School of Engineering
Brown University

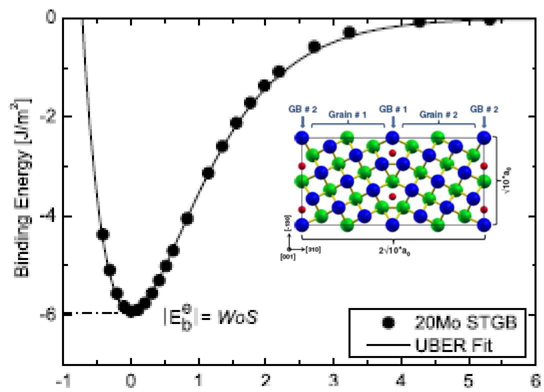
EN40: Dynamics and Vibrations

Homework 4: Work, Energy and Linear Momentum
Due Friday March 1st

1. The figure (from this publication) shows the energy per unit area required to displace two atomic planes of Molybdynum by a distance x from their equilibrium positions. The authors report that:

$$W(x \to \infty) = 6.23 Jm^{-2}$$

$$\frac{d^2W}{dx^2}(x=0) = 43.9 \times 10^{19} N/m^3$$



x(Angstroms)

1.1 Calculate the values of the constants E_0, d in the Universal Binding Energy relation

$$W = E_0 - E_0\left(1 + \frac{x}{d}\right)\exp(-x/d)$$

that will fit this data. Hence, estimate the force per unit area that will cause the planes to separate (i.e. the max force of attraction between the planes).

1.2 Instead of the UBER, the authors decided to fit their calculated work of separation using the more elaborate function (their eq. (9)) of the form

$$W = E_0 - E_0\left(1 + \frac{x}{d} - 0.079\frac{x^3}{d^3} + 0.0063\frac{x^4}{d^4} + 0.00002\frac{x^5}{d^5}\right)\exp(-x/d)$$

use Mupad to plot the attractive force as a function of x for both the universal binding energy function and the modified formula (use the values of E_0, d from part 1.1. Why is this correct?). Calculate the maximum force predicted by the new formula.

2. The Tesla Model S electric vehicle has the following specifications:

- Acceleration from 0 to 60mph in 4.2sec.
- Curb weight of 4647.3 lbs.
- Battery capacity (total energy stored in the battery): 60kWh
- Range at 55mph 244 miles
- Height 56”; width 77”

Assume that air resistance can be calculated from the formula

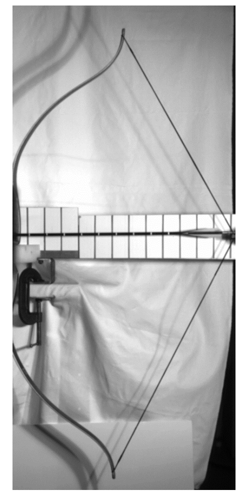
$$F_D = \frac{1}{2}\rho C_D A v^2$$

with drag coefficient C_D, air density $\rho = 1.2 kgm^{-3}$ and projected frontal area A and v the speed

2.1 Assuming that air resistance is the dominant contribution to energy consumption during steady cruise, use the given range and battery capacity to calculate the drag coefficient.

2.2 Calculate the range of the vehicle at 70 mph.

2.3 Assuming that the propulsion system produces a constant power (i.e. independent of velocity) estimate the power necessary to accelerate the vehicle to 60mph in 4.2 sec. You can neglect air resistance to keep the calculation simple.

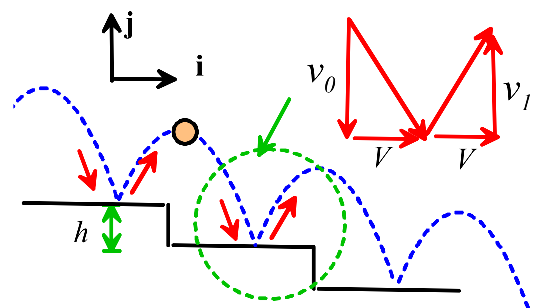


3. The static force-v-displacement measurements for the three bows that you analyzed in Homework 2 can be downloaded from this webpage.

3.1 Use the MATLAB code that you wrote in Homework 2 to calculate the kinetic energy of one (or more) of the arrows just after they leave the bow. Be sure to state which bow!

3.2 The static force-v-draw data for each bow are available as .csv files on the webpage. By integrating the force-v-draw curve, plot a graph of the work done in drawing the bow(s) considered in 2.1 as a function of draw distance *d*. You can use the MATLAB 'trapz' function to do the integral, or a method of your own design. There is no need to submit a copy of your MATLAB code.

3.3 Hence, calculate the *dynamic efficiency* of each bow (the ratio of the kinetic energy of the arrow to the work done in drawing the bow.



4. The figure shows a ball bouncing down a flight of stairs. Assume that the ball travels with constant horizontal speed *V,* and that steady state conditions hold, so that the ball lands on each successive step with the same velocity $\mathbf{v} = V\mathbf{i} - v_0\mathbf{j}$. The goal of this problem is to calculate this special velocity, in terms of the restitution coefficient *e* and the height of the step *h.*

4.1 Write down the velocity vector just after the bounce, in terms of *V,* v_0 and *e*.

4.2 Use energy conservation to find a formula relating v_1 to v_0 and *h*.

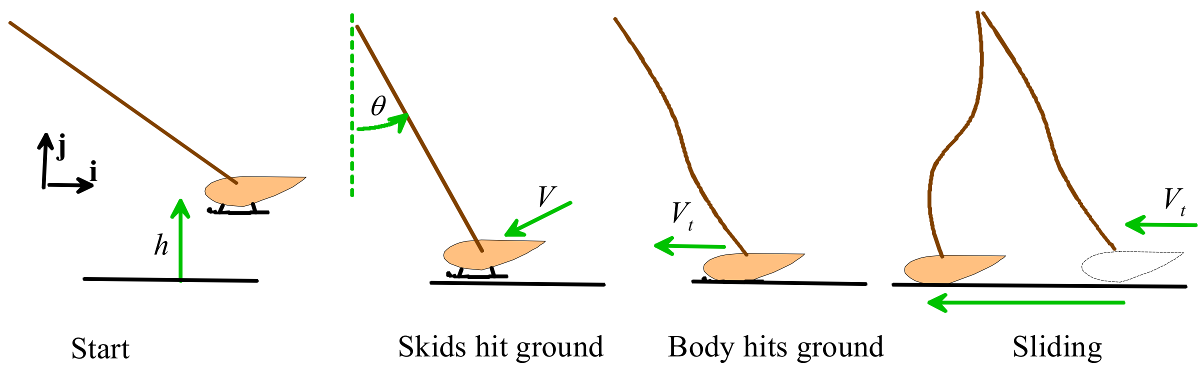
4.3 Hence, show that $v_0 = \sqrt{2gh/(1-e^2)}$

4.4 Calculate the average vertical velocity of the ball (it is easiest to do this by finding the time between two successive bounces).

4.5 **Optional (and quite hard) – for extra credit:** Suppose that the ball is launched at the top of a flight of stairs with velocity $\mathbf{v}_0 = V\mathbf{i} + v_0\mathbf{j}$. Show that it will land on the *nth* step with velocity

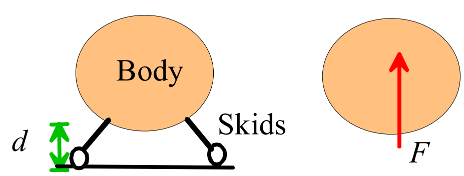
$$\mathbf{v}_n = V\mathbf{i} - \left(\sqrt{e^{2(n-1)} v_0^2 + 2gh \frac{1-e^{2n}}{1-e^2}} \right) \mathbf{j}$$

5 The figure shows a schematic diagram of a helicopter impact test used by NASA. If you watch the movie, you will see that the impact takes place in two stages: (i) the skids hit the ground, and get crushed; and (ii) the main body of the helicopter hits the ground. The goal of this problem is to estimate how much force the skids exert on the helicopter body as they are crushed.



5.1 The helicopter starts at a height h above the ground. Use energy methods to find a formula for its speed V just before the skids hit the ground. Hence, calculate the **i,j** components of velocity just before the skids hit the ground, in terms of the angle θ (this is just trig…).

5.2 As the skids are crushed, they exert a constant vertical force F on the helicopter body (horizontal force can be neglected). The cable exerts no force on the helicopter during this period. Use Newton's laws and the constant acceleration formulas to show that the **i,j** components of velocity of the helicopter body just before it impacts the ground are



$$\mathbf{v} = -\sqrt{2gh}\cos\theta\mathbf{i} - \sqrt{2}\left(\sqrt{g(h\sin^2\theta + d) - Fd/m} \right)\mathbf{j}$$

5.3 After the skids are crushed the body of the helicopter hits the ground. During this impact, the helicopter body is subjected to an impulse $\mu I_N \mathbf{i} + I_N \mathbf{j}$ where μ is the coefficient of friction. It remains in contact with the ground after impact. Show that

$$I_N = m\left[\sqrt{2}\sqrt{g(h\sin^2\theta + d) - Fd/m} \right]$$

and hence deduce that the speed of the helicopter after the body hits the ground is

$$V_t = \left(\sqrt{2gh}\cos\theta - \mu\left[\sqrt{2}\sqrt{g(h\sin^2\theta + d) - Fd/m} \right] \right)$$

5.4 Just after impact the helicopter has a horizontal speed V_t. It then slides over the ground for a time t before coming to rest. Find a formula relating the speed of the helicopter V_t at the start of the skid, to the friction coefficient μ and the time t and g (you can use the straight line motion formula or impulse-momentum).

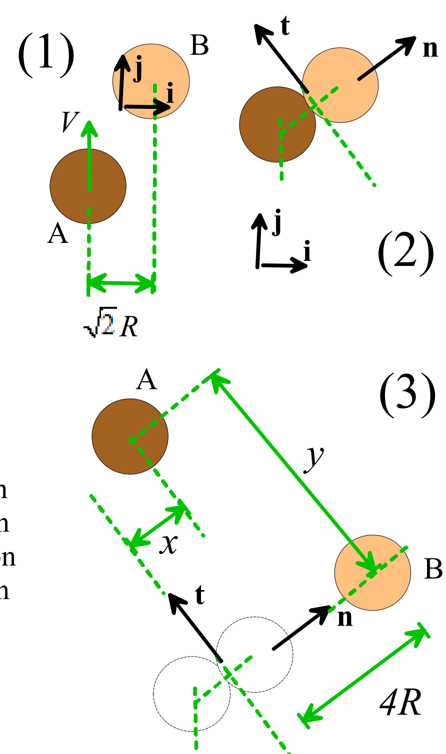
5.5 Use the results of (3) and (4) to show that

$$F = mg(1 + h\sin^2\theta / d) - (mgh/d)\left((\cos\theta/\mu) - t\sqrt{g/2h}\right)^2$$

5.6 Calculate the force for the following parameters:

- Slip time t=2sec
- Friction coefficient $\mu = 0.5$
- Drop height h 12m
- Skid height d 1m
- Mass 1300kg
- Cable angle at impact $\theta = 30^o$

6. The figure shows a collision between two identical spheres with mass m and radius R. The restitution coefficient for the collision is e. The numbers (1), (2), (3) show the sequence of the pictures - (1) is before impact; (2) is impact, and (3) is after impact.



6.1 Write down the total linear momentum of the system before the impact, in {**i,j**} components.

6.2 Find the velocity of sphere A before impact in **n,t** components, and hence write down the total linear momentum of the system in **n,t** coordinates.

6.3 Which of the following are conserved during the impact:

(1) The total linear momentum of the system in the **i** direction
(2) The total linear momentum of the system in the **j** direction
(3) The total linear momentum of the system in the **n** direction
(4) The total linear momentum of the system in the **t** direction
(5) The linear momentum of sphere A in the **i** direction? **j** direction? **n** direction? **t** direction?
(6) The linear momentum of sphere B in the **i** direction? **j** direction? **n** direction? **t** direction?

6.4 Write down the velocities of the two spheres in the **t** direction after impact (you don't need to do any calculations!)

6.5Find a formula for the velocities of the two spheres in the **n** direction, in terms of V and the restitution coefficient e.

6.6 Suppose that sphere *B* travels a distance $4R$. Calculate the coordinates x,y of sphere *A* relative to the impact point in the $\{\mathbf{n},\mathbf{t}\}$ basis at the same instant, in terms of e and R.