EN40: Dynamics and Vibrations

Homework 1:
Mupad review and solving differential equation with MATLAB
Due 12:00 noon Friday January 30

School of Engineering
 Brown University

- Your solution to this homework should consist of two files:
  1. A commented MUPAD .mw file
  2. A commented MATLAB .m file
- Please submit the assignment electronically on the EN40 canvas website. You can log into canvas at http://brown.edu/it/canvas/ (the login link is near the top right of the page). Instructions for uploading are at https://sites.google.com/a/brown.edu/teaching-with-technology/canvas/student-help-site/assignments

Part 1: Use Mupad to solve the following math problems. Be sure to save your work frequently!

1. Use Mupad to solve the simultaneous equations below for x and y in terms of θ

$$x\sin\theta + y\cos\theta = a$$
$$x\cos\theta - y\sin\theta = b$$

2. Plot the function $P(x) = \dfrac{1}{x}\exp(-\log(x)^2)$ in the range $0 < x < 3$ ($P(x)$ is an example of a 'log-normal' probability distribution)

3. Find the value of x that will maximize $P(x) = \dfrac{1}{x}\exp(-\log(x)^2)$ and find the corresponding value of $P(x)$. To do this you will need to (i) Differentiate $P(x)$; (ii) Solve $df/dx = 0$ for x. (iii) Substitute this back into $P(x)$.

4. Compute the first 4 nonzero terms of the Taylor series expansion of $x/(e^x - 1)$ about $x=0$ (use the 'series' function). What is $\lim\limits_{x \to 0}\left[x/(e^x - 1)\right]$?   Compare the coefficients of the series expansion with $B_n/n!$, where B_n is the nth Bernoulli number. In Mupad you can compute the Bernoulli numbers using bernoulli(n)

5. Evaluate the following integral

$$\int_{-a}^{a}\left(\frac{1}{e^x - 1} - \frac{1}{x}\right)dx$$

Can you simplify the answer that Mupad gives by hand? Why do you think Mupad might choose not to simplify the answer?

6. Use Mupad to evaluate and simplify the cross product of the two vectors
$\mathbf{a} = [\sin\theta, \cos\theta, 0]$ $\mathbf{b} = [\cos\theta, -\sin\theta, 0]$

7. The differential equation

$$\frac{1}{\omega_n^2}\frac{d^2x}{dt^2} + x = \sin\omega t$$

is the governing equation for a 'forced harmonic oscillator.' As we shall discuss later in the course, it describes the behavior of an energy conserving system (which could be electrical or mechanical) that vibrates freely at frequency ω_n (radians/sec), and which is excited by an external force at frequency ω. Use MUPAD to solve the equation with initial conditions $x = 0$, $dx/dt = 0$ (simplify the solution, and use IgnoreSpecialCases). Plot the solution for a time interval of 60 sec, with $\omega_n = 1$, $\omega = 1.2$.

Part 2: Please solve the remaining problems using MATLAB (write your code in a matlab .m file). You should make your MATLAB (.m) file a function, so that when the file is executed, it will solve all the homework problems. For example:

```
function domyhomeworkplease
        Solutions to problems 8-11
        Functions for the differential equations in probs 10 and 11
end
function y = prob8function(r,x)
…
end
```
(You might find the solutions to homework 1, 2009-2014 helpful, if you get stuck – you could copy one of those homeworks and use it as a template, if you like.)

8. Write a function that takes as input two numbers *r,x*, and returns as output *y=rx(1-x)*. Test the function by using it to graph *y* as a function of *x* in the range *0<x<1* for *r=0.5*. To do this you will need to (i) Create a vector containing a suitable number (eg 100) of values of *x*; (ii) For each entry in your vector use the function to calculate the value of *y* and store the result in a second vector; and (iii) use the plot() function to graph the y vector against the x vector.

9. Use the function you wrote for Problem 9 to create a vector that contains the first 30 terms in the sequence $x_{n+1} = rx_n(1-x_n)$ starting with $x_1 = 0.5$ (this is called the 'logistic map' and is a discrete version of the Verhulst population growth model). Plot graphs (on the same plot) of x_n as a function of *n* for *r=1.75* and *r=3.6*.

10. The differential equation

$$\frac{dp}{dt} = L - sp + r\frac{p^q}{m^q + p^q}$$

was used by Carpenter, Ludwig and Brock to predict the concentration of phosphorus in a lake. Here *p* is the mass of phosphorus in the lake; *L* is the rate at which phosphorus is added from the watershed; the term *sp* (where *s* is a constant) accounts for phosphorus being absorbed by plants; and the third term in the equation (*m,q* and *r* are constants) accounts for phosphorus recycling. For a particular lake, the authors estimated the following values for the parameters in the model (page 762)
- $s = 0.817$, $r = 731000 kg/year$ $m = 116000$ kg $q = 7.88$

One goal of their paper is to understand how changing *L* will affect the phosphorus concentration. Write a function that calculates *dp/dt* given a value for time *t* and the value of *p* and use the ODE solver ode45 in MATLAB and your solution to 10.1 to calculate *p(t)* with the given values values for *s,r,m,q*, and initial conditions *p=75000* kg. Plot (on the same graph) the solutions for *L=38000kg/year* and 38500 kg/year, for time *0<t< 10* years.

11. The "Rossler attractor" is a system of differential equations of the form

$$\frac{dx}{dt} = -y - z \qquad \frac{dy}{dt} = x + ay \qquad \frac{dz}{dt} = b + z(x - c)$$

where x,y,z are functions of time to be determined, and a,b,c are three constants. The equations were originally intended to describe autocatalytic reactions in collections of biological cells but now are of more interest as a canonical and well characterized example of a system that exhibits chaos for suitable choices of a,b,c. Write a function that computes the vector of time derivatives $d\mathbf{w}/dt=[dx/dt;dy/dt;dz/dt]$ given a value of t and the current values of $\mathbf{w}=[x;y;z]$. Hence, use ode45 to calculate and plot the variation of (x,y,z) as a 3d trajectory (use the plot3 function) for parameters $a=b=0.1$, and $c=18$, (this choice gives a chaotic orbit), initial conditions $x=y=z=1$ and plot the solution for $0<t<300$.