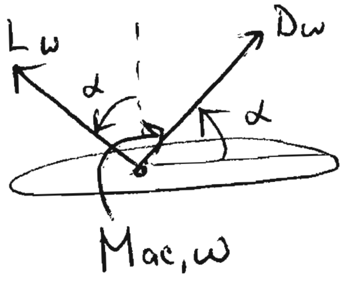
Solutions HW#8

1a)



NOTE COORD. SYSTEM
+M IS CLOCKWISE

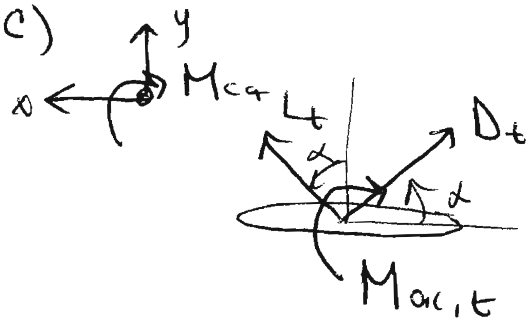
b) Moments about CM.:

$+M_{ac,w}$
$+L_w \cos\alpha \cdot X_{cG}$
$+L_w \sin\alpha \; Z$
$+D_w \sin\alpha \; X_{cG}$
$-D_w \cos\alpha \; Z$

with $\cos\alpha \sim 1$ & $\sin\alpha \sim \alpha$

$$M_w = M_{ac,w} + L_w X_{cG} + L_w \alpha Z + D_w \alpha X_{cG} - D_w Z$$

c)



d) Moments are:

$+M_{ac,t}$
$-L_t \cos\alpha \cdot l_t$
$+L_t \sin\alpha \cdot Z_t$
$-D_t \sin\alpha \; l_t$
$-D_t \cos\alpha \; Z_t$

$$M_t = M_{ac,t} - L_t l_t + L_t \alpha Z_t - D_t \alpha l_t - D_t Z_t$$

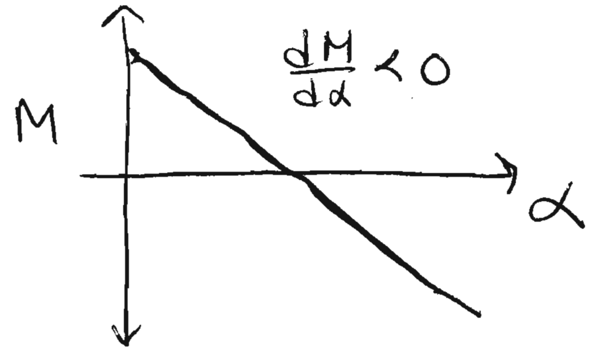
e) $M_{CG} = M_w + M_t$

At cruise, $\Sigma M_{CG} = 0$ (equilibrium)

Main wing will have a $+M_w$, "pitch up" moment
Tail is designed with a "pitch-down" moment

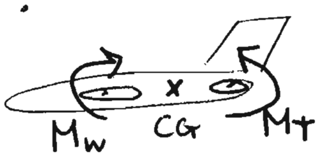
$\boxed{\text{Tail is a longitudinal stabilizer}}$

f)



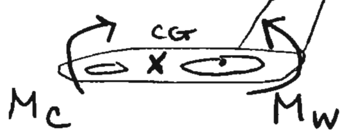
As $\alpha \uparrow$, plane pitches up (due to an increase in L_w)
Need a $-M$ to then stabilize it...
Thus, $\boxed{\frac{dM_{CG}}{d\alpha} < 0}$

g) For the plane in the figure, both wing and tail has a positive angle of attack. Assuming positive Lifting forces on both (true for symmetric airfoils):



The CG must be between the A.C. (aerodynamic center) of the main wing and tail.

Assuming a canard with $+\alpha$ & $+$Lift:



The CG is in front of the main wing.

However, most planes have negative Lift on the tail! This would result in:

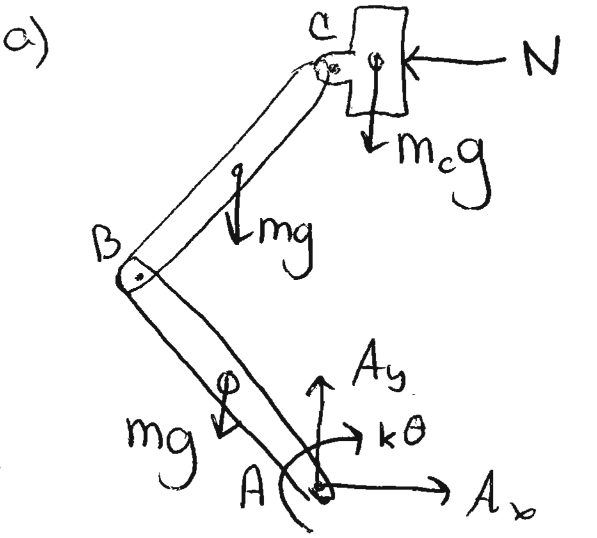
The CG is in front of the A.C. of the main wing.

This is ideal because when the plane stalls, there is no lift on the main wing. Thus $M_w \approx 0$. But the CG will still pitch the nose down. In the top drawing of our hypothetical plane, the CG would pitch the nose up, which is undesirable in stall conditions.

[either answer for tail configuration is correct]

2)

a)



b) $T_{AB} = \underbrace{\frac{1}{2} I_A \omega_{AB}^2}_{\text{rotating about A}} = \frac{1}{2}\left[I_{CG} + \left(\frac{1}{2}l\right)^2 m\right]\omega^2 = \frac{1}{6} m l^2 \omega_{AB}^2$

$T_{BC} = \underbrace{\frac{1}{2} m V_G^2}_{\text{falling}} + \underbrace{\frac{1}{2} I \omega_{BC}^2}_{\text{rotating about CG}} = \frac{1}{2} m V_G^2 + \frac{1}{24} m l^2 \omega_{BC}^2$

$T_c = \frac{1}{2} m_c V_c^2 \quad \Rightarrow \quad T_f = \frac{1}{6} m l^2 \omega_{AB}^2 + \frac{1}{2} m V_G^2 + \frac{1}{24} m l^2 \omega_{BC}^2$

$$\left.\begin{aligned} V_{AB} &= mg \frac{l}{2} \cos\theta \\ V_{BC} &= mg \frac{3}{2} l \cos\theta \\ V_c &= m_c g\, 2l \cos\theta \\ V_{spring} &= \frac{1}{2} k \theta^2 \end{aligned}\right\} \Rightarrow V_f = 2mgl\cos\theta + 2m_c g l \cos\theta + \frac{1}{2} k\theta^2$$

$V_i = m_c g \cdot 2l + \frac{l}{2} mg + \frac{3l}{2} mg$

$\underbrace{\cancel{T_i}}_{0 \text{ (at rest)}} + V_i = T_f + V_f$

initial $\Rightarrow \theta = 0°$

final $\Rightarrow \theta$

$$2mgl + 2m_c g l = \frac{1}{6} m l^2 \omega_{AB}^2 + \frac{1}{2} m V_G^2 + \frac{1}{24} m l^2 \omega_{BC}^2 + \frac{1}{2} m_c V_c^2 + 2mgl\cos\theta + 2m_c g l \cos\theta + \frac{1}{2} k\theta^2$$

c)

$$\underline{V}_B = \cancel{\underline{V}_A}^{\,0} + (\underline{\omega}_{AB} \times \underline{r}_{B/A})$$

$$\underline{V}_B = -l\omega_{AB}\cos\theta\,\hat{\imath} - l\omega\sin\theta\,\hat{\jmath}$$

$$\underline{V}_c = \underline{V}_B + (\underline{\omega}_{Bc} \times \underline{r}_{c/B})$$

only V_{cy}, $V_{cx} = 0$: $\hat{\imath}\,(0 = -l\omega_{AB}\cos\theta - l\cos\theta\,\omega_{BC})$

$\hat{\jmath}\,(V_{cy} = -l\omega_{AB}\sin\theta + l\omega_{BC}\sin\theta)$

$$\begin{cases} \omega_{BC} = -\omega_{AB} & \text{(could determine by inspection)} \\ V_{cy} = -2l\omega\sin\theta \end{cases}$$

$$\underline{V}_{cG} = \underline{V}_B + (\underline{\omega}_{Bc} \times \underline{r}_{G/B})$$

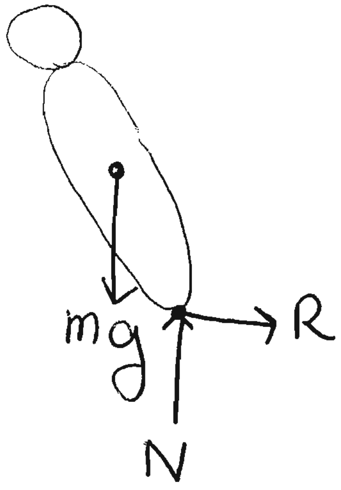
$$\boxed{\underline{V}_{cG} = -\tfrac{1}{2}l\omega\cos\theta\,\hat{\imath} - \tfrac{3}{2}l\omega\sin\theta\,\hat{\jmath}}$$

d) $\omega_{AB} \Rightarrow$ (plug ω_{BC}, V_{cy} & V_{cG} into energy)

$$\omega_{AB} = \left[\frac{2gl(m+m_c)(1-\cos\theta) - \frac{1}{2}k\theta^2}{\frac{1}{3}ml^2 + (m+2m_c)l^2\sin^2\theta}\right]^{1/2}$$

3a) $a_{car} = \dfrac{(2 - (-.5))}{.5} = 14 \, m/s^2$

b)



c) $\sum F_x = R = m a_x$

$\sum F_y = N - mg = m a_y$

$\sum M = I \alpha$

$(-LN\cos\theta + LR\sin\theta)\hat{k} = (I_G \alpha)\hat{k}$

c) $x_G = x_o + L\cos\theta$ & $y_G = y_o + L\sin\theta$

$\dot{x}_G = \dot{x}_o + L\dot{\theta}\cdot(-\sin\theta)$ & $\dot{y}_G = \dot{y}_o + L\dot{\theta}\cos\theta$

$\ddot{x}_G = \ddot{x}_o + L\dot{\theta}^2(-\cos\theta) + L\ddot{\theta}(-\sin\theta) = a_x$

$\ddot{y}_G = \ddot{y}_o + L\dot{\theta}^2(-\sin\theta) + L\ddot{\theta}\cos\theta = a_y$

d) $R = m\ddot{x}_G \Longrightarrow$ substitute $\ddot{x}_G$ from above!

$N - mg = m\ddot{y}_G$

$$\left\{\begin{array}{l} -LN\cos\theta + LR\sin\theta = I_G\ddot{\theta} \\ \ddot{x}_o = a_{car} \\ \ddot{y}_o = 0 \\ R = m(\ddot{x}_o + L\dot{\theta}^2(-\cos\theta) + L\ddot{\theta}(-\sin\theta)) \\ N - mg = m(L\dot{\theta}^2(-\sin\theta) + L\ddot{\theta}\cos\theta) \end{array}\right.$$

5 unknowns: θ, x_o, y_o, R, N

e) To solve, set up a matrix eqn:

$$MX = f$$

$$M = \begin{bmatrix} 1 & 0 & L\sin\theta \\ 0 & 1 & -L\cos\theta \\ L\sin\theta & -L\cos\theta & -I_g/m \end{bmatrix}$$

$$X = \begin{bmatrix} R/m \\ N/m \\ \dot{\omega} \end{bmatrix} \qquad f = \begin{bmatrix} \dot{V}_{ox} - L\dot{\theta}^2\cos\theta \\ g + \dot{V}_{oy} - L\dot{\theta}^2\sin\theta \\ 0 \end{bmatrix}$$

Only 1st derivatives for matlab, so $\ddot{\theta} \Rightarrow \dot{\omega}$

$\ddot{x}_o \Rightarrow \dot{V}_{ox}$

$\ddot{y}_o \Rightarrow \dot{V}_{oy}$

For ODE45 we have 6 variables:

$$W = \begin{bmatrix} x_o \\ y_o \\ \theta \\ V_{ox} \\ V_{oy} \\ \omega \end{bmatrix} \qquad \frac{dw}{dt} = \begin{bmatrix} V_{ox} \\ V_{oy} \\ \omega \\ a_{ox} \\ a_{oy} \\ \dot{\omega} \end{bmatrix}$$

where $a_{ox} = \begin{cases} -5 & t \leq .5\,s \\ 0 & t > .5\,s \end{cases}$

$a_{oy} = 0$

$\dot{\omega}$ is found from solving matrix equation above: $X = M \backslash f$

$\dot{\omega} = X(3)$

$$W_i = \begin{bmatrix} 0 \\ 0 \\ 5\pi/12 \\ -5 \\ 0 \\ 0 \end{bmatrix}$$

f) $V_{head} = 5.89\ m/s$

Alternative Solution Method for e) & f):

$$R = m(\ddot{x}_0 - L(\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta)) \qquad (1)$$

$$N = m[g + L(-\dot{\theta}^2\sin\theta + \ddot{\theta}\cos\theta)] \qquad (2)$$

Substitute (1) & (2) into $-LN\cos\theta + LR\sin\theta = I_G\ddot{\theta}$

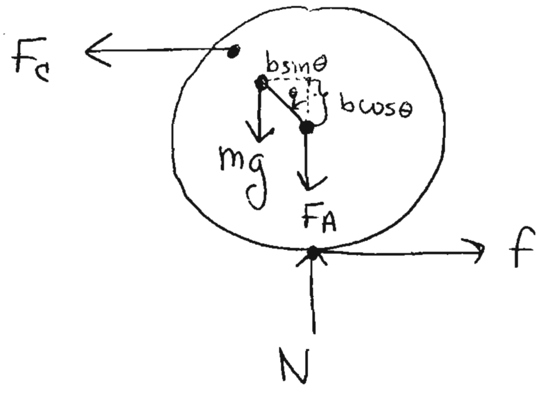
$$\boxed{\begin{aligned}&-Lm\cos\theta[g + L(-\dot{\theta}^2\sin\theta + \ddot{\theta}\cos\theta)] + \\ &Lm\sin\theta[\ddot{x}_0 - L(\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta)] = I_G\ddot{\theta}\end{aligned}}$$

Solve 2nd Order (system of 2 1st Order) ODE in matlab, for $\theta(t)$

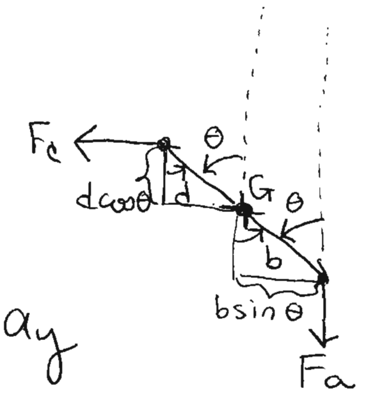
Solutions HW 8

problem 4: unbalanced wheel

a)



b)



$$f - F_c = m a_x$$

$$N - mg - F_A = m a_y$$

$$F_c \cdot d\cos\theta - F_A b\sin\theta + N b\sin\theta + f(b\cos\theta + R) = I\alpha$$

c) Find a kinematic relationship:

$$\underline{a}_G = \underline{a}_A + (\underline{\alpha} \times \underline{r}_{G/A}) + (\underline{\omega} \times (\underline{\omega} \times \underline{r}_{G/A}))$$

$$\underline{r}_{G/A} = (-b\sin\theta)\hat{\imath} + (b\cos\theta)\hat{\jmath}$$

$$\underline{\alpha} = \alpha\hat{k}$$

$$\underline{\omega} = \omega\hat{k}$$

$$\underline{a}_A = -R\alpha\hat{\imath}$$

Evaluating cross products:

$$\underline{\omega} \times \underline{r}_{G/A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & \omega \\ -b\sin\theta & b\cos\theta & 0 \end{vmatrix} = -\omega b\cos\theta\,\hat{\imath} - \omega b\sin\theta\,\hat{\jmath}$$

$\underline{\omega} \times (-\omega b\cos\theta\,\hat{i} - \omega b\sin\theta\,\hat{j})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ -\omega b\cos\theta & -\omega b\sin\theta & 0 \end{vmatrix} = +\omega^2 b\sin\theta\,\hat{i} - \omega^2 b\cos\theta\,\hat{j}$$

$$\underline{\alpha} \times \underline{r}_{G/A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \alpha \\ -b\sin\theta & b\cos\theta & 0 \end{vmatrix} = -b\alpha\cos\theta\,\hat{i} - b\alpha\sin\theta\,\hat{j}$$

Putting it together, relating $\hat{i}$ components only:

$$(a_x = -R\alpha - b\alpha\cos\theta + \omega^2 b\sin\theta)\,\hat{i}$$

$\hat{j}$ components:

$$(a_y = 0 - b\alpha\sin\theta - \omega^2 b\cos\theta)\,\hat{j}$$

d) $I\alpha = F_c d\cos\theta - F_A b\sin\theta + N b\sin\theta + f(b\cos\theta + R)$

but $f = m a_x + F_c$

$a_x = -R\alpha - b\alpha\cos\theta + \omega^2 b\sin\theta$

$N = m a_y + F_A + mg$

$a_y = -b\alpha\sin\theta - \omega^2 b\cos\theta$

5 eqns, 5 unknowns $(\omega = \dot{\theta},\ \alpha = \ddot{\theta})$

θ, f, N, a_x, a_y

↳ see mathematica

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In[12]:= f = m * ax + Fc

Out[12]= Fc + m (-alpha R - alpha b cos[theta] + b omega^2 sin[theta])

In[13]:= ax = -R * alpha - b * alpha * cos[theta] + omega^2 * b * sin[theta]

Out[13]= -alpha R - alpha b cos[theta] + b omega^2 sin[theta]

In[14]:= FullSimplify[f]

Out[14]= Fc - alpha m R - alpha b m cos[theta] + b m omega^2 sin[theta]

In[15]:= Nf = m * ay + FA + m * g

Out[15]= FA + g m + m (-b omega^2 cos[theta] - alpha b sin[theta])

In[17]:= ay = -b * alpha * sin[theta] - omega^2 * b * cos[theta]

Out[17]= -b omega^2 cos[theta] - alpha b sin[theta]

In[19]:= FullSimplify[Nf]

Out[19]= FA + g m - b m (omega^2 cos[theta] + alpha sin[theta])

         FA + g m - b m omega^2 cos[theta] + alpha b m sin[theta]

In[20]:= Solve[IG * alpha ==
           Fc * d * cos[theta] - FA * b * sin[theta] + Nf * b * sin[theta] + f * (b * cos[theta] + R), alpha]

Out[20]= {{alpha -> (Fc R + b Fc cos[theta] + d Fc cos[theta] + b g m sin[theta] + b m omega^2 R sin[theta]) /
            (IG + m R^2 + 2 b m R cos[theta] + b^2 m cos[theta]^2 + b^2 m sin[theta]^2)}}

In[21]:= FullSimplify[%]

Out[21]= {{alpha -> (Fc R + (b + d) Fc cos[theta] + b m (g + omega^2 R) sin[theta]) /
            (IG + m R^2 + b m (2 R cos[theta] + b cos[theta]^2 + b sin[theta]^2))}}
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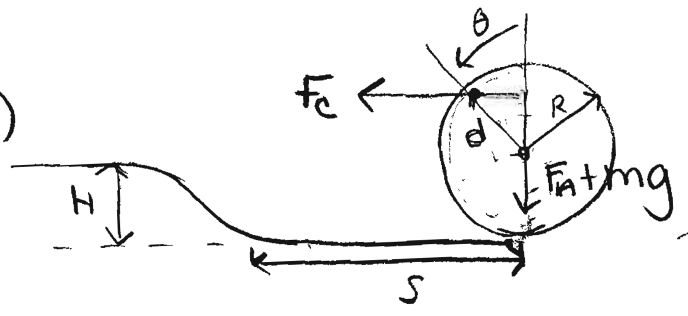
e) $\alpha = \dfrac{F_c (R + d\cos\theta)}{3/2\, m R^2}$

$$a_x = -R\alpha = -R\frac{F_c (R + d\cos\theta)}{3/2\, m R^2} = \frac{-2}{3}\frac{F_c (R + d\cos\theta)}{mR}$$

f)



$\mathcal{U}$ = total work done

$\mathcal{U} = T_f - T_i = 0$ assuming wheel comes to rest at top of step.

Work done by gravity = $-mgH$

work done by F_A = $-F_A H$

work done by force F_c = $F_c(s + d\sin\theta)$

$$s = R\theta \quad \text{OR} \quad \theta = s/R$$

$$F_c\,(s + d\sin(s/R)) = (mg + F_A)\,H$$

$$F_c = \frac{(mg + F_A)\,H}{(s + d\sin(s/R))}$$