School of Engineering
Brown University

**EN40: Dynamics and Vibrations**

**Homework 1:**
**Mupad review and solving differential equation with MATLAB**
**Due 12:00 noon Friday January 31**

- Your solution to this homework should consist of two files:
  1. A commented MUPAD .mw file
  2. A commented MATLAB .m file
- Please submit the assignment electronically on the EN40 canvas website. You can log into canvas at http://brown.edu/it/canvas/ (the login link is near the top right of the page). Instructions for uploading are at https://sites.google.com/a/brown.edu/teaching-with-technology/canvas/student-help-site/assignments

**Part 1: Use Mupad to solve the following math problems. Be sure to save your work frequently!**

**1.** Find whether 9999943 is a prime number (use the 'isprime' function)

**2.** Calculate the value of $e^{i\pi}$ (here $i=\sqrt{-1}$ - Mupad uses capital I to denote $i$, and PI to denote $\pi$ )

**3.** Calculate $\sum_{n=1}^{\infty}\frac{1}{n^2}$ (see the command bar to the right of the mupad window for the summation function. It works just like the int function. 'infinity' is mupad's convention for $\infty$ )

**4.** Compute the first 5 nonzero terms of the Taylor series expansion of $\sin(x)/x$ about $x=0$ (use the 'series' function). What is $\lim_{x\to 0}\sin(x)/x$ ?

**5.** Plot the function $x\exp(-x^2)$ in the range $-4<x<4$

**6.** Find the value of $x$ that will maximize $f(x)=x\exp(-x^2)$ . To do this you will need to (i) Differentiate $f(x)$ ; (ii) solve $df/dx=0$ for $x$.

**7.** Evaluate the following definite integral (find a numerical value)

$$\int_0^{2\pi}\frac{\sin(x)}{x}dx$$

**8.** Solow's economic growth model is a famous economic model that predicts the influence of the rate of investment on the long-term standard of living. 'Standard of living' is quantified by the ratio $r=K/L$ of the total 'capital' $K$ (the time integral of 'income') to the labor force $L$. The standard of living evolves according to the differential equation

$$\frac{dr(t)}{dt}=sr(t)^{1/3}-gr(t)$$

Here, $s$ and $g$ are constants characterizing the savings rate and the rate of growth of the labor force, respectively. Use Mupad to solve the differential equation with initial condition $r=r_0$ at time $t=0$. To make Mupad give a simple solution, use the statements `assume(s>0): assume(g>0): assume(r0>0):` and `solve(ode({diffeq,ICs}, r(t)),IgnoreSpecialCases)` (you must define diffeq and ICs). There will still be an arbitrary constant in the solution -this can be set to zero.

**Part 2: Please solve the remaining problems using MATLAB** (write your code in a matlab .m file). You should make your MATLAB (.m) file a function, so that when the file is executed, it will solve all the homework problems. For example:

```
function ihavenolife
       Solutions to problems 9-12
       Functions for the differential equations in probs 11 and 12
end
function number_of_vars = count_variables(vector,value)
…
end
```

(You might find the solutions to homework 1, 2009-2013 helpful, if you get stuck)

**9.** Create two vectors v and w containing 1000 random integers that lie between 1 and 100 (use MATLAB's randi function –see last year's homework 1 for an example.)

**10.** Write a function that will check your two vectors v and w, and count the number of entries of the two vectors that are equal (for example, if v=[1,2,5,3,1] w=[1,3,5,2,6] the first and third entries v(1)=w(1); v(3)=w(3) are equal). See HW1 from 2013 for an example of counting variables in a vector.

**11.** The differential equation

$$\frac{dr}{dt} = \sqrt{\frac{a}{r} - k}$$

arises in cosmology to describe the expansion of a simplified universe (see these notes from OSU, for example) . Write a MATLAB code that will calculate the size of the universe *r(t)* given values for *a,k* and the value of *r* at time *t=0*. Plot the solution for the following parameter values:

(i) *r(0)=0.1* , *a=10, k=-1*

(ii) *r(0)=0.1* , *a=10, k=1*. For the latter case try the solution both with the default value of tolerance in MATLAB, and also with `options = odeset('RelTol',0.000001);`
Extra credit problem – can you make the peculiar error message in part (ii) go away?

**12.** The 'SIR' model is used to predict the spread of an infectious disease. It assumes that a population can be divided into three fractions *S*, *I*, and *R*, (which each vary between 0 and 1 (100%)) where

- *S* is the fraction of the population that has not yet been infected, and so is susceptible
- *I* is the fraction of the population that is infected and capable of spreading the disease
- *R* is the fraction of the population that has recovered and hence is immune.

They obey the following equations

$$\frac{dS}{dt} = -\beta SI \qquad \frac{dI}{dt} = \beta SI - \gamma I \qquad \frac{dR}{dt} = \gamma I$$

where $\beta, \gamma$ are two constants. To interpret these equations, notice that the fraction of infected people increases in proportion to (i) the fraction *S* who have not yet been infected; and (ii) the fraction of infected people *I* that are spreading disease, and decreases at the rate that people recover. Write a MATLAB code that will calculate *S(t), I(t)* and *R(t)*, given values for $\gamma, \beta$ and the values of $S, I, R$ at time *t=0*. Calculate and plot the solution for *0<t<90* days as a function of time for the following cases:

(i) $S(0) = 0.99$ , $I(0) = 0.01$ , $R(0) = 0$, $\beta = 0.4$ days$^{-1}$ $\gamma = 0.3$ days$^{-1}$

(ii) $S(0) = 0.99$ , $I(0) = 0.01$ , $R(0) = 0$, $\beta = 0.4$ days$^{-1}$ $\gamma = 0.05$ days$^{-1}$

**13. Optional (extra credit problem for experienced programmers)** Write a matlab function that will compute all the prime numbers up to a maximum value using the 'Sieve of Eratosthenes'.