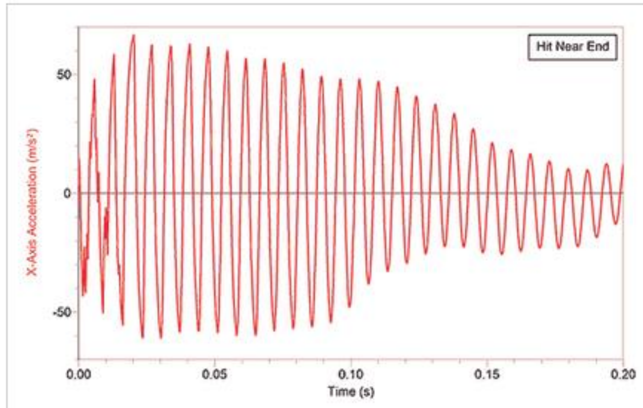
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 5: Free Vibrations
Due Friday March 21st

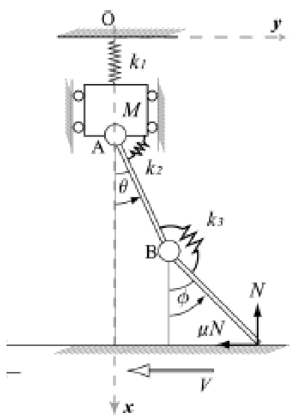
1. The figure shows a vibration measurement from an accelerometer attached to a baseball bat (from this website). For the time interval between 0.02sec and 0.06sec, estimate:



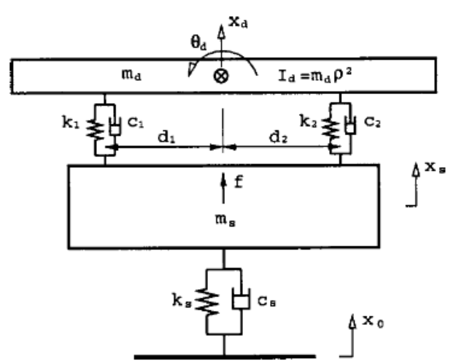
1.1 The amplitude and frequency of the acceleration

1.2 The amplitude of the displacement of the bat (at the point where the accelerometer is attached).

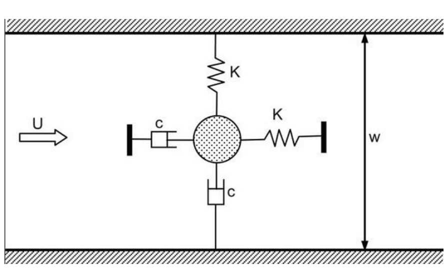
2. State the number of degrees of freedom and the number of natural frequencies of vibration for each of the systems shown below (The articles linked give some practical examples of vibration). Note that the first two show the coordinates so this is sort of trivial)!



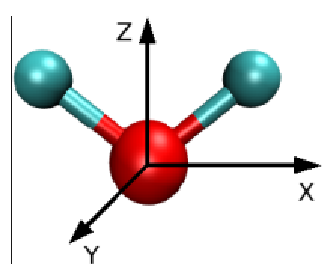
(a) 2D Laser printer wiper



(b) 2D Model of a tuned vibration absorber
The top mass is a rigid body,
the bottom mass is a particle
The base vibration is prescribed (not a DOF)



(c) Model of a vibrating pipe in a fluid flow

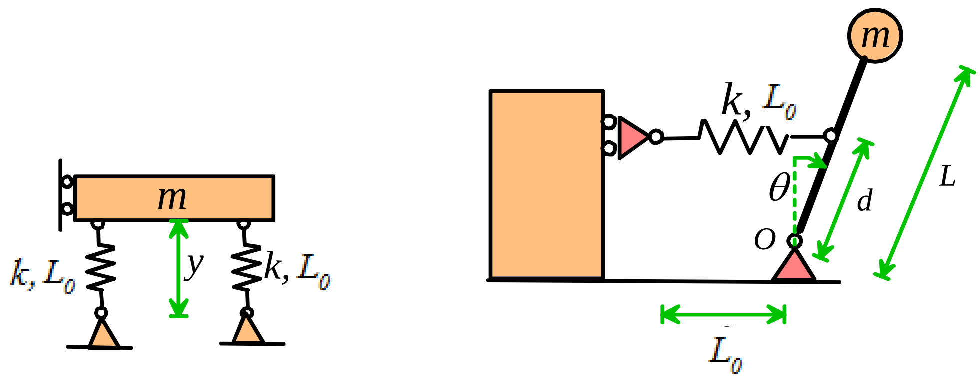


(d) Water molecule

3. Solve the following differential equations (use the Solutions to Differential Equations)

3.1 $4\frac{d^2y}{dt^2}+16y=16 \qquad y=1 \quad \frac{dy}{dt}=2 \quad t=0$

3.2 $4\frac{d^2y}{dt^2}+4\frac{dy}{dt}+16y=0 \qquad y=0.1 \quad \frac{dy}{dt}=0 \quad t=0$



4. For the two conservative single-degree of freedom systems shown in the figure:

4.1 Derive the equation of motion (use energy methods, and include gravity). State whether the equation of motion is linear or nonlinear.

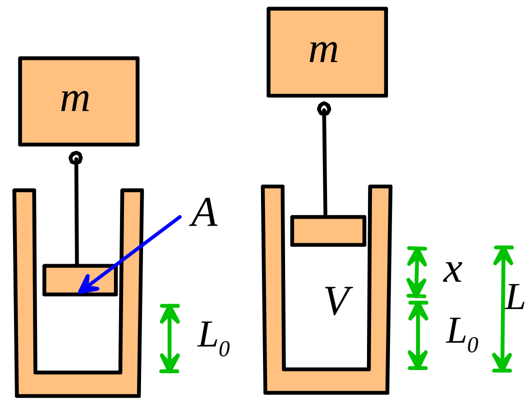
4.2 If appropriate, linearize the equation of motion for small amplitude vibrations (that means doing that Taylor series stuff discussed in class. "Linearizing" means replacing the nonlinear function of the variable with an approximate linear function)

4.3 Arrange the (linearized) equation of motion into standard form, and find an expression for the natural frequency of vibration. Identify conditions where the natural frequency cannot be calculated and explain what the system will do if disturbed from $\theta=0$ in this case.

5. A pneumatic vibration isolator works by using a compressed air column as a spring. The air pressure p in the piston obeys (approximately) the adiabatic ideal gas law

$$\frac{p}{p_0}\left(\frac{V}{V_0}\right)^{\gamma} = 1$$

Where p_0, V_0 are the air pressure and volume in the cylinder when it is fully extended (i.e. just constants), V is the (variable) volume of the cylinder, and $\gamma \approx 7/5$ for air. The goal of this problem is to derive a formula for the natural frequency of vibration of a mass supported by the isolator (the specification sheet gives formulas, but it is not easy to see where they come from).



5.1 Show that the force exerted by the isolator (air pressure x cylinder area) is related to the air cylinder area A and length L by

$$F = \frac{p_0 V_0^{\gamma}}{A^{\gamma-1} L^{\gamma}}$$

5.2 Suppose that the isolator supports an object with mass m and is in static equilibrium (no motion or vibration). Show that the static length L_0 of the actuator satisfies the equation

$$\frac{p_0 V_0^{\gamma}}{A^{\gamma-1}} = mgL_0^{\gamma}$$

5.3 Suppose that as the mass vibrates, its length changes to $L = L_0 + x$. Use Newton's law of motion for the mass, and the results of 5.1 and 5.2 to show that x must satisfy the equation of motion

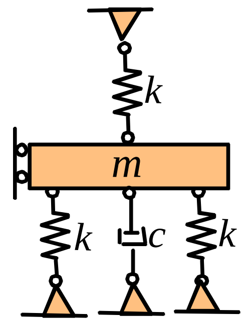
$$m\frac{d^2x}{dt^2} = mg\frac{L_0^{\gamma}}{(L_0 + x)^{\gamma}} - mg$$

5.4 Linearize the equation of motion for $x << L_0$, and hence show that the natural frequency of vibration is

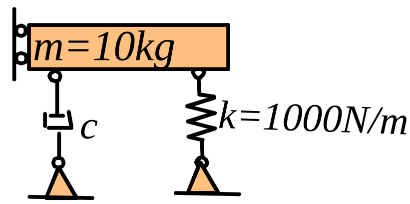
$$\omega_n = \sqrt{\frac{\gamma g}{L_0}}$$

In many applications the actuator is pressurized so as to maintain a constant static length L_0 by an automatic leveling system. If this is done, the natural frequency of the system is independent of the mass m.

6. Replace the system shown in the figure with an equivalent spring-mass system consisting of a mass with only one spring and dashpot. Hence, determine a formula for the undamped natural frequency and the damping factor for the system.

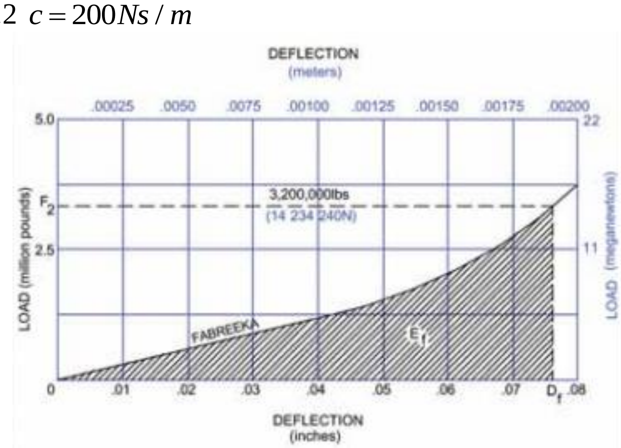


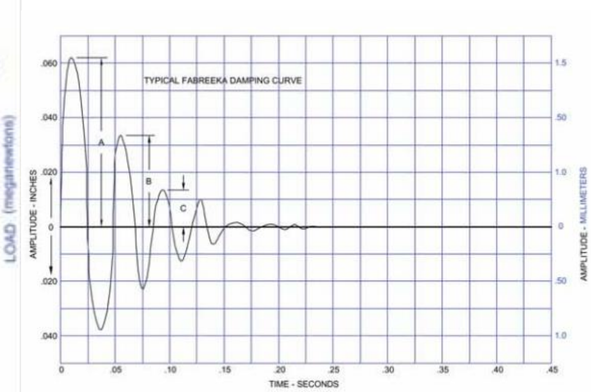
7. The spring-mass system shown in the figure is at rest for time $t<0$. The mass is then displaced from its equilibrium position vertically by a distance $x_0 = 10mm$ and released (from rest). Find formulas for the subsequent motion of the mass $x(t)$ for each of the following cases (you don't need to re-derive the equations of motion, since this is a standard system. Note also that since x is the displacement from equilibrium the constant C in the standard equations of motion is zero).



7.1 $c = 20Ns/m$

7.2 $c = 200Ns/m$





8. The figures show the results of (a) a static test and (b) a dynamic test on a shock absorbing pad (details here). In the dynamic test, a mass (with unknown value) was placed on the pad and struck to start the mass vibrating on the pad, and the displacement of the mass was measured.

8.1 Use the results of the static test to estimate the stiffness of the pad (you can assume that the deflection is less than 1mm).

8.2 Use the dynamic test to determine the log decrement, and hence determine the natural frequency ω_n and damping factor ζ

8.3 Hence, calculate the value of the mass used in the test, and determine a value for the dashpot coefficient c that would model the energy dissipation.

8.4 What value of mass on the pad would lead to the system being critically damped?