency?

5. Our atmosphere is full of many different gases---nitrogen, oxygen, carbon dioxide, and water vapor. Water vapor and carbon dioxide are greenhouse gases, whereas nitrogen and oxygen are not. In this problem we will see why.



- 5.1 Model the CO₂ molecule as a system made up of a central mass m_2 connected by equal springs of spring constant k to two masses m_1 and m_3 , with $m_3=m_1$. Supposing the masses only move horizontally (see the figure above), write the equations of motion for the three coordinates x_1 , x_2 , and x_3 .
- 5.2 We will suppose the center of mass of the molecule is initially at rest. Using conservation of momentum, relate x_2 to x_1 and x_3 .
- 5.3 Since the result of 5.2 gives the motion of the carbon atom in terms of the motion of the oxygen atoms, we will no longer consider the equation of motion for x_2 . Eliminate x_2 and write two coupled differential equations for x_1 and x_3 .
- 5.4 Take the sum and difference of the two equations to get equations for the two normal mode coordinates $q_1=x_1-x_3$ and $q_2=x_1+x_3$. Remember that in a normal mode, all parts of the system vibrate at the same frequency. What are the normal mode frequencies?
- 5.5 Sketch the normal modes. Note that in first normal mode $q_2=0$, and in the second normal mode, $q_1=0$.
- 5.6 Write the solutions for x_1 and x_3 , assuming that at $t=0$, the system is at rest, $x_1=X_0$ and $x_3=0$.
- 5.7 Visible light from the sun mostly passes through the atmosphere. What isn't reflected by clouds or by the earth heats the earth, and the warm earth re-radiates the energy back into atmosphere in the form of infrared radiation. Greenhouse gases absorb in the infrared. Infrared radiation, like all electromagnetic radiation, is a propagating electromagnetic field. A molecule at a given point sees electromagnetic radiation as an oscillating electric field. Thus, a CO₂ molecule in the atmosphere sees an oscillating electric field $E=E_0\cos\omega t$. From chemistry, you know that the electrons in a molecule aren't necessarily equally shared among the atoms. In O₂ or N₂ they are (why?), but not in CO₂. Using the fact that the electric force on a particle with charge q is qE , can you explain why infrared radiation can excite CO₂ or H₂O molecules, but not N₂ or O₂?

6. Two equal masses are connected as shown below with two identical massless springs of spring constant k .



6.1 Considering only motion in the vertical direction, show that the frequencies of the two normal modes are given by

$$\omega^2 = (3 \pm \sqrt{5})k / (2m).$$

6.2 Find the ratio of the amplitudes of the motions of the masses in each normal mode.

