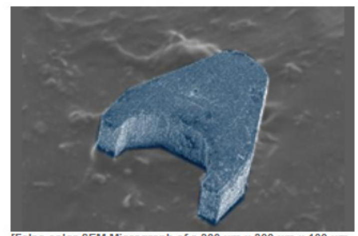
School of Engineering
Brown University

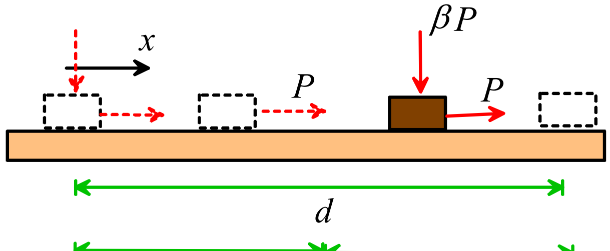
EN40: Dynamics and Vibrations

Homework 2: Kinematics and Dynamics of Particles
Due Friday Feb 7, 2014

1. An experimental 'micro-robot' (see a description here – most people would call the robot a speck of dirt…) is moved around on a surface by subjecting it to a constant magnetic force P that acts tangent to the surface. In addition, an electrostatic force with magnitude βP can be applied to attract the micro-robot to the surface. Friction with coefficient μ acts between the robot and the surface, so the particle can be moved around by turning the electrostatic force on and off. Gravity may be neglected.


[False-color SEM Micrograph of a 300 μm x 300 μm x 100 μm Mag-μBot with a claw.]

Suppose that the micro-robot has mass *m*, and is at rest at *x=0* at time *t=0*, with the electrostatic force turned on. To move the robot through a distance *d*, the electrostatic force is turned off for a time $0 < t < t_1$, and then turned back on again. The goal of this problem is to derive a formula relating t_1 to *d*.

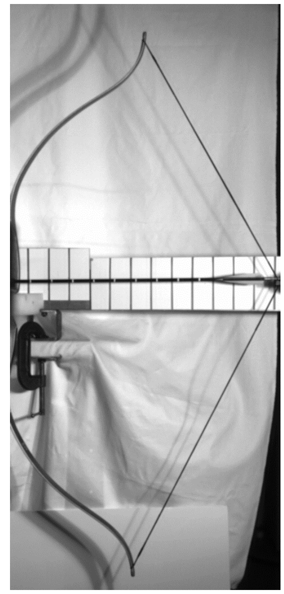


1.1 Use Newton's law to find the acceleration of the micro-robot during the time when the electrostatic force is switched off $0 < t < t_1$, and hence find formulas that give the speed and position of the micro-robot as functions of time for $0 < t < t_1$, in terms of *P* and *m*.

1.2 Similarly, find formulas that predict the speed and position of the micro-robot for time $t > t_1$

1.3 Use the conditions that the robot comes to rest at position *d* to determine t_1 in terms of *P*, *d* μ, β and *m*.

2. This webpage shows some high speed videos of three different bow designs as they fire an arrow (Courtesy of Ahyoung Choe, 15). Position-v-time data for the arrow has been extracted from the images, using the MATLAB image processing toolbox. Your mission is to write a MATLAB script that will analyze this data to plot graphs showing the velocity and acceleration of the arrow.



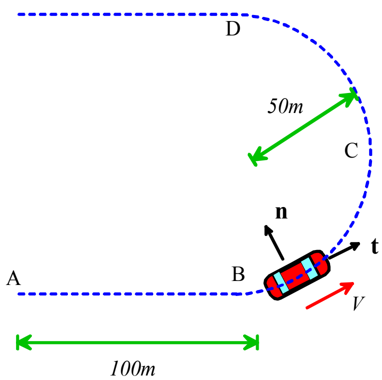
- The position-v-time data are stored in three .csv ('comma separated value') files that can be downloaded from this webpage.
- You can read the data files directly into MATLAB using the 'csvread' command – for example
```
compoundbow_data = csvread('compound_data.csv');
```
The variable 'compoundbow_data' is a matrix, with the first column (compoundbow_data(:,1)) containing time values, in seconds, and the second column (compoundbow_data(:,2)) containing the distance moved by the arrow, in m. You can also open the files with excel or any text editor.
- The easiest approach to calculating velocity and acceleration is to numerically differentiate the data. For example, let the N time and position values be t_i, x_i with $i=1...N$. You can estimate the velocity at the midpoint of a time interval between two successive data points as $v = \Delta x / \Delta t$, giving
$$\overline{t_i} = (t_i + t_{i+1})/2 \qquad v_i = (x_{i+1} - x_i)/(t_{i+1} - t_i) \quad i = 1....N-1$$
You can calculate vectors of time and velocity using these formulas and a loop, and then plot them. You can then calculate the accelerations by differentiating the velocity $a = \Delta v / \Delta \overline{t}$. The acceleration will be quite noisy. If you would like to try something more sophisticated you can explore MATLAB's 'Curve Fitting Toolbox,' which has a lot of capabilities for curve-fitting, smoothing, and computing numerical derivatives.

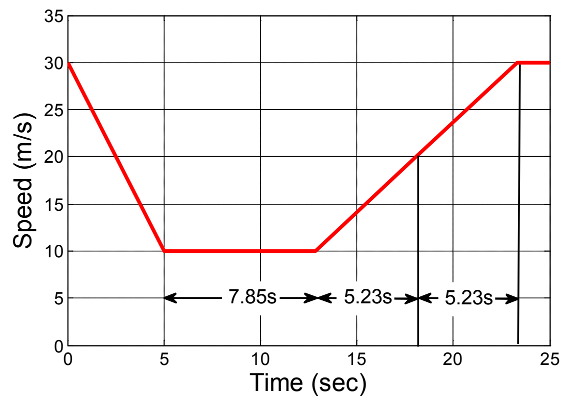
2.1 Submit graphs that show the velocity and acceleration of at least **ONE** bow as a function of time (you can plot them all if you are curious but this is not required). Be sure to specify which bow you analyzed. Also include a brief description that will help the grader understand how you processed the data (numerical differentiation, curve fitting, etc). You do not need to submit MATLAB code.

2.2 Estimate the maximum force exerted by the bow on the arrow as it is fired. For comparison, the *static* force-v-draw curves (i.e. the force required to draw the bow through a distance *d*) for the three bows are shown on the webpage. Why do you think the dynamic force differs from the static force? Would you expect the average dynamic force as the arrow is fired to be greater, or less than the static force?

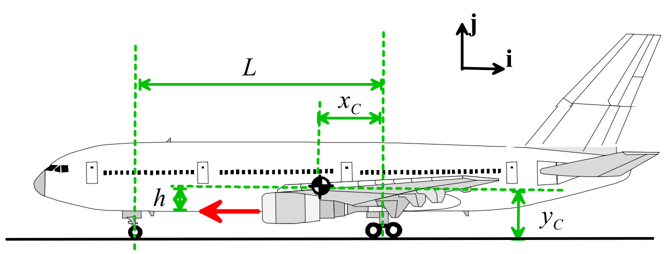
2.3 **Optional – for extra credit** Plot graphs that compare the static force exerted by the bow as a function of draw distance *d* to the actual, dynamic force exerted on the arrow while it is fired (plot draw distance on the horizontal axis and the force on the vertical axis)

3. The path and speed of a vehicle driving around a sharp bend is shown in the figure below (the vehicle is at point A at time $t=0$).
   3.1 Sketch graphs of the normal and tangential acceleration of the vehicle. Explain briefly how you calculated relevant quantities.
   3.2 **Optional – extra credit** Calculate the minimum friction coefficient required to prevent the vehicle from slipping (Draw a FBD; use Newton's laws to calculate the reactions; and use the friction law to find μ)





4. The figure shows an aircraft just starting its take-off roll. The engines provide a total thrust F_T that act a height h below the center of mass, producing an acceleration $a_x = -(g/2)\mathbf{i}$. Since the aircraft is not yet moving lift and drag forces are zero.



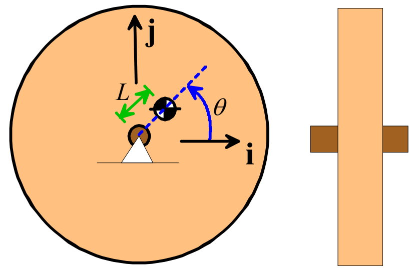
4.1 Draw a free body diagram showing the forces acting on the aircraft.

4.2 Write down Newton's law of motion and the equation of rotational motion for the aircraft (assume straight line motion without rotation)

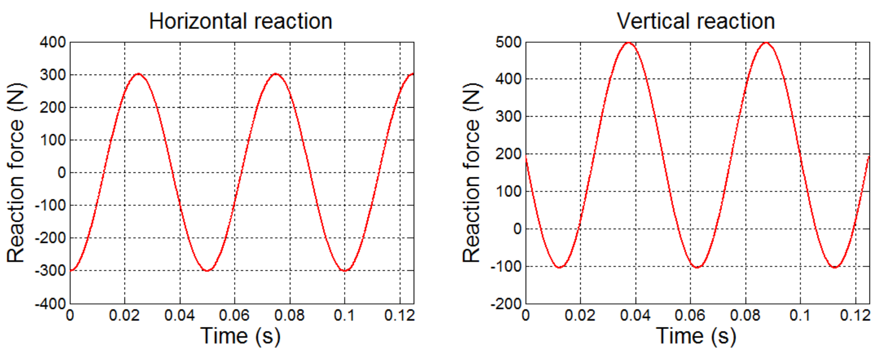
4.3 Hence, find formulas for the reaction forces on the wheels (for the rear wheels, calculate the total force).

4.4 Hence, show that the front wheel will lose contact with the ground if h exceeds a critical value, and find a formula for this critical value of h.

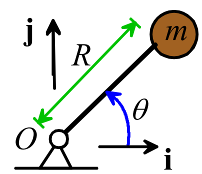
5. The figure shows an unbalanced rotor that spins at constant angular speed $d\theta / dt = \omega = 40\pi$ rad/s. The center of mass of the rotor is a distance L from the axle. As a result, large fluctuating reaction forces develop at the axle. The horizontal and vertical reaction forces acting on the wheel are plotted in the graphs shown (the forces act in the positive **i** and **j** directions). At time $t=0$ the angle θ is zero.



A 100 gram mass is to be added to the disk to balance it. The goal of this problem is to calculate the position of the mass on the disk.



5.1 As a preliminary step, consider a mass m that rotates at constant angular rate $\frac{d\theta}{dt} = \omega$ at the end of a massless link with radius R (see the figure). Find a formula for the acceleration vector of the mass in terms of R, ω, θ , expressing your answer as components in the **i,j** basis.



5.2 Draw a free body diagram showing the forces and moments acting on the link and mass together (include reaction forces/moments at O and gravity)

5.3 Use Newton's laws to calculate a formula for the reaction forces acting on the link, in terms of ω, θ, R, m, g .

5.4 Hence, find the distance R for which the 100 gram mass will produce the same horizontal reaction force as the disk (see the figure) when rotating at the same 40π rad/s.

5.5 Draw a sketch showing where the mass m should be located on the disk (show the location of the center of mass of the disk, together with the added mass).

5.6 What are the reaction forces acting on the disk after the disk has been balanced?