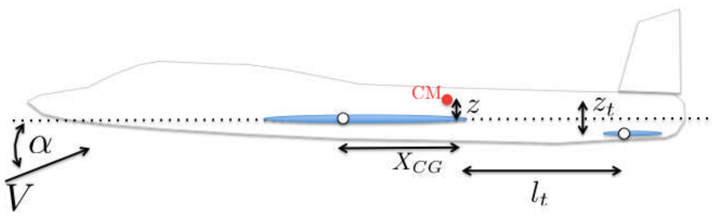
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 8: Rigid Body Dynamics

1. Stability of an Aircraft

Below is a two-dimensional schematic of an airplane wing and horizontal tail, and the relevant dimensions with respect to the center of mass of the entire plane. The relative wind vector is depicted by V. The lift force on the wing, L_w, is perpendicular to the wind velocity vector, and the drag on the wing, D_w, is parallel to V. The lift and drag resultant vectors act at the aerodynamic center of the wing, as depicted in the figure by a white circle. There is also a moment, $M_{ac,w}$, on the wing section due to the aerodynamic forces. Likewise, the aerodynamic forces and moment acting on the tail are L_t, D_t, and $M_{ac,t}$



a) Draw a FBD of the aerodynamic forces and moments acting on the wing section.

b) Find an expression for the moment of the wing about the center of mass of the entire plane. Simplify your expression using the small angle approximation.

c) Draw a FBD of the aerodynamic forces and moments acting on the tail section.

d) Find an expression for the moment of the tail about the center of mass. Note that the total moment about the center of mass, due to the aerodynamic forces, is $M_{CG} = M_t + M_w$.

e) What is the purpose of the tail?

f) Static stability is defined as whether the plane can "right itself" after a small disturbance, or a small change in α (for example, due to a wind gust). What criterion must be met for the plane to be statically stable?

g) Look at the two airplane configurations below. The left figure has a canard, and the right has a traditional tail. What design difference must the canard plane have for it to be statically stable?

airliners.net canard

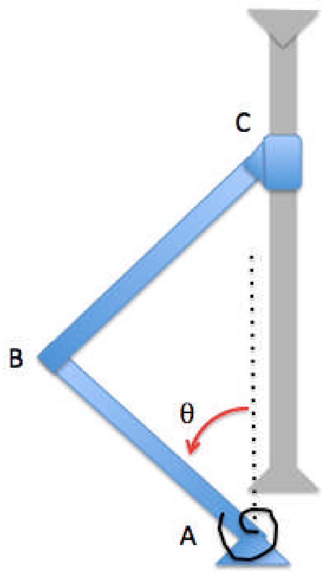
2. Bars AB and BC each have a mass of *m* and a length *l*. The moment of inertia about the center of mass is $1/12ml^2$. The collar has a mass m_c and slides with negligible friction. A torsional spring at A exerts a clockwise couple, $k\theta$, on bar AB. The system is released from rest at $\theta=0$.

a) Draw a FBD

b) Find an energy expression between $\theta=0$ and another arbitrary value of θ.

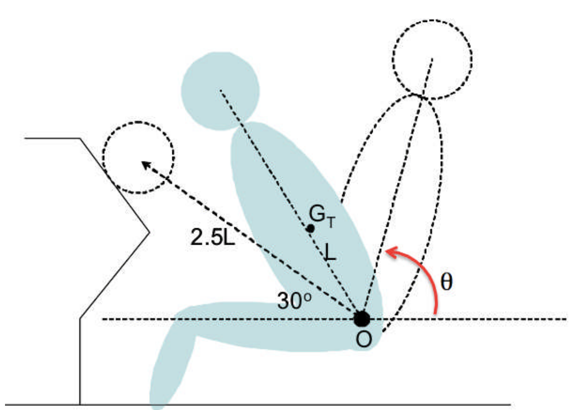
c) Use the kinematics equations to find ω_{BC}, v_c, and v_{CG} of bar BC.

d) Find an expression for ω as a function of θ.



3. Analysis of Car Crash

A 2500 kg car crashes into a fixed barrier wall at speed v_c=5 m/s and is observed to rebound with a velocity of v_f=2 m/s. The total impact time is 0.5s. The passenger's torso and head is treated as a single rigid body, rotating about the point O as shown in the diagram below, where L is the distance from O to the center of mass. $I_G=1.2mL^2$ for the passenger.



a) What is the average deceleration of the car?

b) Draw a FBD for the passenger.

c) Write the 3 appropriate equations of motion.

c) What is the position and acceleration of point G in terms of point O?

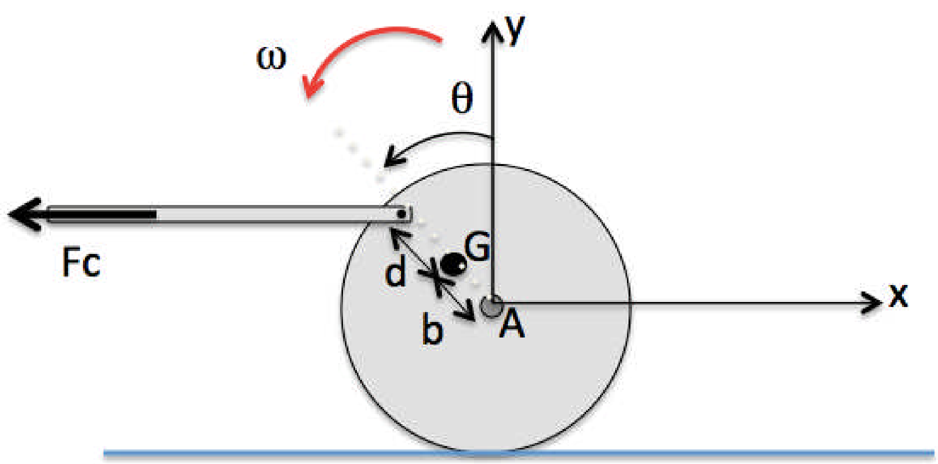
d) Write a system of equations for θ(t), xo(t), yo(t), and the 2 reaction forces at point O.

e) Write a system of equations such that Matlab can solve the problem.

f) Solve the system in matlab and find the velocity at which the head strikes the dashboard (relative to the moving car) using L=0.3m, m=70kg, and θ =75degrees.

4. Unbalanced Wheel

The drive wheel below is pulled by a rigid bar along a horizontal surface. The wheel is connected to an axis that exerts a downward force $\mathbf{F_A}$ at the point A. The horizontal force of the connecting rod is $\mathbf{F_c}$. The wheel has a radius *R*, a mass *m*, and the moment of inertia about its center of mass is *I*. However, the center of mass is offset from the geometric center by a distance *b*.



a) Draw a FBD for the wheel

b) Write out the 3 equations of motion in terms of the forces above. The linear acceleration of the center of mass is $\mathbf{a_G}=a_x\mathbf{i}+a_y\mathbf{j}$, and the angular acceleration is $\boldsymbol{\alpha}=\alpha\mathbf{k}$.

c) Find 2 kinematic equations by relating $\mathbf{a_G}$ to $\mathbf{a_A}$.

d) Formulate a single equation of motion in terms of θ and its derivatives (Mathematica is ok here).

e) Set b=0, (move the center of mass to the geometric center), and $I=mR^2/2$ (solid cylinder), and write an expression for the linear acceleration vector.

f) Beginning from rest, what is the minimum constant force $\mathbf{F_c}$ required to roll the wheel (without slipping) up a small step of height *H* located a distance *s* away?