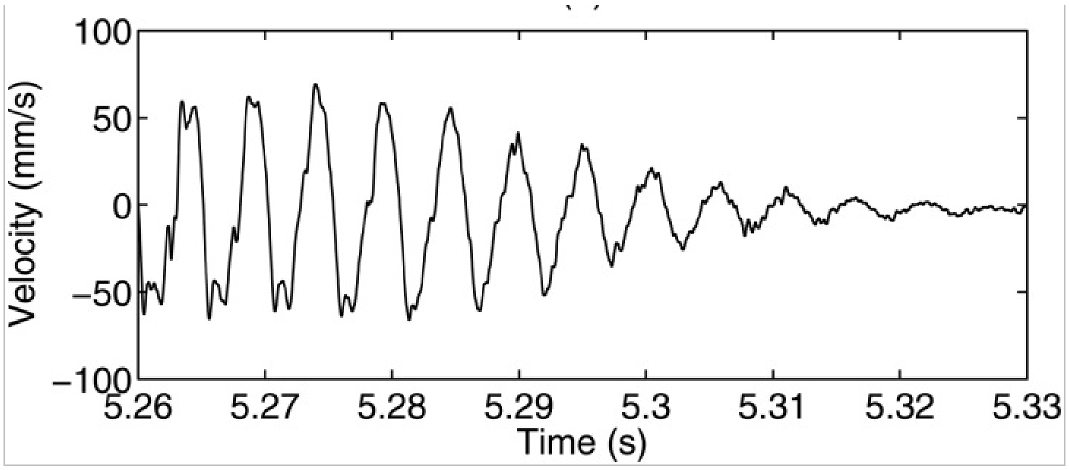
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 5: Free Vibrations - Solutions
Due Friday March 20th 2015
50 POINTS TOTAL



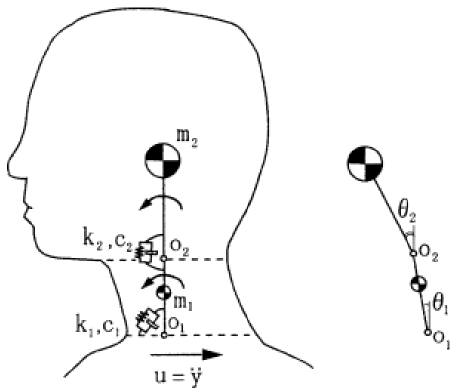
1. The figure shows a laser-vibrometry measurement of the velocity of a vocal-fold (from this paper). For the time interval 5.26s<t<5.28s, estimate

1.1 The amplitude and frequency of the vibration (give the frequency both in Hertz and in radians per second)

1.2 The amplitude of the displacement.

1.3 The amplitude of the acceleration.

2. State the number of degrees of freedom and the number of natural frequencies of vibration for each of the systems shown below



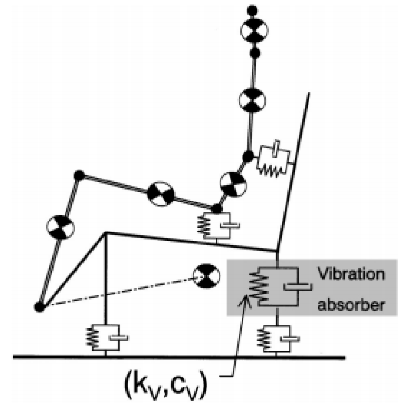


Fig. 6 **Simulation model with vibration absorber**

(a) Model of a human head/neck The figure on the right shows that the neck and head are idealized by two rigid bodies connected by pin joints.

(b) 2D Model of a patient on a wheelchair (The seat can only move vertically, and each link in the chain is a rigid body that is connected to its neighbors by a pin joint)

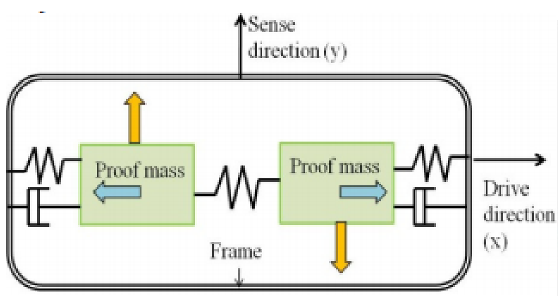
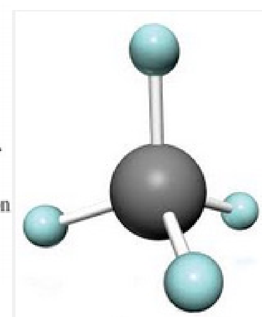


Figure 3: Two Proof Mass-Spring-Damper System



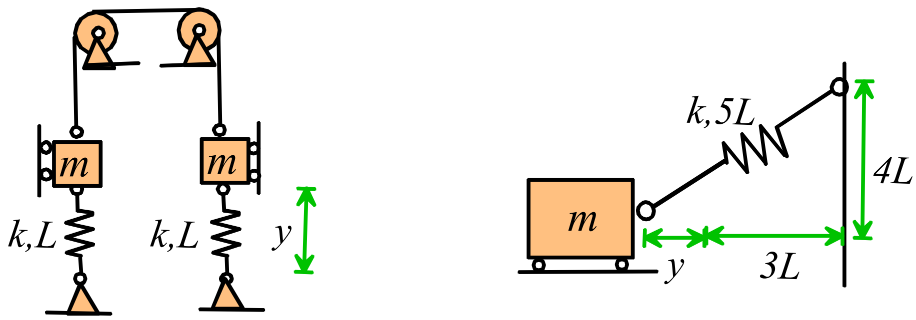
(c) Model of a MEMS gyroscope (the masses are particles, and move in the *x,y* plane)

(d) Methane molecule (the balls are particles, the rods are springs)

3. Solve the following differential equations (use the Solutions to Differential Equations)

3.1 $\frac{d^2y}{dt^2} + 4y = 0 \qquad y = 1 \quad \frac{dy}{dt} = 0 \quad t = 0$

3.2 $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 16y = 16\sin(4t) \qquad y = 0 \quad \frac{dy}{dt} = 0 \quad t = 0$

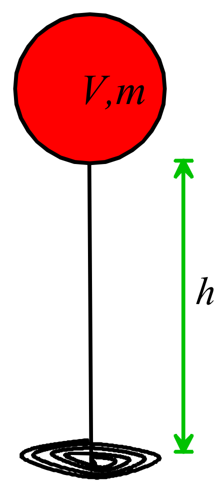


4. For the two conservative single-degree of freedom systems shown in the figure:

4.1 Derive the equation of motion (use energy methods, and include gravity. The pulleys and cable are massless). State whether the equation of motion is linear or nonlinear.

4.2 If appropriate, linearize the equation of motion for small amplitude vibrations (that means doing that Taylor series stuff discussed in class. "Linearizing" means replacing the nonlinear function of the variable with an approximate linear function)

4.3 Arrange the (linearized) equation of motion into standard form, and find an expression for the natural frequency of vibration.



5. A helium balloon with total mass m and volume V is supported by a tether with mass per unit length μ . Part of the cable is coiled on the ground.

5.1 Assuming vertical motion, write down the total potential and kinetic energy of the balloon and cable together as a function of its height h. (The buoyancy force acting on the balloon is $g\rho V$ where ρ is the mass density of air. Since the balloon floats $mg < \rho V g$. Assume that the air current surrounding a balloon moving with velocity $\mathbf{v}$ has kinetic energy $\rho V |\mathbf{v}|^2 / 4$ - this comes from a fluid mechanics calculation of flow past a moving sphere)

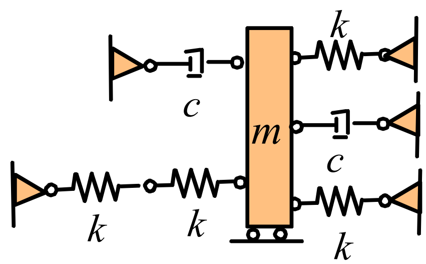
5.2 Hence, find the equation of motion for h

5.3 Find the value of h for which the balloon is in static equilibrium.

5.4 Assuming that the balloon is disturbed by a small distance δx from the equilibrium height found in

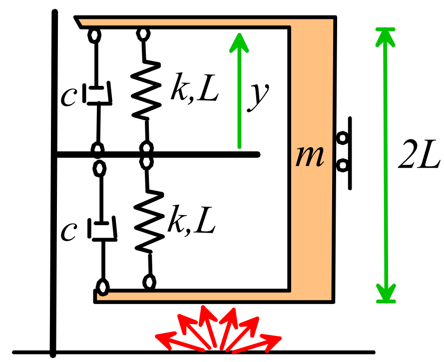
5.3, linearize the equation and hence find the natural frequency (to linearize you will need to assume that displacement, velocity and acceleration are all sufficiently small to neglect quadratic or higher order terms. Products of δx and acceleration can also be neglected).

6. Replace the system shown in the figure with an equivalent spring-mass system consisting of a mass with only one spring and dashpot. Hence, determine a formula for the undamped natural frequency and the damping factor for the system.



7. The figure (from this paper) shows an instrument that is designed to measure the vertical impulse *I* exerted by the explosion of a buried charge. A mass *m* is supported by springs and dampers attached to a rigid frame. The vertical displacement of the mass is measured after the explosion.

7.1 Derive the equation of motion for the length y (during the vibration after the explosion)



7.2 Calculate a formula for y when the system is in static equilibrium (before the charge is fired – assume the mass does not touch the ground).

7.3The system starts at rest with y at its equilibrium position (calculated in 7.2). The explosive charge exerts an impulse I on the mass m. The system is designed to be under-damped. Write down the velocity of the mass just after the explosion, and hence show that

$$y(t) = L - \frac{mg}{2k} + \frac{I}{\sqrt{2km(1-\zeta^2)}} \exp(-\zeta\omega_n t)\sin\omega_d t$$

and give formulas for $\omega_n, \zeta \ \omega_d$

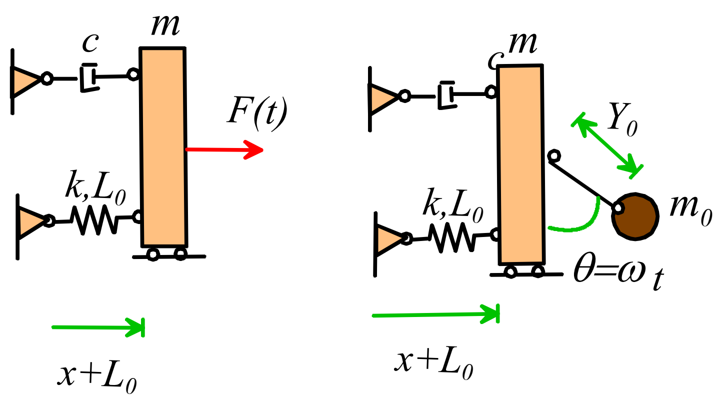
7.4 The actual design has the following parameters:

- k=20 MN/m
- c=50 kNs/m
- m=25000 kg

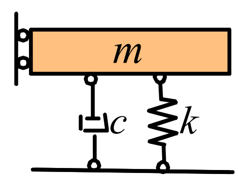
The maximum displacement of the frame (measured relative to its static equilibrium position) was found to be 4.63 cm. Find the values of ω_n, ζ for the instrument and hence determine the magnitude of the impulse.

8. Determine the steady-state amplitude of vibration for the spring-mass systems shown in the figure (you don't need to derive the equations of motion – these are standard textbook systems and you can just use the standard formulas). In each case the mass $m=20$kg, the stiffness $k=2000N/m$ $c=20Ns/m$.

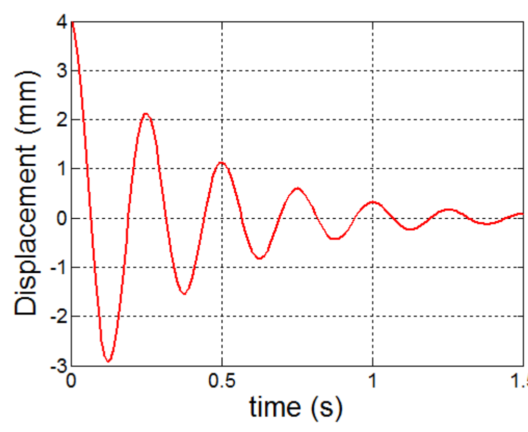
The force is $F(t) = 20\sin 20t$ N ;, the length of the rotor is 5cm; the eccentric mass $m_0 = 4kg$ and the angular velocity of the rotor is $\omega = 40$ rad/s.



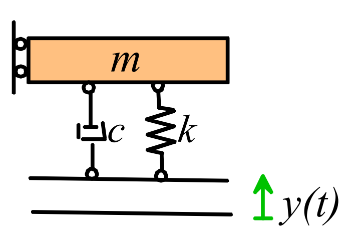
9. A vibration isolation platform can be idealized as a spring-mass-damper system as shown in the figure. In a free vibration test on the table, the base is held fixed and the platform is disturbed slightly from its equilibrium position. The subsequent displacement of the table is plotted in the figure below as a function of time.



9.1 Use the graph provided to estimate the period of oscillation and the log decrement. Hence, calculate the natural frequency ω_n and damping factor ζ that characterize the vibration isolation table.



9.2 The base of the platform is subjected to a harmonic displacement $y(t) = Y_0 \sin\omega t$ with amplitude 5mm and frequency $(25/\pi)$ Hz. Calculate the amplitude of vibration of the platform.



9.3 It is necessary to modify the vibration isolation system to further reduce the vibration amplitude by a factor of two. Recommend changes to the values of *k*,*m*, and/or *c* necessary to achieve this (e.g. recommend that *k* should be increased by some factor, *m* should be reduced by some factor, etc).

The figure shows the plot of magnification –v- frequency for a base-excited system - you can use this to find a graphical solution; or use the formulas for *M*.