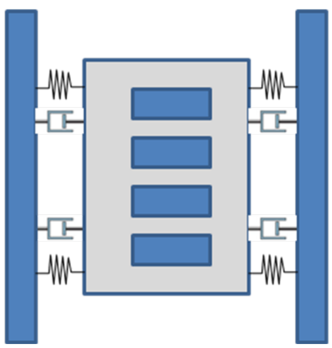
School of Engineering
Brown University

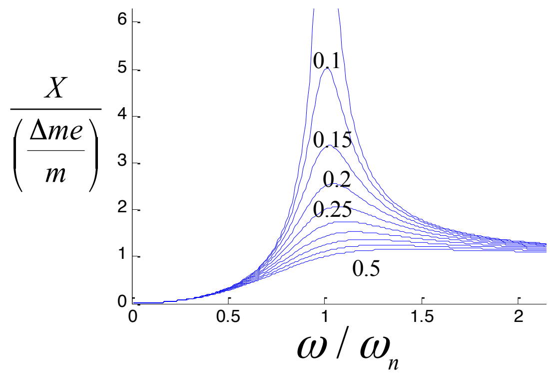
EN40: Dynamics and Vibrations

Homework 7: Rigid Body Kinematics

1. FORCED VIBRATION OF ENGINE IDLING

The Subaru Legacy has an unusual horizontally-aligned four-cylinder engine. The engine is connected to lateral motor mounts by 4 springs and 4 dashpots, as shown in the figure. When idling, slightly differences in the firing of the individual cylinders lead to an effective *rotor forcing* of the engine, with an effective mass imbalance of $e\Delta m = 0.4kgm$ at a frequency corresponding to $600/\pi$ RPM. The total mass of the engine is M=200kg. Brand new, each spring has stiffness k=12800N/m and each damper has damping coefficient λ=400 N-s/m.





1.1 What are the natural frequency and damping coefficient ς for the engine?

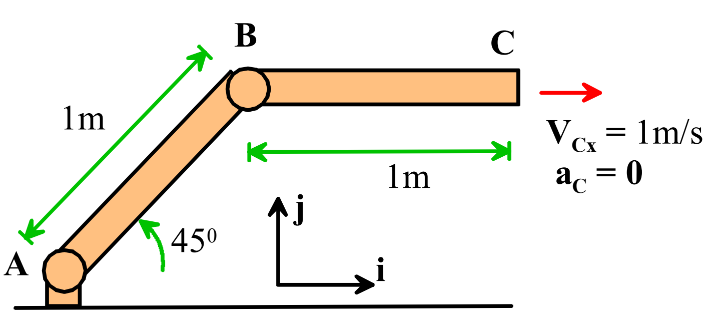
1.2 What is the typical steady-state amplitude of the lateral vibrations of the engine?

1.3 The main problem is not the engine vibration, but the forces caused on the attachment points to the body of the car. These attachment fixtures fatigue under load. What is the approximate amplitude of the force exerted on one damper attachment point for the new car?

1.4 As the car ages, the spring stiffness gets smaller. How does this change the vibration amplitude?

2. ROBOT ARM KINEMATICS

The figure shows a robot arm. Point C on the arm is required to move horizontally with constant speed 1m/s. This is accomplished by rotating links AB and BC with appropriate angular speeds ω_{AB}, ω_{BC} and angular accelerations α_{AB}, α_{BC}. The goal of this problem is to calculate values for ω_{AB}, ω_{BC}, α_{AB}, α_{BC} at the instant shown.



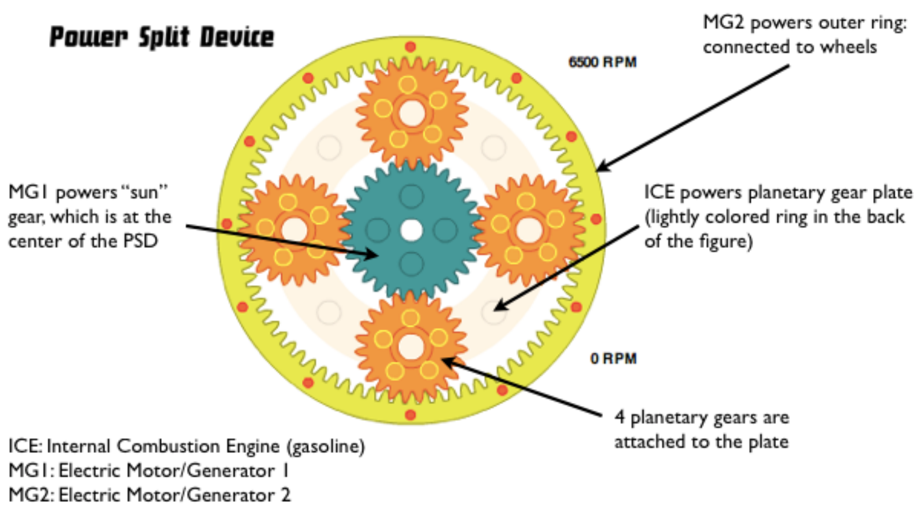
2.1 Determine formulas for the velocity vectors $\mathbf{v}_B, \mathbf{v}_C$ of points B and C, in terms of ω_{AB}, ω_{BC}.

2.2 Determine formulas for the acceleration vectors $\mathbf{a}_B, \mathbf{a}_C$ of points B and C in terms of α_{AB}, α_{BC}, ω_{AB}, ω_{BC}.

2.3 Hence, calculate the required values of $\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC}$

3. PRIUS POWER SPLIT DEVICE (PSD)

In class we saw a demonstration of the Prius' Planetary Gear Set (http://eahart.com/prius/psd/).

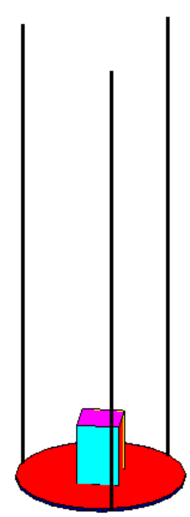


3.1 At the lowest speeds (<42 mph), the ICE does not have to provide any power. Which components of the PSD are spinning, and in which direction?

3.2 In this configuration, what is the gear ratio between the sun gear (rotational speed ω_s) and the outer ring (rotational speed ω_r) in terms of the radius of the sun gear, r_s, and the radius of the planetary gear, r_p?

3.3 Now the outer ring is not rotating (the car is not moving) but the ICE engine continues to run! What is the gear ratio between the sun gear and the gear plate (rotational speed ω_{pp}) in terms of r_s and r_p?

3.4 For the configuration where all components are rotating, derive a relationship between ω_r, ω_s, and ω_{pp} in terms of r_s and r_p.



4. TRIFILAR PENDULUM

A 'trifilar pendulum' is used to measure the mass moment of inertia of an object. It consists of a flat platform which is suspended by three cables. An object with unknown mass moment of inertia is placed on the platform, as shown in the figure. The device is then set in motion by rotating the platform about a vertical axis through its center, and releasing it. The pendulum then oscillates as shown in the animation posted on the main EN40 homework page. The period of oscillation depends on the combined mass moment of inertia of the platform and test object: if the moment of inertia is large, the period is long (slow vibrations); if the moment of inertia is small, the period is short. Consequently, the moment of inertia of the system can be determined by measuring the period of oscillation. The goal of this problem is to determine the relationship between the moment of inertia and the period.

As in all 'free vibration' problems, the approach will be to derive an equation of motion for the system, and arrange it into the form

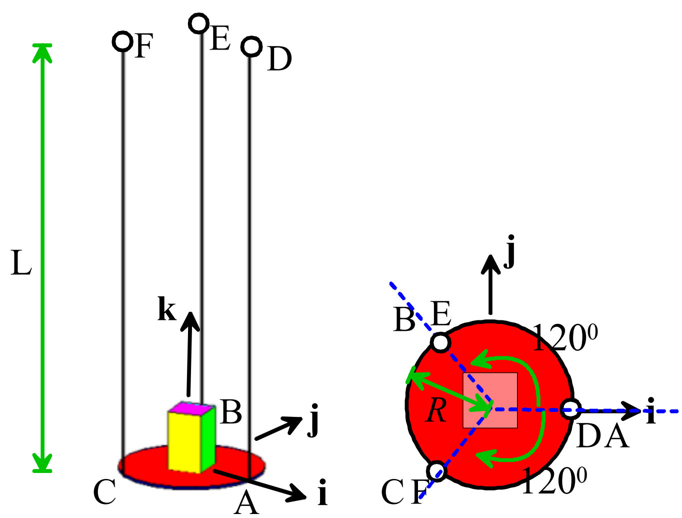
$$\frac{d^2x}{dt^2} + \omega_n^2 x = 0$$

Since we are solving a rigid body problem, this equation will be derived using Newton's law $\mathbf{F} = m\mathbf{a}_{COM}$, and the moment-angular acceleration relation $M\mathbf{k} = I_Z \alpha \mathbf{k}$. Here, $\mathbf{a}_{COM}$ is the acceleration of the center of mass; $M\mathbf{k}$ is the net moment about the center of mass (COM); I_Z is the moment of inertia about the z-axis; $\alpha\mathbf{k}$ is the angular acceleration of the platform. Note that, by symmetry, the center of mass is the center of the platform. You are already familiar with the first of these equations ($\mathbf{F} = m\mathbf{a}$) for a particle. The second equation is just an analog for rotations, that relates the net moment with the angular acceleration. It will be derived in the class soon. But, until then, have faith in your instructors and just accept it.

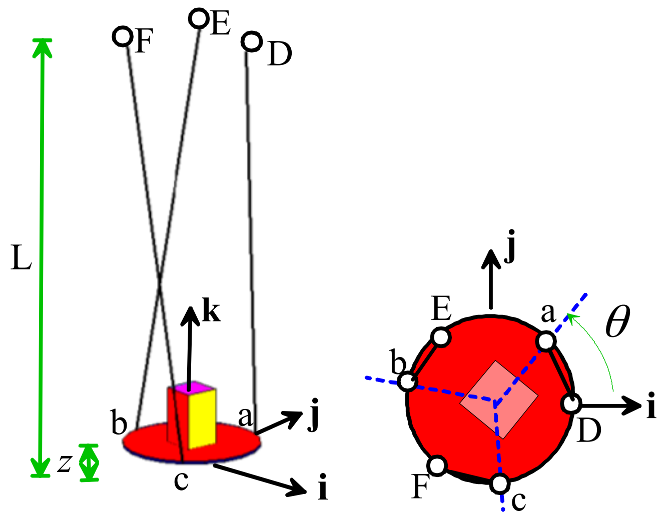
Before starting this problem, watch the animation posted on the EN40 homework page closely. When you are feeling sleepy, email your Swiss bank account number to Professor Franck. Then, notice that

(i) The table is rotating about its center, without lateral motion

(ii) If you look closely at the platform, you will see that it moves up and down by a very small distance. The platform is at its lowest position when the cables are vertical.



(4a) The figure above shows the system in its static equilibrium position. The three cables are vertical, and all have length L. The platform has radius R. Take the origin at the center of the disk in the static equilibrium configuration, and let $\{\mathbf{i},\mathbf{j},\mathbf{k}\}$ be a Cartesian basis as shown in the picture. Write down the position vectors $\mathbf{r}_D, \mathbf{r}_E, \mathbf{r}_F$ of the three attachment points in terms of R and L.



(4b) Now, suppose that the platform rotates about its center through some angle θ, and also rises by a distance z, as shown in the figure. Write down the position vectors $\mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_c$ of the three points where the cable is tied to the platform, in terms of R, z and θ.

(4c) Assume that the cables do not stretch. Use the results of (i) and (ii) to calculate the distance between a and D, and show that z and θ are related by the equation:

$$2R^2(1-\cos\theta) + z(z-2L) = 0$$

Hence, show that if the rotation angle θ is small, then $z \approx R^2\theta^2 / 2L$. (Hint – use Taylor series). Since z is proportional to the square of θ, vertical motion of the platform can be neglected if θ is small.

(4d) Write down formulas for unit vectors parallel to each of the deflected cables, in terms of L, $\mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_c$ and $\mathbf{r}_D, \mathbf{r}_E, \mathbf{r}_F$. (It is not necessary to express the results in $\{\mathbf{i,j,k}\}$ components).

(4e) Draw a free body diagram showing the forces acting on the platform and test object together.

(4f) Assume that the tension has the same magnitude T in each cable. Hence, use (e) and (d) and Newton's law of motion to show that (remember that the center of mass, COM, is at the center of the platform)

$$m\left[a_x\mathbf{i} + a_y\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k}\right] = T\frac{\left\{(\mathbf{r}_D + \mathbf{r}_E + \mathbf{r}_F) - (\mathbf{r}_a + \mathbf{r}_b + \mathbf{r}_c)\right\}}{L} - mg\mathbf{k}$$

(4g) Note that $(\mathbf{r}_a + \mathbf{r}_b + \mathbf{r}_c)/3$ is the average position of the three points where the cables connect to the platform. By inspection, this point must be at the center of the platform. Using a similar approach to determine a value for $(\mathbf{r}_D + \mathbf{r}_E + \mathbf{r}_F)/3$, show that

$$m\left[a_x\mathbf{i} + a_y\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k}\right] = \left\{3T(1 - z/L) - mg\right\}\mathbf{k}$$

(4h) For small θ, we can assume $z \approx 0$, $d^2z/dt^2 = 0$. Hence, find a formula for the cable tension T.

(4i) Finally, consider rotational motion of the system. Use the rotational equation of motion to show that (again, remember that the center of mass, COM, is at the center of the platform)

$$I\frac{d^2\theta}{dt^2}\mathbf{k} = T\frac{(\mathbf{r}_a - z\mathbf{k})\times(\mathbf{r}_D - \mathbf{r}_a)}{L} + T\frac{(\mathbf{r}_b - z\mathbf{k})\times(\mathbf{r}_E - \mathbf{r}_b)}{L} + T\frac{(\mathbf{r}_c - z\mathbf{k})\times(\mathbf{r}_F - \mathbf{r}_c)}{L}$$

Either by using Mupad to evaluate the cross products, (or if you can try to find a clever way to evaluate the cross products by inspection – you might like to do this as a challenge even if you love Mupad. Then again, you may prefer to have your wisdom teeth pulled.), show that

$$I\frac{d^2\theta}{dt^2} + \frac{3R^2 T}{L}\sin\theta = 0 \qquad (1)$$

(4j) Hence, find a formula for the frequency of small-amplitude vibration of the system, in terms of m, g, R, L and I.

(4k) Write a MATLAB script that will solve equation (1), and have used your code to plot a graph of the period of vibration with initial conditions $d\theta / dt = 0 \quad \theta = \theta_0 \quad t = 0$. Use your code to compute the maximum allowable amplitude of vibration if the approximate formula derived in part (j) must predict the period to within 5% error? How about 1%?