School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 4: Work, Energy and Linear Momentum
Due Friday March 6th

1. The Rydberg potential is a simple model of atomic interactions. It specifies the potential energy of a bond between two atoms in the form

$$V = -D(1 + a\rho)\exp(-a\rho) \qquad \rho = (r / R) - 1$$

where r is the distance between atoms; R is the separation between atoms at minimum energy, and D and a are constants. The table below (from Molecular Physics, 106, 2008, p.753) gives values for D, a and R for several bonds.

Table 1. Continued.

Diatoms	D(eV)	R (Å)	a_1(Å)	a_2(Å^{-2})	a_3(Å^{-3})	a by Method 1	a by Method 2	a by Method 3
NN	9.905	1.0977	5.396	7.328	4.988	5.396	3.802738	4.006181
NO	6.614	1.1508	5.398	7.041	4.823	5.398	3.880258	4.0476
NP	6.443	1.4909	4.491	5.165	2.882	4.491	3.136731	3.322883
NS	4.875	1.4940	4.926	6.677	4.539	4.926	3.303252	3.546221
NSi	5.701	1.5718	3.732	2.975	1.460	3.732	2.824504	2.875488
OO	5.213	1.2075	6.080	11.477	11.003	6.08	3.743314	4.171992
OP	6.226	1.4759	4.275	4.399	2.717	4.275	3.078575	3.178364
OS	5.430	1.4811	4.748	6.504	5.228	4.748	3.087961	3.308623
OSi	8.337	1.5097	3.208	1.685	1.217	3.208	2.63083	2.517422
PP	5.081	1.8934	3.920	4.266	2.246	3.92	2.614269	2.821203
SiSi	3.242	2.2460	2.957	2.300	0.962	2.957	2.035645	2.151074
SS	4.414	1.8892	3.954	4.312	2.332	3.954	2.647662	2.846695
SSi	6.466	1.9293	2.773	1.462	0.647	2.773	2.183009	2.16445

1.1 Plot the variation of V with r/R for the Si-Si bond (compare the curves for the three values of a in the paper). Use eV for the units of energy, and $0.5<r/R<3$

1.2 Find a formula for the force of attraction between the atoms, in terms of D, a, R and ρ (if you use Mupad to do the derivative be careful not to use capital D as a variable – D means a derivative to mupad).

1.3 Hence, calculate a formula for the force required to break the bond (i.e. the maximum value of F)

1.4 Find a value for the strength of the Si-Si bond for the three possible choices of the value of a. You can use units of $eV / \overset{o}{\mathrm{A}}$ for the units of force – this is a funny unit but often used in atomistic calculations.

2. The Zero SR ZF12.5 electric motorcycle has the following specifications:

- Acceleration from 0 to 60mph in 3.3 sec.
- Curb weight of 188kg.
- Nominal Battery capacity (total energy stored in the battery): 11kWh (kilo-watt-hours)
- Maximum engine power 50kW
- Range at 55mph (89 km/hr) 151 km
- Height 56"; width 77"

Assume that air resistance can be calculated from the formula

$$F_D = cv^2$$

with c a constant.

2.1 Assuming that air resistance is the dominant contribution to energy consumption during steady cruise, use the given range and battery capacity to calculate c. (find the total work done against air drag in terms of c, and set this equal to the battery capacity).

2.2 Assume that the power produced by the electric motor is related to the vehicle's speed by $P = 4P_{\max}(1 - v/v_0)(v/v_0)$. Assuming that all the power developed by the motor is available to increase its kinetic energy during acceleration (neglect drag), show that the acceleration satisfies

$$a = \frac{4P_{\max}}{mv_0}(1 - v/v_0)$$

Hence calculate a formula for the speed as a function of time, and use the given power and time to reach 60mph to find a value for v_0 . Assume a rider weight of 70 kg. Use Mupad to solve the equation.

2.3 Use the solution to 2.1 and 2.2 to estimate the maximum possible speed of the motorcycle.

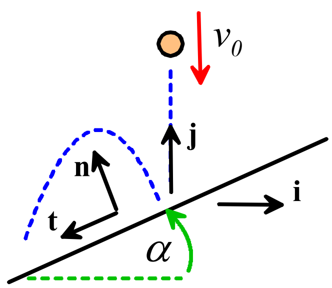
3. The Orion launch vehicle is placed in an initial orbit whose perigee (closest to earth's surface) and apogee (furthest) have altitudes 185 km and 888 km above the earth's surface, respectively. The velocity of the satellite at perigee is 7.989 km/s. Take the earth's radius as 6370 km and the gravitational constant $G = 6.67384 \times 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$.

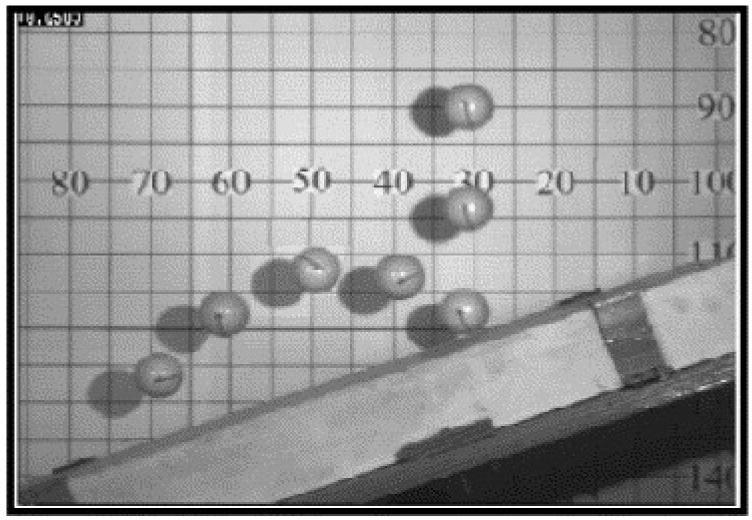
3.1 Calculate the speed of the vehicle at the apogee of this orbit

3.2 When the vehicle reaches its perigee a rocket burn increases the speed of the rocket (without affecting its altitude). This changes the orbit to -23km x 5800km which has a 4.74 km/s velocity at apogee. Calculate the speed of the vehicle just after the rocket burn.

3.3 Assuming that the vehicle has a constant mass of 50 000 kg, calculate the impulse exerted by the rocket burn

4. The figure shows a sequence of images recording the position of a spherical piece of rock as it bounces off an inclined plane. The image (from this paper) is part of an experiment to measure the restitution coefficient of collisions between rocks, which is of interest to geologists and civil engineers studying rock-slides). The scale is in cm, the rock has mass 204.33g and the time interval between frames is 0.04s.





4.1 Estimate the angle of the slope α

4.2 Suppose the specimen is dropped from some (unknown) initial height h at time t=0, and impacts the slope at time t_1 Write down a formula for the height of the specimen y above the impact point as a function of time, for $t < t_1$.

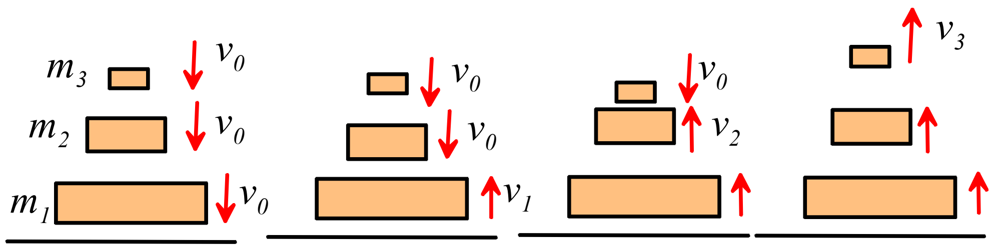
4.3 Assume that the first two images are taken at times $t = t_0$ and $t = t_0 + \Delta t$, where $\Delta t = 0.04\ s$. Use your solution to 4.2 and the figure to estimate t_0 . Hence use the height of the first image above the ramp to find h

4.4 Use energy conservation to determine the speed of the specimen just before it impacts the ramp.

4.5 Assume that the 5th frame gives the maximum height of the rebound. Use its position to estimate the horizontal and vertical components of velocity after the impact.

4.6 Calculate the normal and tangential components of impulse exerted on the specimen during the impact.

4.7. Calculate the restitution coefficient for normal impact



5. The figure shows an approximate model of an EN40 project 1 mass launcher with three masses. The bottom and top masses m_1, m_3 are fixed. The goal of this problem is to determine the value of m_2 that will maximize the launch velocity. Instead of considering the springs directly, we approximate the interaction between masses (and the lowest mass with the ground) as perfectly elastic (*e=1*) rigid body collisions. The collisions occur in sequence: first, mass 1 hits the ground; then collides with mass 2; and finally mass 2 collides with 3 to launch it. The stack is dropped with mass 1 a height *h* above the ground. The distance between the masses is very small compared to *h*.

5.1 Calculate the velocity of the masses just before the first mass hits the ground, in terms of *g* and *h*.

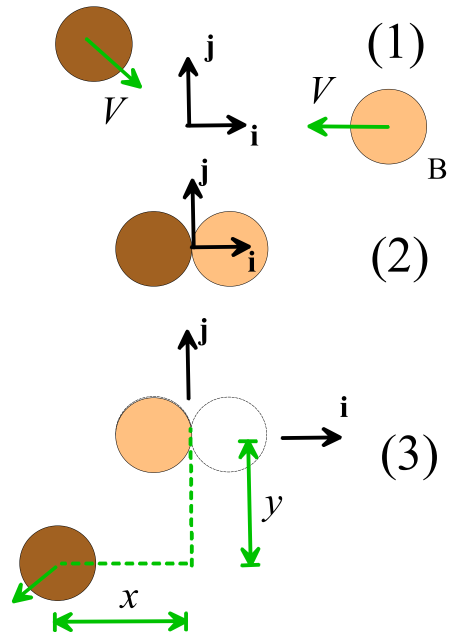
5.2 Write down the velocities of each mass after mass m1 has rebounded, but has not yet collided with mass *m2*.

5.3 Calculate the velocity of mass 2 after its collision with mass 1, but before its collision with mass 3, in terms of *g*, *h* and m_1, m_2

5.4 Calculate the velocity of mass 3 after its collision with mass 2, in terms of *g*, *h* and the masses.

5.5 Hence, find a formula for the value of mass m_2 that will maximize the launch velocity, in terms of m_1, m_3. Compare the values with the predictions of the MATLAB code from your project.

6. The figure shows a frictionless collision between two identical spheres with mass m and radius R. The restitution coefficient for the collision is e. The numbers (1), (2), (3) show the sequence of the pic tures - (1) is before impact; (2) is impact, and (3) is after impact. At the instant (1) particle A has velocity $\mathbf{v} = V(\mathbf{i}-\mathbf{j})/\sqrt{2}$ and particle B has velocity $\mathbf{v}_B = -V\mathbf{i}$



6.1 Write down the total linear momentum of the system before the impact, in {**i**,**j**} components.

6.2 Write down the total linear momentum of the system after impact, in terms of V

6.3 Explain why sphere B must continue to move parallel to the **i** direction after impact

6.4 Write down **j** component of velocity of sphere A after impact.

6.5 Use momentum conservation and the restitution formula parallel to the **i** direction to find the velocities of the two spheres after impact

6.6 Assuming that $e < (\sqrt{2}-1)/(\sqrt{2}+1)$, find the distances x,y at the instant (3) (at this instant particle B is located at the point occupied by particle A at the instant of collision).