



EN40: Dynamics and Vibrations

Homework 1: Introduction to Mupad and solving differential equation with MATLAB Due 12:00 noon Friday February 1st

School of Engineering
Brown University

- Your solution to this homework should consist of two files:
 1. A commented MUPAD .mw file
 2. A commented MATLAB .m file
- The assignment should be submitted electronically to Stephanie_gesualdi@brown.edu
- Please put en4_hw1 in the subject of your email
- PLEASE NAME THE ATTACHED FILES *lastname_firstname.mn* and *lastname_firstname.m*

Part 1: Use Mupad to solve the following math problems. Be sure to save your work frequently!

1. Calculate $100!/50!$.
2. Determine the first 400 digits of pi (you will have to read the Mupad help documentation to find how to do this – you can find it in Mupad>Getting Started>Perform Computations> Compute with numbers)
3. Solve the equation $x^3 + x^2 + x + 1 = 0$ for x
4. Plot the function $y = x^4 - 300x^2 - 500x - 1$ in the range $-20 < x < 20$.
5. Find the values of x for which $y = x^4 - 300x^2 - 500x - 1$ is either a local maximum or a local minimum (use Mupad to differentiate the function and solve for the points where the derivative is zero. Mupad will return the answer in the RootOf() format – you can convert the solution into numbers by putting float(%) on the next line). Determine the values of y corresponding to each solution.
6. Evaluate the following indefinite integrals

$$\int \sqrt{\frac{x+k}{x}} dx \quad \int \sqrt{x + \frac{k}{x}} dx$$

(The solution to the second one is given in terms of a ‘Hypergeometric Function’ – Mupad Help will tell you what this is, if you are curious)

7. The differential equation

$$\frac{dN}{dt} = \frac{rN(K - N)}{K}$$

is called the ‘Logistic equation’ or the ‘Verhulst’ model of population growth in mathematical biology. N is the population of a species; r is the reproduction rate and K is the maximum population.

7.1 Use Mupad to solve the differential equation with initial condition $N = N_0$ at time $t=0$. (Remember to tell Mupad that N is a function of time by entering it as $N(t)$, and use the IgnoreSpecialCases option and the simplify() command to make the solution as simple as possible). Ignore the funny conditional statement – it has no effect.

7.2 Plot the solution for $K = 10000, N_0 = 1800, r = 0.02$ for $0 < t < 200$ (these parameters roughly describe the growth of the global population, in millions of people, since year 1900).

Part 2: Please solve the remaining problems using MATLAB (write your code in a matlab .m file). You should make your MATLAB (.m) file a function, so that when the file is executed, it will solve all the homework problems. For example:

```
function give_me_an_A
    Solutions to problems 8-11
    Functions for the differential equations in probs 10 and 11
end
function number_of_vars = count_variables(vector,value)
...
end
```

If you get stuck on the MATLAB problems, you might find the solutions to homework 1, 2009-2012 helpful (if you haven't taken EN3 you will may it helpful to do some of the easier MATLAB problems on old homeworks as MATLAB practice). If you find this homework too easy, you could make it more interesting by looking for more sophisticated solutions to the problems that make use MATLAB's ability to manipulate vectors. Or you could try a social life – I am told that it can be quite pleasant.

8. Create a vector containing 100 random integers that lie between 1 and 10 (use MATLAB's randi function – there is an example in the help documentation. We note in passing that only a computer scientist could seriously consider calling a function 'randi...').

9. Write a function that counts the number of variables in a vector that are equal to a prescribed value. Your function could look something like this:

```
function number_of_vars = count_variables(vector,value)
% Function to count the number of entries in vector with
% value exceeding threshold
    enter your calculation here (Hint: use the function length(vector)
to compute the number of elements in v,
and use a loop and a conditional statement to count
the values of v ). Advanced programmers might like to explore MATLAB
for a more elegant solution...
end
```

and should appear at the end of your homework file. Use your function to

- (i) count the number of occurrences of 5 in the random integer you created in problem 10.
- (ii) Count the number of the first 17 digits of pi that equal 1. You can use the following method to create a vector containing the first 17 digits of pi

```
for i=1:17
    pivec(i) = floor(pi*10^(i-1))-10*floor(pi*10^(i-2));
end
```

If you are up for a challenge, write a code that will count the frequency of each of the digits 0-9 in the first n digits of pi, and plot the graph for some suitably large value of n . You can find a function to compute an arbitrary number of digits of pi in the solutions to last year's HW1.

10. Solow's economic growth model (which was one of the contributions that led to his [Nobel prize](#)) is a famous economic model that predicts the influence of the rate of investment on the long-term standard of living. 'Standard of living' is quantified by the ratio $r=K/L$ of the total 'capital' K (the time integral of 'income') to the labor force L . The standard of living evolves according to the equation

$$\frac{dr}{dt} = sr^{1/3} - \lambda r$$

Here, s and λ are constants characterizing the savings rate and the rate of growth of the labor force, respectively. The variable r is a function of time t , which must be determined using MATLAB.

10.1 Write a MATLAB function that will compute the value of dr/dt given the value of r . You will need to follow the procedure outlined in section 13.2 of the Matlab tutorial. Check your function by using it to plot a graph of dr/dt as a function of r , with $s=0.05$ and $\lambda=0.2$ (both variables have units of 1/years), and $0 < r < 0.2$. You should use a loop that evaluates your function and stores the values of dr/dt corresponding to a range of values of r in a vector.

10.2 Hence, add lines to your Matlab script that will compute r using the ODE solver `ode45`, and then plot a graph showing the variation of r with time. Use $s=0.05 \text{ years}^{-1}$ and $\lambda=0.2 \text{ years}^{-1}$, and plot solutions that start with $r=0.01$ and $r=0.3$ at $t=0$, for a period of 20 years.

11. A Chemostat is a device for harvesting bacteria. It consists of a culture chamber, which is supplied with nutrients through an inflow pipe; while bacteria are harvested from the chamber through an outflow pipe (you can find a detailed description and discussion of its applications [here](#)). When designing a chemostat, it is important to control both the concentration of nutrients as well as the concentration of bacteria. The following mathematical model describes how the nutrient concentration N and bacteria concentration B change with time

$$\begin{aligned} \frac{dB}{dt} &= \alpha \left(\frac{N}{1+N} \right) B - B \\ \frac{dN}{dt} &= - \left(\frac{N}{1+N} \right) B - N + \beta \end{aligned}$$

where α, β are two constants, while N and B are both functions of time, and are to be determined by MATLAB. As always, we can only calculate N and B if we know their values N_0, B_0 at time $t=0$.

11.1 Write a MATLAB function which will compute the values of dN/dt and dB/dt given values of N and B . The function will need to follow the procedure outlined in Section 13.5 of the Matlab tutorial – store the variables N, B and their time derivatives as vectors.

11.2 Hence, add lines to your Matlab script to compute N and B using the ODE solver `ode45`, and then plot a graph showing the variation of N and B with time. Plot the solution for initial conditions: $B_0=0.1, N_0=0.25$ at time $t=0$, and try $\alpha=2.7, \beta=0.5$ (in this case the nutrient supply is insufficient to sustain the bacteria); and $\alpha=2.7, \beta=0.8$ (now the system is stable with $B > 0$). Plot both solutions for a time period of 20 (the units are all arbitrary).