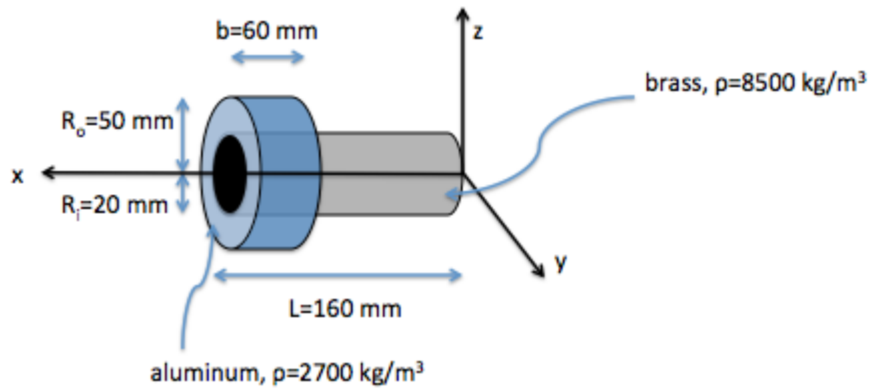
ENGN 0040: Dynamics and Vibrations, Spring 2014

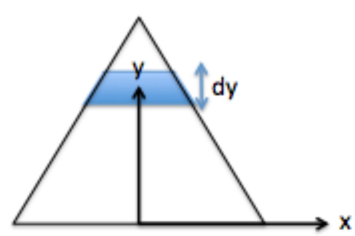
Homework 8: Rigid Body Kinematics
Due Friday April 25th

Problem 1: For the rigid body shown below,



1.1. Determine the mass moment of inertia about the x-axis. 1.2. Determine the mass moment of inertia about the y-axis.

Problem 2: For an equilateral triangle of uniform density ρ, length b and negligible width, derive the mass moment of inertia about the x-axis as shown below:

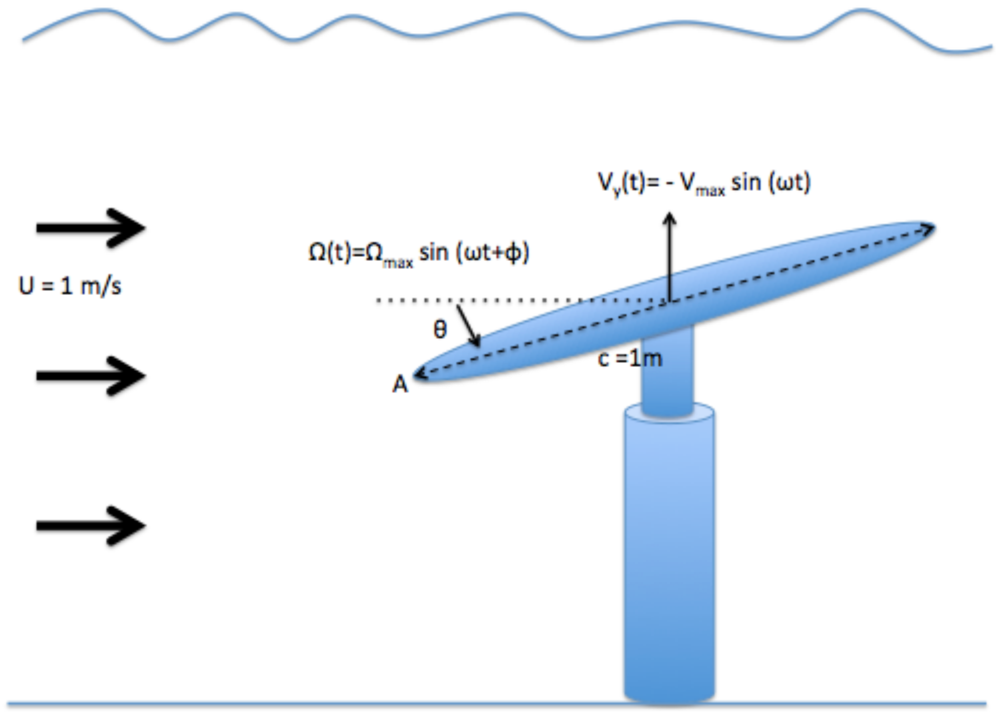


Problem 3: Tidal Energy Harvesting System A device to harvest kinetic energy from tidal flows is designed as a long wing, or hydrofoil, with an elliptical cross-section of aspect ratio 10, with a chord (leading edge to trailing edge) $c = 1m$ and span $b = 10m$ (into the page). At the center of the hydrofoil there is a motor/generator which can generate vertical and rotational motion. Thus, hydrofoil oscillates vertically in *heave* and oscillates about the center point in *pitch* as indicated in the diagram below. Also see movie here: `http://news.brown.edu/pressreleases/2014/03/waterwing`. As the hydrofoil oscillates in the incoming tide, indicated by the velocity vector $U = 1m/s$, there is a net Force and Moment applied to the hydrofoil. By optimizing the kinematic motion of the hydrofoil one can extract the kinetic energy from the oncoming flow and convert it into electrical power.

Given the kinematic parameters:

$$\begin{aligned} \Omega_{max} &= 1.23 rad/s \\ V_{max} &= 0.471 m/s \end{aligned}$$

$$\omega = 0.942rad/s$$
$$\phi = \pi/2$$



3.1. Plot the position of the leading edge, A, as a function of time.

3.2. What is the frontal area (perpendicular to the oncoming flow) that the hydrofoil sweeps during 1 complete stroke, Y_p? (Hint: Find the position vector of the leading edge, point A, and use its max and min value.)

3.3. What is the highest (linear) velocity magnitude of any point on the hydrofoil? Compare this to the highest velocity on a rotating turbine of radius 2.5 m and rotational speed of 40 RPM (see `http://www.verdantpower.com/kinetic-hydropower-system.html`.

3.4. In the absence of fluid forces/moments and frictional forces/moments, what is the max torque needed by the motor to generate the peak angular acceleration? Assume a uniform density of the hydrofoil of $\rho = 150kg/m^3$.

3.5. The class homework website has matlab data for the vertical force and moment about the CG for an elliptical hydrofoil for 1 complete cycle in an oncoming fluid flow of 1 m/s. Using this data, plot the mechanical power of the turbine as a function of time.

3.6. What is the average power over a cycle? If the total power available from the tidal flow is $1/2\rho U^3 Y_p$, $\rho = 1000kg/m^3$, then what is the efficiency of the hydrofoil?

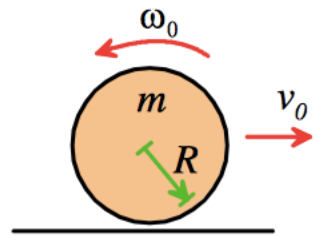
Problem 4: A thin uniform disk of radius R and mass m is placed on the ground with a positive velocity v_o in the horizontal direction, and a counterclockwise rotational velocity (a backspin) ω_o. The contact between the disk and the ground has a friction coefficient μ. The disk initially slips on the ground, and for suitable a range of values of ω_o and v_o its direction of motion may reverse.

4.1. Draw a FBD showing the forces on the disk just after it hits the ground.

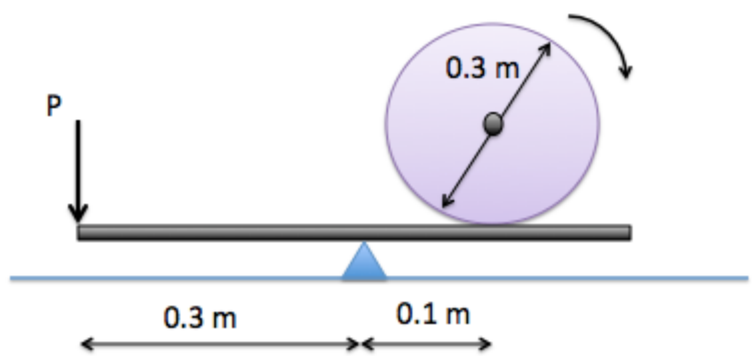
4.2. Find formulas for the initial acceleration a and angular acceleration α in terms of g, R, and μ. Note that the contact point is slipping.

4.3. At what time t will the disk reverse its direction?

4.4. Find a formula for the time at which the disk begins to roll without slip. Hence, show that the disk will reverse its direction only if $v_o < \omega_o R/2$.



Problem 5: A thin, circular brake drum of mass $m = 60kg$ and diameter $D = 0.3m$ is rotating on frictionless bearings at 125 RPM. The drum is brought to rest by the braking lever shown below. The coefficient of friction is 0.25.



5.1. Draw a FBD of the brake drum, and of the braking lever.
5.2. Determine the force, P, which will bring the drum to rest in 10 seconds.