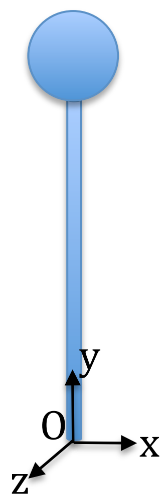
School of Engineering
Brown University

EN40: Dynamics and Vibrations

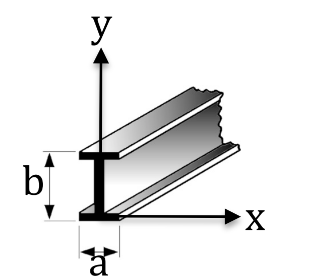
Homework 8: Rigid Body Dynamics

1. CALCULATE THE MASS MOMENT OF INERTIA



You may use the mass moment of inertia tables in the slides provided online.

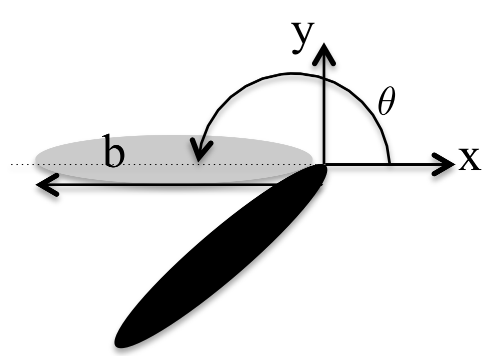
1A. A thin disk of radius R and mass m_d connected to a slender rod of length L and mass m_r. It rotates around the z-axis.



1B. An I-beam is composed of 3 sections of equal thickness t, density ρ, and length L. It is rotating around the y-axis.

1C. A hollow cylinder of length L, outer radius R, inner radius r, and density ρ. It is rotating around the x-axis.

2. ELLIPSOID PENDULUM



A rigid body pendulum is composed of an ellipsoid whose length is b and whose width and height are a. The ratio $b/a=10$. The ellipsoid is free to pivot about a fixed point at the origin (point O) in the x-y plane. It has a mass of m.

2A. What is the mass moment of inertia, I_o in terms of m and b?

2B. Draw a FBD.

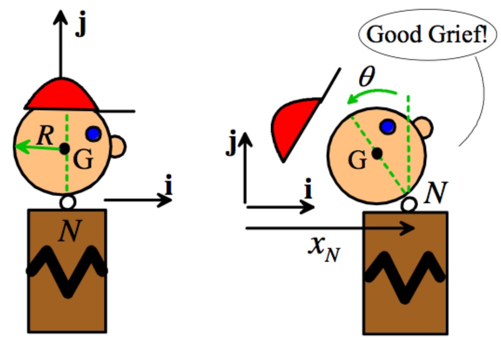
2C. Write 3 appropriate equations of motion and identify the unknowns.

2D. By eliminating the extra unknown variables, write a single equation of motion for the rigid body pendulum in terms of b, m, g, I_G, and θ.

2E. For small oscillations (about the negative y-axis), what is the natural freqency in terms of b, m, g, I_G, and θ?

3. PASSENGER WITH NO HEADREST

The figure shows a passenger in a vehicle with no headrests. The car is initially at rest, and is hit from behind by another vehicle, giving the car and the passenger's torso a forward acceleration of $\mathbf{a} = a_N\mathbf{i}$. The acceleration bends the passenger's neck through an angle θ as shown in the figure.



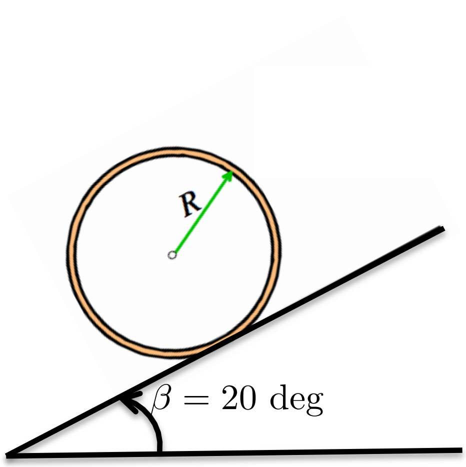
- Assume the head is a sphere with radius R, mass m, and mass moment of inertia of $I_G=2mR^2/5$.

- There is a moment applied by neck on the head. Assume the next acts as a torsional spring with spring constant *k*.

3.1 Draw a FBD of the head.

3.2 Show that the equation of motion is $\frac{7}{5}R^2m\alpha + k\theta - Rmgsin(\theta) - Rma_Ncos(\theta) = 0$

4. ROLLING RING



Consider a ring of radius mass M and radius R, initially released from rest on an incline of 20 degrees. The coefficient of kinetic friction is μ_k, and the coefficient of static friction is μ_s.

4.1 Assuming a thickness *t* that is much smaller than *R*, derive an expression for the mass moment of inertia in terms of *M* and *R*.

4.2 Let's first assume the ring rolls without slipping. Derive an expression for the linear acceleration of the center, plus the angular acceleration soon after it begins to roll.

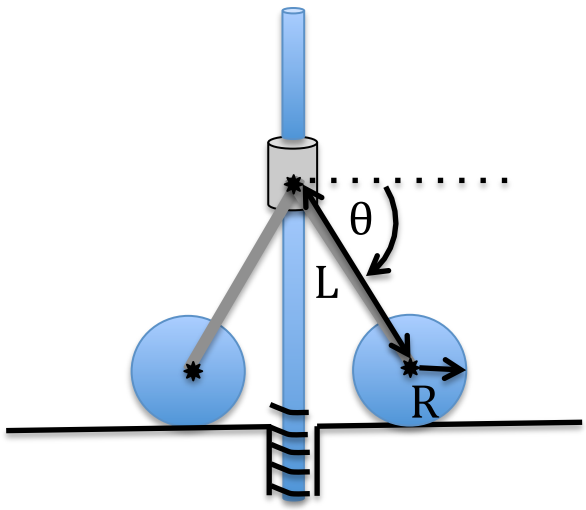
4.3 If R=0.5m, μ_k=0.12, and μ_s=0.15, does our assumption of rolling without slip hold? Why or why not?

4.4 For the given values in 4.3, how long will it take the ring to travel 5 meters down the incline?

4.5 Now let's assume it is *rolling without slip*. Using energy methods, calculate the rotational velocity after it has travelled 5 meters.

4.6 In problem 4.5, is there work done by friction? Why/Why Not?

5. Spring Collar Wheel System



In the mechanism shown, each of the two wheels (mass=m_w, radius=R) is connected to a collar via a slender bar (length=L, mass=m_b). The collar slides (mass=m_c) frictionlessly on a vertical shaft, and hits a spring (spring constant=k) when the bars are in a horizontal configuration. The wheels roll without slipping.

If the collar is released from rest at θ=45 deg:

5.1 Calculate the velocity of the collar as it first hits the spring.

5.2. Calculate the maximum deformation of the spring.