School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 4: Work, Energy and Linear Momentum
Due Friday March 1st

1. The 'Buckingham Potential' is used to approximate the forces acting between atoms in a diatomic molecule. The potential energy of the force of interaction between the atoms is expressed as

$$V(d) = A\exp(-Bd) - \frac{C}{d^6}$$

The table (from J. Bicerano "Computational Modeling of Polymers" Marcel Dekker, 1992) gives values for *A*,*B* and *C* for various bonds.

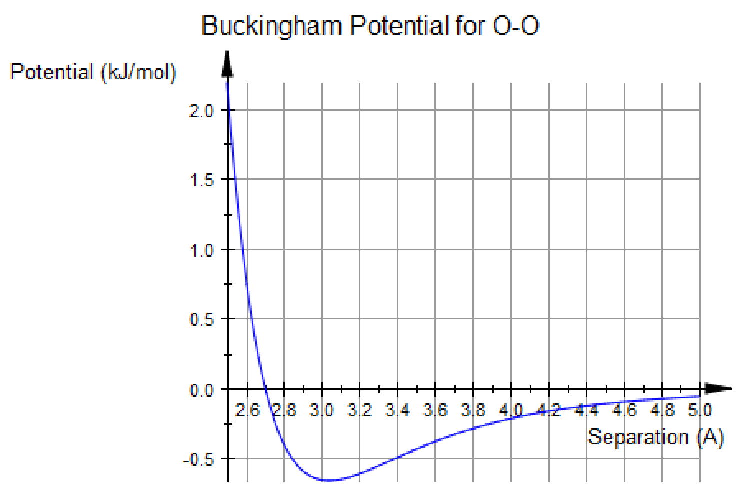
Table 3 Selected Values[a] of the Buckingham Potential Parameters *A*, *B*, and *C*.

Interaction	$10^{-3}A^b$	B^b	C^b
C···C[c]	541.4	4.59	363.0
C···C[d]	1820	4.59	556.7
N···N	393.2	4.59	547.3
O···O	135.8	4.59	217.2
S···S	906.3	3.90	3688
H···H	7.323	4.54	47.1

[a]Taken from Ref. 48.
[b]Units are such as to give energy in kcal mol^{-1} for *r* in Å.
[c]Aliphatic carbon atoms.
[d]Aromatic carbon atoms.

1.1 Plot a graph of the energy as a function of *d* for the O-O bond. Use kJoules /mol for the units of energy, and the separation *d* in Angstroms, with 2.5<d<5. Note that 1 kcal/mol is 4.2 kJ/mol. Note also that the value of *A* in the table needs to be multiplied by 10^3

The plot is shown below.



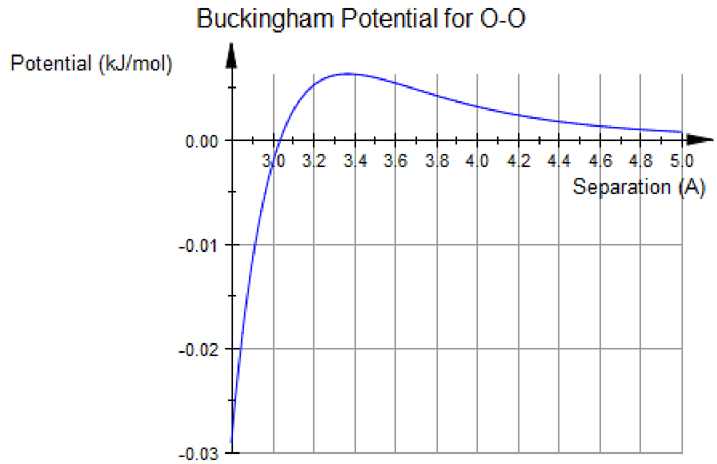
[2 POINTS]

1.2 Find an expression for the magnitude of the force acting between two atoms, and plot the force as a function of *d*, for the same range. Use eV/A (electron-volts per Angstrom – a common unit for atomic forces) for the force unit. Google will tell you the conversion factors from kJ to eV, and recall that one mol is 6.02×10^{23} molecules.

The force follows as

$$F(d) = -\frac{dV}{dd} = -AB\exp(-Bd) + \frac{6C}{d^7}$$

The raw numbers give the force in $kJ / A / mol$ - to convert to N we need to multiply by 1000 (to get Joules/A/mol), divide by 1.60 10^{-19} (to get eV/A/mol) and divide by Avogadro's number 6.02x10^{23} to get eV/A.



[3 POINTS]

1.3 Hence, calculate values for the following quantities for the O-O bond:

(i) The equilibrium bond length (the length of the bond when the bond force is zero)

We must just solve for $F=0$ using Mupad:

```
eqsep := solve(F=0,r,Real)[2]
```

3.039843951

Hence the equilibrium bond length is $r = 3.04A$

[1 POINT]

(ii) The bond strength (the force required to break the bond)

The separation corresponding to the maximum force can be computed by differentiating F with respect do d, setting the result to zero, and solving for d. Substituting this back into the expression for F then gives the maximum force.

```
Fderiv := diff(F,r)
```

$$124599.2501\, e^{-4.59\, r} - \frac{397.2824667}{r^8}$$

```
maxFsep := float(solve(Fderiv=0,r,Real)[2])
```

3.369644534

```
Fmax := float(subs(F,r=maxFsep))
```

0.006298374943

The maximum force is therefore $F_{\max} = 0.0063 eV / A$

[2 POINTS]

(iii) The binding energy (the total energy required to pull the atoms apart from their equilibrium spacing to infinity – you can express this in kJ/mol)

The work of separation is the difference between the minimum energy and the energy when the bond length is stretched to infinity. The energy at infinity is zero by inspection, so

```
Vmin := float(subs(V,r=eqsep))
-0.6589645167
```

The work of separation is thus $0.658 kJ / mol$

[1 POINT]

(iv) The stiffness of the bond (i.e. the slope of the force-separation relation at the equilibrium spacing. You can express your answer in eV/A^2

```
stif := float(subs(Fderiv,r=eqsep))
0.05411977801
```

The stiffness is thus 0.054 eV/A^2. In *N/m* this is $1.602 \times 10^{-19} \times 10^{10} \times 10^{10} \times 0.054 = 0.864$ N/m.

[1 POINT]

2. Japan's N700I Shinkansen 'Bullet train' has the following specifications:

- Max power output: 9760 kW
- Max weight 365 Tonnes (metric)
- Cruising speed 330 km/h

Assume that the air resistance can be approximated as

$$F_D = \frac{1}{2} \rho C_D A v^2$$

with drag coefficient $C_D = 0.1$, air density $\rho = 1.2 kg m^{-3}$ and projected frontal area $A = 10 m^2$

2.1 Estimate the power consumption of the train at cruise speed on level grade.

We idealize the train (upstream of its propulsion system) as a particle that is subjected to the air drag force; gravity; and vertical reaction forces. Since the train has no vertical velocity, the rate of work done by gravity and the vertical reaction forces is zero. The air drag force acts in the opposite direction to the velocity and so does work

$$W = -F_d v = \frac{1}{2} \rho C_D A v^3 = -\frac{1}{2} \times 1.2 \times 0.1 \times 10 \times (330000/3600)^3 = -462 kW$$

Since the train moves at constant speed, its kinetic energy is constant, and so no net work is done by the forces acting on the train. The forces exerted by the propulsion system must do work at rate of 462kW (just where these forces act is a bit mysterious – it depends which part of the propulsion system you regard as part of the particle representing the train, and which part you regard as external to the particle).

[2 POINTS]

2.2 Use the energy-power relation to calculate a formula for the acceleration of the train as a function of its speed *v*. Hence, calculate the shortest possible time that the train can take to accelerate to cruise speed.

The energy equation now gives $\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = P - \frac{1}{2}\rho C_D A v^3$. Hence

$$mv\frac{dv}{dt} = P - \frac{1}{2}\rho C_D A v^3$$

$$\Rightarrow \int_0^{v_{\max}} \frac{mvdv}{P - \frac{1}{2}\rho C_D A v^3} = t$$

$$= \int_0^{v_{\max}} \frac{365\times 10^3 vdv}{9760 - 0.6v^3} = 160\sec$$

Note that air drag has very little effect on the time – if you set the drag to zero it takes 157 sec to reach full speed.

[3 POINTS]

2.3 Estimate the total energy expended in overcoming air resistance in a 100km trip (you can assume constant speed). Compare the energy expended with the kinetic energy of the train at maximum speed.

A 100km trip takes 100/330 hours=1091 sec. The total work done is 462x1091=512MJ

The total KE of the train is $\frac{1}{2}mv^2 = \frac{1}{2}\times 365\times 10^3 \times \left(330000/3600\right)^2 = 1533MJ$. The KE is very significant – if all this energy were lost to heat during braking, it would drastically reduce the efficiency of the train. For this reason the Shinkansen has a fully regenerative braking system – most of the KE is recovered during deceleration.

[2 POINTS]

3. The standards for motorcycle and bicycle helmets are set by the Snell memorial foundation. Their latest motorcycle helmet standards can be found here. Among several other criteria, they specify that the helmets must be tested by fitting them to a head-form with mass 3.1kg, and the head-form/helmet assembly must then be dropped onto a flat anvil from a height that leads to a 7.5m/s impact velocity (see the figure for a representative experimental apparatus). The peak acceleration of the head-form during the impact must be less than 275*g* (where *g* is the gravitational acceleration)

3.1 Use energy methods to calculate the required drop-height.

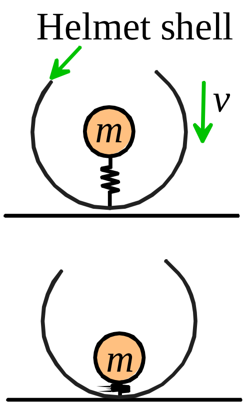
Energy conservation gives

$$V_0 + T_0 = V_1 + T_1$$

$$mgh + 0 = 0 + \frac{1}{2}mv^2 \Rightarrow h = \frac{v^2}{2g} = \frac{7.5^2}{2\times 9.81} = 2.87$$

[2 POINTS]

3.2 The protective effect of the helmet in this impact test comes primarily from its padding. Idealize the helmet padding as a spring with stiffness *k*. Use energy methods to find a formula relating the maximum compression of the spring δ to the impact velocity v_0, the helmet mass *m* and the gravitational acceleration *g* (assume that the outer shell of the helmet does not rebound from the anvil). Does increasing the foam stiffness *k* increase or decrease δ?



Again, energy conservation, but this time between the instant just before impact and the time at which the foam is crushed to its maximum depth (where the headform has zero velocity) gives

$$V_0 + T_0 = V_1 + T_1$$

$$0 + \frac{1}{2}mv^2 = \frac{1}{2}k\delta^2 - mg\delta + 0$$

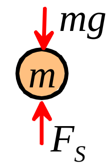
$$\Rightarrow \delta = \frac{mg}{k}\left(1 + \sqrt{1 + \frac{kv^2}{mg^2}}\right)$$

(Graders – note that solutions may be given in a different algebraic form)

Increasing the stiffness reduces δ.

[3 POINTS]

3.3 Find a formula for the maximum acceleration of the head-form in terms of the padding stiffness k, the head form mass m and the gravitational acceleration g.

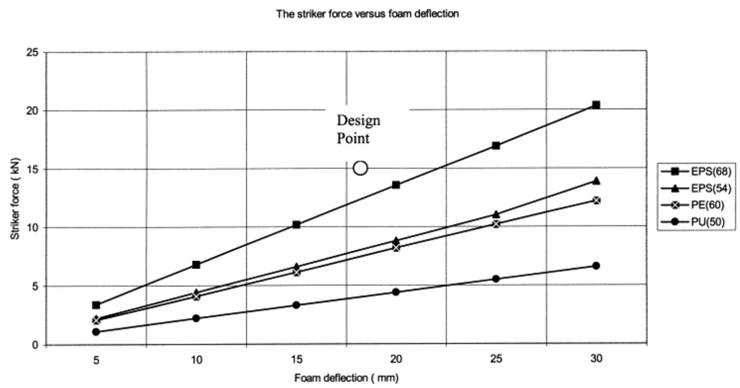


A free body diagram for the head-form is shown in the figure. Newton's law gives

$$F_s - mg = ma \Rightarrow k\delta - mg = ma_{\max} \Rightarrow a_{\max} = \frac{k}{m}\delta - g$$

[2 POINTS]

3.4 Force-deflection curves for several candidate helmet pad foams (from Shuaeib *et al*, JMEP **123,** 422, 2002) are shown in the figure (ignore the 'Design Point'). Select the foam that will minimize the padding thickness (which must exceed δ) while still meeting the Snell standard, and find the foam thickness required and the maximum predicted acceleration of the head-form.



Eliminating δ from the solutions to the two preceding problems, we can calculate the maximum allowable stiffness

$$\delta = \frac{mg}{k}\left(1+\sqrt{1+\frac{kv^2}{mg^2}}\right) \qquad a_{\max} = \frac{k}{m}\delta - g$$

$$\Rightarrow k \le \frac{m}{v^2}\left[a_{\max}^2 - g^2\right] = \frac{3.1\times 9.81^2}{7.5^2}\left[275^2 - 1\right] = 0.4MN/m$$

We can estimate the stiffnesses of the foams shown in the figure (the slope) – starting with the one with the largest slope:

- EPS(68) *k=0.66 MN/m*
- EPS(54) and EPS(60) are about the same at *k=0.45MN/m* and *k=0.4MN/m* – the latter might be just borderline acceptable, but does not leave any safety factor so should be rejected.
- PU(50) is 0.25 MN/m is safe.

The required thickness is

$$\delta = \frac{mg}{k}\left(1+\sqrt{1+\frac{kv^2}{mg^2}}\right) = \frac{3.1\times 9.81}{0.66\times 10^6}\left(1+\sqrt{1+\frac{0.25\times 10^6\times 7.5^2}{3.1\times 9.81^2}}\right) = 10mm$$

A 1.5 cm thickness of foam would be a good choice, to leave a bit of a factor of safety.

The maximum acceleration is

$$a_{\max} = \frac{k}{m}\delta - g = \frac{0.25\times 10^6}{3.1}\times 10\times 10^{-3} - 9.81 = 796m/s^2 = 81g$$

Again, there is a good safety factor.

(Graders - you can accept sols with EPS60) [4 POINTS]

4. One way to measure the restitution coefficient e of an object colliding with a flat surface is to measure the total time T taken for the object to stop bouncing (of course, this only works if the object's shape permits it to bounce at all, and if the restitution coefficient is large enough for the object to bounce a fairly large number of times). In this problem you will calculate the relationship between T and e.

4.1 Suppose that the object falls from a height h above the flat surface. Use energy conservation to calculate its velocity just before impact.

Energy conservation gives

$$PE_1 + KE_1 = PE_2 + KE_2$$
$$mgh + 0 = 0 + \frac{1}{2}mv_0^2$$
$$\Rightarrow v_0 = \sqrt{2gh}$$

[2 POINTS]

4.2 Use the restitution coefficient to determine the velocity just after impact.

Let v_1 denote the upwards speed after the impact, and note that the ground remains stationary both before and after impact. The restitution formula gives

$$\frac{v_{B1} - v_{A1}}{v_{A1} - v_{B1}} = e$$
$$\frac{v_1}{v_0} = e \Rightarrow v_1 = ev_0$$

[2 POINTS]

4.3 Use impulse-momentum to find the time between the first and second bounce, in terms of v_1 and g.

This can be done various ways: we could calculate the time between the bounce and the top of the trajectory after the bounce (where the velocity is zero) – the momentum at the start and end are known, and so we can calculate the impulse using the impulse-momentum formula as

$$-mgt\mathbf{j} = 0 - mv_1\mathbf{j} \Rightarrow t = v_1 / g$$

It must take the same amount of time for the particle to drop back to the ground again so $\Delta t = 2v_1 / g$.

Alternatively we could note that because of energy conservation the magnitude of the velocity just before the second bounce must equal to that just after the first one, but the direction has changed. Applying momentum conservation then gives

$$-mg\Delta t\mathbf{j} = -mv_1\mathbf{j} - mv_1\mathbf{j} \Rightarrow \Delta t = 2v_1 / g$$

[2 POINTS]

4.4 Assume that the first impact occurs at $t=0$. Show that the remaining (infinite) number of bounces occur during a total time interval

$$T = \sqrt{\frac{8h}{g}} \sum_{n=1}^{\infty} e^n$$

We can add up all the successive times

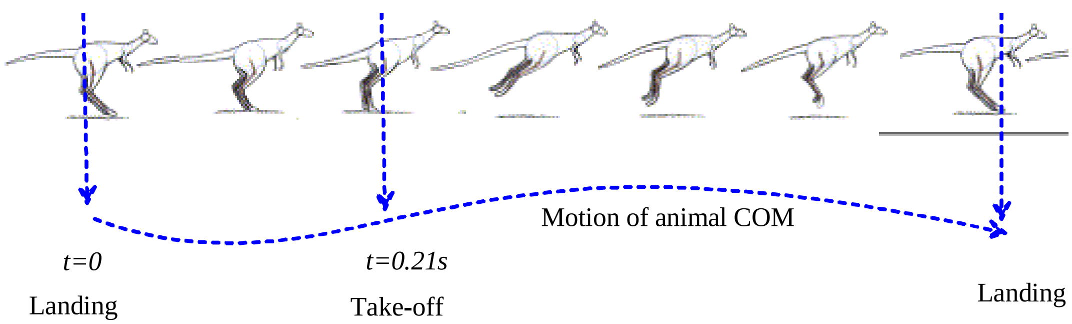
$$\begin{aligned} T &= 2\frac{v_1}{g} + 2\frac{v_2}{g} + 2\frac{v_3}{g} + \ldots \\ &= 2e\frac{v_0}{g} + 2e\frac{v_1}{g} + 2e\frac{v_2}{g} + \ldots \\ &= 2e\frac{v_0}{g} + 2e^2\frac{v_0}{g} + 2e^2\frac{v_1}{g} + \ldots \\ &= 2e\frac{v_0}{g} + 2e^2\frac{v_0}{g} + 2e^3\frac{v_0}{g} + \ldots. = 2\frac{v_0}{g}\sum_{n=1}^{\infty} e^n = \sqrt{8\frac{h}{g}}\sum_{n=1}^{\infty} e^n \end{aligned}$$

[2 POINTS]

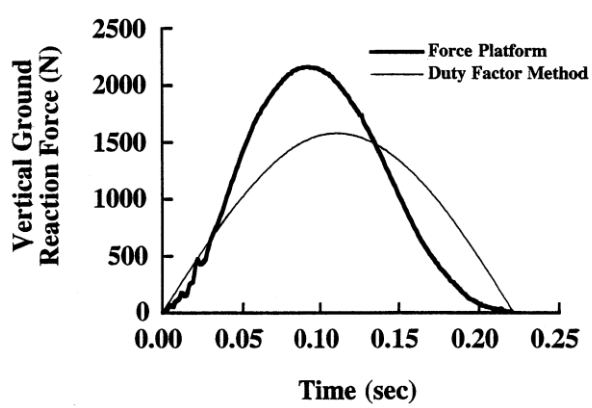
4.5 Sum the series (mupad will do it for you) and hence find an expression for *e* in terms of *T*.

Mupad gives $\sum_{n=1}^{\infty} e^n = \frac{1}{1-e} - 1 = \frac{e}{1-e}$ so $e = \left(1 + \frac{1}{T}\sqrt{\frac{g}{8h}}\right)$

[2 POINTS]



5. The figure (from Kram *et al* Comparative Biochemistry and Physiology, **B**120 41-49, 1998) shows the variation of the contact force with time between the foot and ground of a red kangaroo as it hops. The kangaroo has a mass of 46kg and hops at a steady horizontal speed of 3.9 ms^{-1}



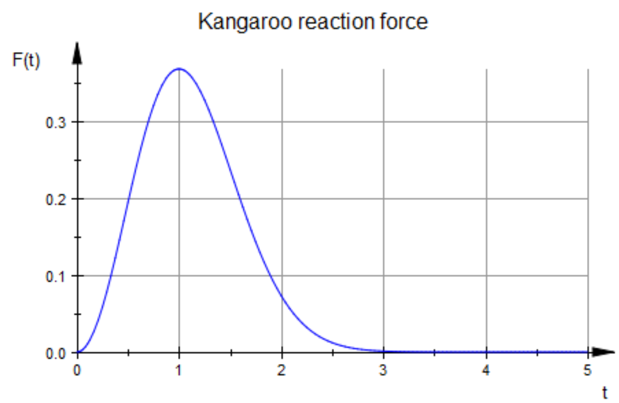
Suppose that the variation of contact force with time can be approximated using the equation

$$F(t) = F_0 \left(\frac{t}{t_0}\right)^2 \exp\left(-\left(\frac{t}{t_0}\right)^2\right)$$

where t_0, F_0 are two numbers that can be adjusted to give the best fit to the experimental data.

5.1 Use Mupad to plot $F(t)$, for $t_0 = 1, F_0 = 1$.

The plot is shown below.



[2 POINTS]

5.2 Find a formula for the time at which $F(t)$ is a maximum, in terms of t_0, and determine the corresponding maximum force, in terms of F_0.

We can find the max in the usual way by differentiating *F* with respect to *t*, setting the result to zero, and solving. This can be done trivially by hand but here's a Mupad

```
solve(diff((t/t0)^2*exp(-(t/t0)^2),t)=0,t)
{0, t0, -t0}
```

The maximum force follows as $F_{max} = F_0 \exp(-1) = 0.3678 F_0$

[2 POINTS – Mupad not required, of course]

5.3 Find a formula for the impulse exerted by the force, in terms of t_0, F_0

By definition, the impulse is $I = \int_0^T F(t)dt$. In this case we can take the upper limit T to be infinite.

The integral can be done in Mupad, but if you do the integral blindly you get a nasty answer that looks like this .

```
simplify(int((t/t0)^2*exp(-(t/t0)^2),t=0..infinity))
```

$$\lim_{t\to\infty} \frac{-\frac{t\,t0^2 e^{-\frac{t^2}{t0^2}}}{2} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(t\sqrt{-\frac{1}{t0^2}}\,i\right) i}{4\left(-\frac{1}{t0^2}\right)^{3/2}}}{t0^2}$$

It is better to change variables and let $\tau = t/t_0$, in which case

```
impulse := t0*int(F0*(tau)^2*exp(-(tau)^2),tau=0..infinity)
```

$$\frac{F0\,\sqrt{\pi}\,t0}{4}$$

[3 POINTS]

5.4 Determine values for t_0, F_0 that will approximate the experimental data (ignore the curve labeled 'duty factor method – that's a rough theoretical prediction – see the paper for details).

The peak force occurs at about 0.95ms so $t_0 = 0.095s$. The peak force magnitude is about 2100N, so
$F_0 = 2100/(0.3678) = 5708N$.

[3 POINTS]

5.5 Hence, use impulse-momentum and energy conservation to estimate how high the kangaroo bounces during the jump (neglect air resistance, and assume that the kangaroo's horizontal speed is constant. Don't forget to include gravity in the impulse-momentum calculation)

We assume steady-state, so the downward velocity of the kangaroo just before impact is equal to its upward velocity just after impact. Impulse-Momentum gives

$$m\Delta v\mathbf{j} = \mathbf{I}$$

$$\Rightarrow 2mv_0 = \frac{\sqrt{\pi}}{4} F_0 t_0 - mg\Delta t = 240.3 - 46 \times 9.81 \times 0.21 Ns$$

$$\Rightarrow v_0 = \frac{240.3 - 46 \times 9.81 \times 0.21}{2 \times 46} = 1.58 m/s$$

Energy conservation during the time interval between just after take-off and the highest point of the jump gives

$$\frac{1}{2} m(v_x^2 + v_0^2) = \frac{1}{2} m v_x^2 + mgh$$

where v_x is the constant horizontal speed, and h is the jump height. Thus $h = v^2/2g = 0.128m$.

[Graders – numbers will vary a bit depending on the value selected for Δt - not important]

[4 POINTS]

5.6 Estimate the time that the kangaroo is airborne during one hop.

The trajectory is symmetric – it takes the same length of time to rise from take-off to the peak of the trajectory as it takes to fall from the peak to landing. Applying impulse-momentum from take-off to the peak of the trajectory, and noting the force acting on the kangaroo during flight is $-mg\mathbf{j}$, we get

$$m\Delta\mathbf{v} = m\left(v_x\mathbf{i} - (v_x\mathbf{i} + v_0\mathbf{j})\right) = -mg\Delta t\mathbf{j}$$
$$\Rightarrow \Delta t = v_0 / g = 1.58/9.81 = 0.161s$$

The total time is twice this value – 0.322 sec.

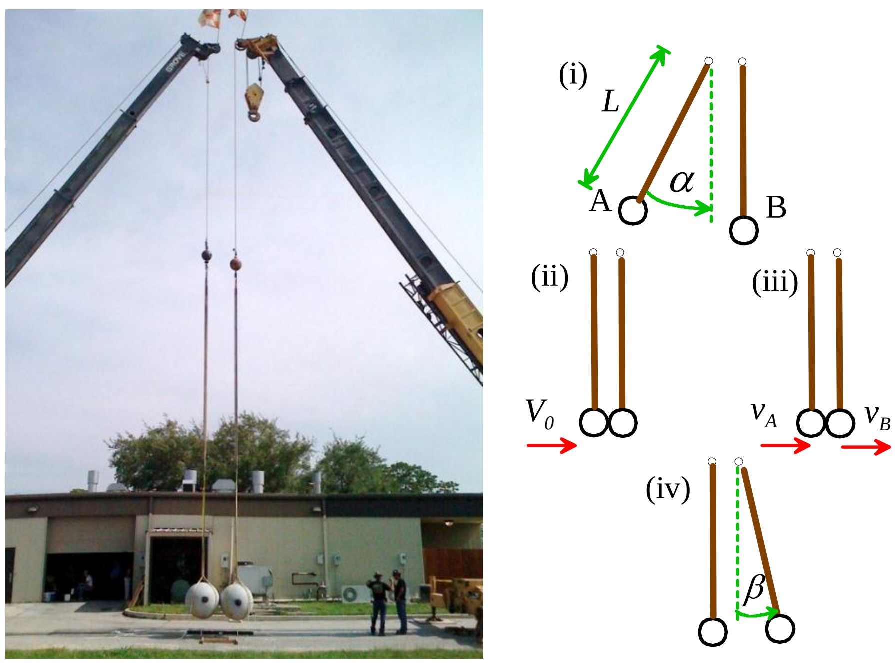
Of course, **F=**m**a** and the constant acceleration formula gives the same answer – impulse-momentum is just a different way of writing **F**=m**a.**

[3 POINTS]

5.7 Calculate the length of the jump (the distance traveled while airborne).

$$\Delta x = \Delta t v_x = 0.322 \times 3.9 = 1.26m$$

[2 POINTS]



6. The figure (from Durda *et* al, Icarus 211 849-855, 2011, see also the videos here) shows an experiment conducted by planetary geologists to determine the restitution coefficient between large masses of rock. Two large granite spheres with identical mass m are suspended from cranes to form large pendula. One pendulum is released from rest at an angle α to the vertical. It then collides with the second sphere, causing it to swing through an angle β before coming to rest. The goal of this problem is to find a formula relating α and β to the restitution coefficient.

6.1 Using energy methods, find a formula for the speed V_0 of sphere A just before impact, in terms of g and α.

Conservative system, so total energy is constant. This shows that
$$mV_0^2/2 = mgL(1-\cos\alpha)$$
$$\Rightarrow V_0 = \sqrt{2gL(1-\cos\alpha)}$$

[2 POINTS]

6.2 By considering the collisions, find a formula for the speed v_B of sphere B just after impact, in terms of V_0 and e.

Momentum is conserved during the impact, which yields $mV_0 = mv_A + mv_B$.
The restitution coefficient formula gives $eV_0 = v_B - v_A$
Dividing the first equation through by m and adding to the second gives $v_B = (1+e)V_0/2$

[2 POINTS]

6.3 Find a formula for the angle β in terms of v_B. Hence, determine the required relationship between $e,\ \alpha, \beta$.

This is 4.1 in reverse - $v_B = \sqrt{2gL(1-\cos\beta)}$. From 4.2 we see that $e = 2v_B/V_0 - 1 = 2\sqrt{\dfrac{1-\cos\beta}{1-\cos\alpha}} - 1$

[2 POINTS]

7. A recent article in Nature Physics reported observations of the collision between two 'Coronal Mass Ejections' (huge balls of magnetized plasma ejected from the sun). The authors suggest that the collision between the CMEs can be modeled using the classical theory of impact between two frictionless spheres.

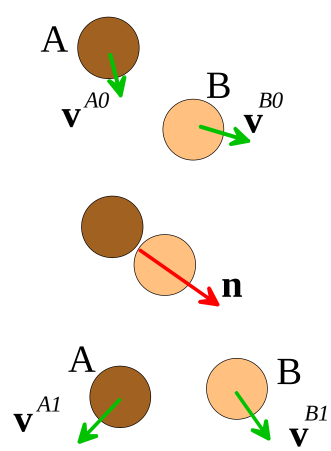


Table 1 of the article gives velocity components (in km/s) for the two colliding CMEs before (0) and after (1) collision as follows:

$$\mathbf{v}^{A0} = 237\mathbf{i} + 332\mathbf{j} \qquad \mathbf{v}^{B0} = 205\mathbf{i} + 130\mathbf{j}$$
$$\mathbf{v}^{A1} = 116\mathbf{i} + 332\mathbf{j} \qquad \mathbf{v}^{B1} = 288\mathbf{i} + 130\mathbf{j}$$

The direction of contact (**n**) is parallel to the **i** direction. The two masses are given as $m_B = 1.8\times10^{12}kg \qquad m_A = 1.2\times10^{12}kg$.

7.1Calculate the total linear momentum vector before and after the collision, and calculate the value of the restitution coefficient e.

The linear momentum before impact is
$$1.2\times10^{12}(237\mathbf{i} + 332\mathbf{j})\times10^3 + 1.8\times10^{12}(205\mathbf{i} + 130\mathbf{j})\times10^3$$
$$= (653.4\mathbf{i} + 632.4\mathbf{j})\times10^{15} kgm/s$$

After impact

$$1.2\times10^{12}\left(116\mathbf{i}+332\mathbf{j}\right)\times10^{3}+1.8\times10^{12}\left(288\mathbf{i}+130\mathbf{j}\right)\times10^{3}$$
$$=\left(657.6\mathbf{i}+632.4\mathbf{j}\right)\times10^{15}\,kgm/s$$

Momentum is conserved to within experimental error.

The formula for the restitution coefficient is

$$\frac{\left(\mathbf{v}^{B1}-\mathbf{v}^{A1}\right)\cdot\mathbf{n}}{\left(\mathbf{v}^{B0}-\mathbf{v}^{A0}\right)\cdot\mathbf{n}}=-e$$

$$e=-\frac{288-116}{207-237}=5.38$$

(Table 1 in the paper gives *e=5.4* so they evidently did the calculation correctly!)

[2 POINTS]

7.2 Suppose that the CMEs have the same initial velocities as those measured experimentally, but *e=1*. Calculate the predicted velocities after impact.

The component of velocity perpendicular to **n** is not affected by the value of *e*. The **j** component of velocity is therefore unchanged. The component parallel to **n** is determined by the restitution formula and momentum conservation. These give

$$\frac{v_x^{B1}-v_x^{A1}}{v_x^{B0}-v_x^{A0}}=-e \qquad m_A v_x^{A0}+m_B v_x^{B0}=m_A v_x^{A1}+m_B v_x^{B1}$$
$$\Rightarrow\frac{v_x^{B1}-v_x^{A1}}{207-237}=-1 \qquad 653.4=1.2v_x^{A1}+1.8v_x^{B1}$$
$$\Rightarrow v_x^{A1}=199km/s \qquad v_x^{B1}=229.8km/s$$

Thus

$$\mathbf{v}^{A0}=237\mathbf{i}+332\mathbf{j} \qquad \mathbf{v}^{B0}=205\mathbf{i}+130\mathbf{j}$$
$$\mathbf{v}^{A1}=199\mathbf{i}+332\mathbf{j} \qquad \mathbf{v}^{B1}=230\mathbf{i}+130\mathbf{j}$$

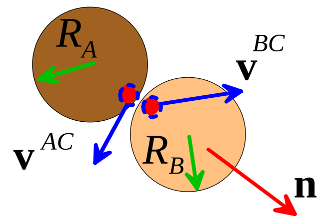
It's possible to do this calculation by just plugging numbers into the long formulas for velocities after impact as well

$$\mathbf{v}^{B1}=\mathbf{v}^{B0}-\frac{m_A}{m_B+m_A}(1+e)\left[\left(\mathbf{v}^{B0}-\mathbf{v}^{A0}\right)\cdot\mathbf{n}\right]\mathbf{n}$$
$$\mathbf{v}^{A1}=\mathbf{v}^{A0}+\frac{m_B}{m_B+m_A}(1+e)\left[\left(\mathbf{v}^{B0}-\mathbf{v}^{A0}\right)\cdot\mathbf{n}\right]\mathbf{n}$$

Another way would be to use the fact that energy is conserved when *e=1*

[3 POINTS]

7.3 The authors of the article suggest that the unusually large value for the apparent restitution coefficient is because the two CMEs are expanding at the time of collision. To check this idea, we could try to extend the 'standard' collision formulas by saying that the two points AC and BC that collide on the *surfaces* of the spheres obey



$$\frac{\left(\mathbf{v}^{BC1}-\mathbf{v}^{AC1}\right)\cdot\mathbf{n}}{\left(\mathbf{v}^{BC0}-\mathbf{v}^{AC0}\right)\cdot\mathbf{n}}=-e$$

(the argument for this is that the energy lost in a collision is determined primarily by deformation in the material in the two solids near the point of collision). We can re-write the equation for the relative velocities of the centers of the spheres as

$$\frac{\left(\mathbf{v}^{B1}-\mathbf{v}^{A1}\right)\cdot\mathbf{n}-\frac{d}{dt}(R_A+R_B)}{\left(\mathbf{v}^{B0}-\mathbf{v}^{A0}\right)\cdot\mathbf{n}-\frac{d}{dt}(R_A+R_B)}=-e$$

where R_A, R_B are the radii of the spheres. The article states that the sum of the expansion speeds of the CMEs is 117 km/s. Calculate the value of the restitution coefficient *e* with this definition.

We can just substitute numbers into the formula

$$\frac{\left(\mathbf{v}^{B1}-\mathbf{v}^{A1}\right)\cdot\mathbf{n}-\frac{d}{dt}(R_A+R_B)}{\left(\mathbf{v}^{B0}-\mathbf{v}^{A0}\right)\cdot\mathbf{n}-\frac{d}{dt}(R_A+R_B)}=\frac{288-116-117}{207-237-117}=-e$$

$$\Rightarrow e=0.37$$

This is a more usual value for e - the expansion of the CMEs is very likely to be responsible for the strange value of e that was measured.

[2 POINTS]