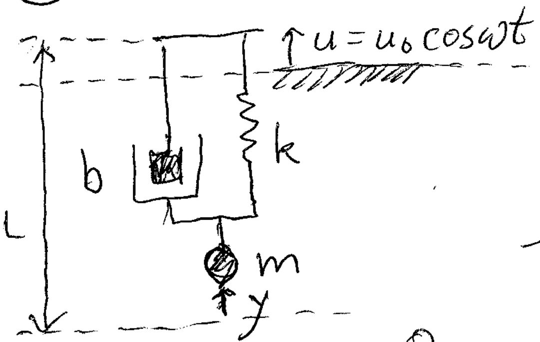
(4.12) mistake on original soln: $A(\omega_n) = \frac{u_0\omega_n^2}{2\zeta\omega_n^2} = Q u_0 = (15)(1\,mm) = \boxed{15\,mm}$

(4) Better explanation for 4.3



← origin in an inertial frame

The acceleration of the mass in the inertial frame is $\ddot{u} + \ddot{y}$.

Thus $-ky - b\dot{y} = m(\ddot{u} + \ddot{y})$

Or $\boxed{\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = -\ddot{u} = \omega^2 u_0 \cos\omega t}$

Mean power input required to maintain the oscillation is $\langle P\rangle = \langle \underbrace{(-ky - b\dot{y})}_{\text{force required to shake top of spring/damper combination}} \underbrace{\dot{u}}_{\text{velocity of top}} \rangle = m\langle \ddot{u}\dot{u}\rangle + m\langle \ddot{y}\dot{u}\rangle$

But $u = u_0\cos\omega t$
$\Rightarrow \langle \ddot{u}\dot{u}\rangle = u_0\omega^3\langle\cos\omega t\sin\omega t\rangle = 0$

so $\langle P\rangle = m\langle\ddot{y}\dot{u}\rangle$. The steady-state solution to the boxed equation above is $y = A\cos(\omega t + \phi)$, with ϕ as in lecture and A as in lecture with $\frac{F_0}{m}$ replaced by $u_0\omega^2$:

$A = \frac{u_0\omega^2}{[(\omega^2-\omega_n^2)^2 + 4\zeta^2\omega^2\omega_n^2]^{1/2}}$. Thus $\langle P\rangle = m\omega^3 A u_0\langle\cos(\omega t+\phi)\sin\omega t\rangle$

$= -m\omega^3 A u_0\langle\sin^2\omega t\rangle\sin\phi$

$= -\frac{u_0}{2}m\omega^3 A\sin\phi$

From lecture, $\sin\phi = \frac{-2\zeta\omega\omega_n}{[(\omega^2-\omega_n^2)^2 + 4\zeta^2\omega^2\omega_n^2]^{1/2}}$ recall $Q = \frac{1}{2\zeta}$

$\Rightarrow \langle P\rangle = \frac{m}{2}\frac{u_0^2\omega^5\cdot 2\zeta\omega\omega_n}{(\omega^2-\omega_n^2)^2 + 4\zeta^2\omega^2\omega_n^2} = \boxed{\frac{m}{2Q}u_0^2\omega^4\frac{\omega_n}{(\omega - \frac{\omega_n^2}{\omega})^2 + \omega_n^2/Q^2}}$

which is the expression in the original solution