HW 7 Solutions ENGN 0040 2015

(40 pts)

1.1. (3 pts)

$$I_{DISK} = 1/2\, m(L/4)^2$$

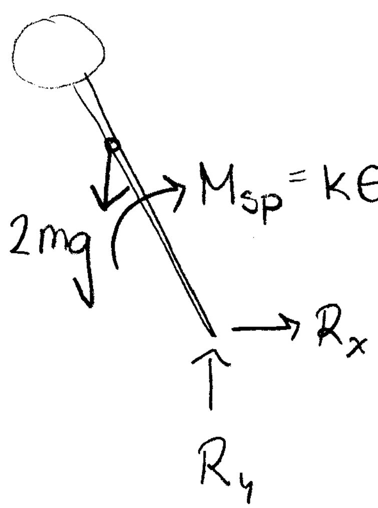
$$I_{DISK,A} = 1/2\, m(L/4)^2 + m(L + L/4)^2 = 51\, mL^2/32$$

$$I_{ROD} = 1/12\, mL^2$$

$$I_{ROD,A} = 1/12\, mL^2 + m(L/2)^2 = 1/3\, mL^2$$

$$I_A = I_{DISK} + I_{ROD} = 185/96\, mL^2$$

1.2. (4 pts)

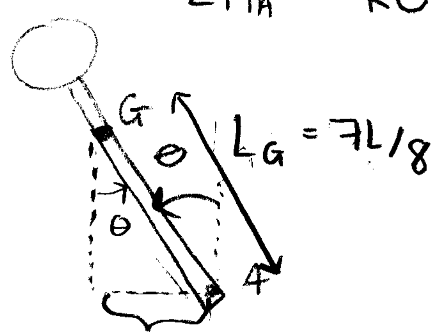


$$\Sigma F_x = R_x = m a_{Gx}$$

$$\Sigma F_y = R_y - mg = m a_{Gy}$$

$$\Sigma M_A = -M_{sp} + 2mgL_G \sin\theta = I_A \alpha$$

$$\Sigma M_A = -k\theta + 2mg\frac{7L}{8}\sin\theta = I_A\alpha$$



1.3. (3 pts)

$$-K\theta + 2mg\frac{7L}{8}\sin\theta = \frac{185}{96} mL^2 \cdot \ddot{\theta}$$

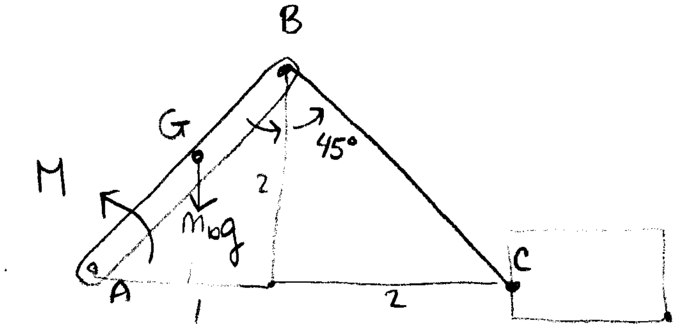
Simplifying & $\sin\theta \sim \theta$

$$-K\theta + 2mg\frac{7L}{8}\cdot\theta = \frac{185}{96} mL^2\, \ddot{\theta}$$

$$\frac{185}{96} mL^2\, \ddot{\theta} + \left(2mg\frac{7}{8}L\theta - K\theta\right) = 0$$

$$\omega_n \approx \sqrt{\frac{\frac{7mgL\theta}{4} - K\theta}{\left(185/96\, mL^2\right)}}$$

2.1 (7 pts)



geometry

$r_{B/A} = 1\hat{\imath} + 2\hat{\jmath}$

$r_{C/B} = 2\hat{\imath} - 2\hat{\jmath}$

Tension mag. T, direction $e_T = \frac{2\hat{\imath} - 2\hat{\jmath}}{\sqrt{8}}$

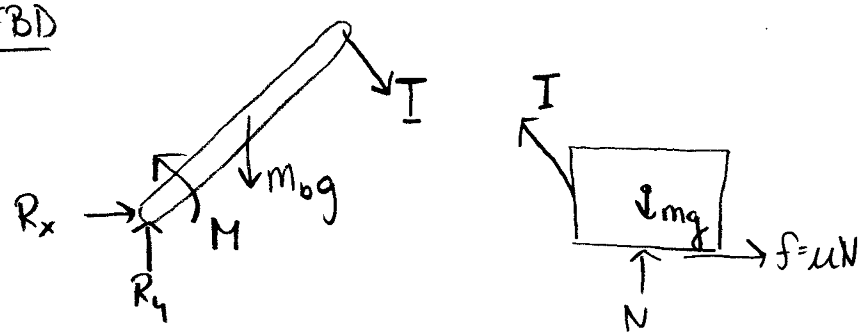
$e_T = \frac{\hat{\imath} - \hat{\jmath}}{\sqrt{2}}$

$\underline{T} = T e_T$

$I_A = I_G + m_b (L/2)^2, \quad L = \sqrt{5}$

$I_A = 1/3\, m_b L^2$

FBD



EOM

$$\boxed{\begin{aligned} R_x + \frac{T\sqrt{2}}{2} &= m_b a_{xG} \\ R_y - m_b g - \frac{T\sqrt{2}}{2} &= m_b a_{yG} \end{aligned}}$$

$$M \underbrace{- m_b g \frac{L}{2}\cos 63.4^\circ}_{\hat{k} \text{ moment caused by } m_b g} + \underbrace{(\underline{r}_{B/A} \times \underline{T})}_{\hat{k} \text{ moment caused by } \underline{T} = -3/\sqrt{2}\, T} = I_A \alpha_{AB}$$

$$\boxed{M - m_b g \frac{L}{2}\cos 63.4^\circ - \frac{3}{\sqrt{2}} T = I_A \alpha_{AB}} \quad (2)$$

EOM (crate)

$-mg + N + T\sqrt{2}/2 = 0$

$$\boxed{-T\sqrt{2}/2 = m_c a_{cx}} \quad (1)$$

Kinematics

$\underline{a}_B = \underline{\alpha}_{AB} \times \underline{r}_{B/A} = -2\alpha_{AB}\hat{\imath} + \alpha_{AB}\hat{\jmath}$

$\underline{a}_C = \underline{a}_B + \underline{\alpha}_{BC} \times \underline{r}_{C/B} = -2\alpha_{AB}\hat{\imath} + \alpha_{AB}\hat{\jmath} + 2\alpha_{BC}\hat{\imath} + 2\alpha_{BC}\hat{\jmath}$

constraint $a_{cy} = 0 \Rightarrow \alpha_{BC} = -1/2\, \alpha_{AB}$

$$\boxed{a_{cx} = -3\alpha_{AB}} \quad (3)$$

solving (1), (2) & (3)

$$\boxed{\begin{aligned} a_{cx} &= -2.26\ \text{m/s}^2 \\ \alpha_{AB} &= 0.75\ \text{rad/s}^2 \end{aligned}}$$

2.2. (3 pts)

$$\boxed{\begin{aligned} &\frac{T\sqrt{2}}{2} + N - m_c g = 0 \\ &-\frac{T\sqrt{2}}{2} + \mu N = m_c a_{cx} \end{aligned}}$$

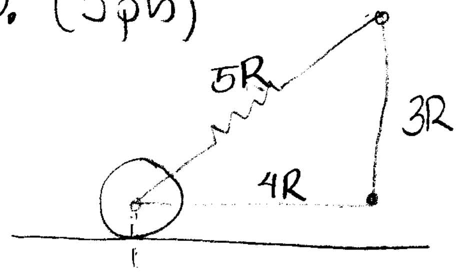
New eom for crate

$$N = m_c g - \frac{T\sqrt{2}}{2}$$

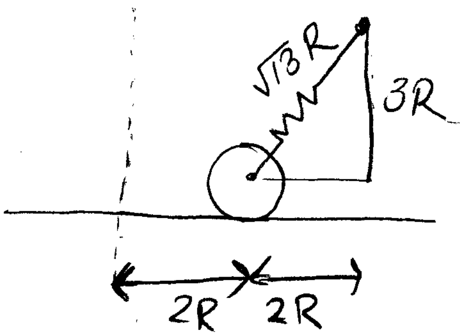
$$-\frac{T\sqrt{2}}{2} + \mu\left(m_c g - \frac{T\sqrt{2}}{2}\right) = m_c a_{cx} \Rightarrow$$ use instead of 3 & solve for a_x

$$\boxed{a_{cx} = -.836 \text{ m/s}^2}$$

3. (5 pts)



$t=0$: $V_i = (3R)^2 \frac{1}{2} K \qquad T_i = 0$



At time t: $V_f = (\sqrt{13}-2)^2 R^2 \, 1/2 K = 2.58 R^2 \frac{1}{2} K$

$T_f = 1/2 \, m V_G^2 + 1/2 \, I_G \omega^2$

Energy Balance: $9/2 \, R^2 K = \frac{2.58 R^2 K}{2} + \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2$

Kinematics: $V_G = -R\omega$, $I_G = 1/2 \, m R^2$

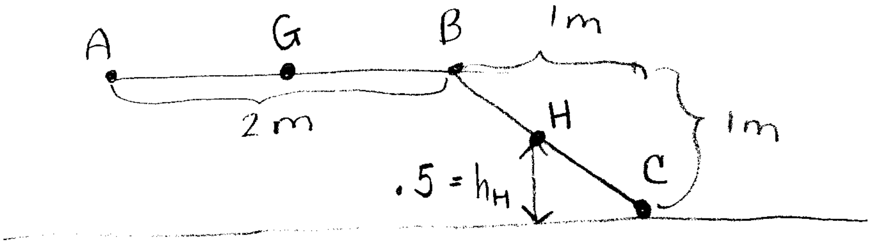
$9/2 \, R^2 K - 2.58/2 \, R^2 K = 1/2 \, m(-R\omega)^2 + 1/2 (1/2 \, m R^2) \omega^2$

$6.42 K = + m\omega^2 + 1/2 \, m\omega^2$

$6.42 K \cdot \frac{2}{3} = m\omega^2 \Rightarrow \omega^2 = 4.28 \, K/m$

$\boxed{2.07 \sqrt{\frac{K}{m}} = \omega}$

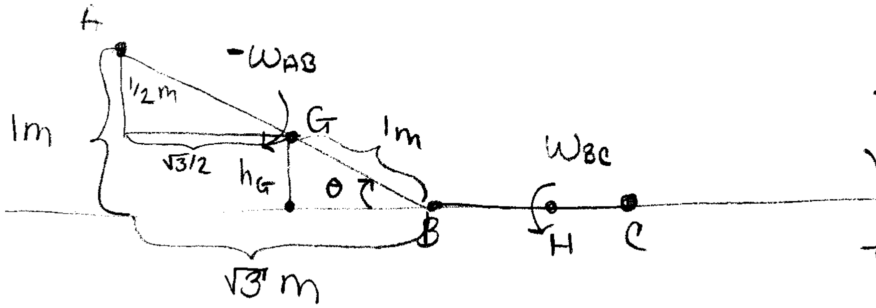
4. (10 pts)



$t=0$:

$V_i = m_{AB}\, g\, h_G + m_{BC}\, g\, h_H$

$V_i = 5g + 3/2\, g = \frac{13}{2} g$



$\theta = 30°$, $h_G = \sin 30° = 1/2$

t: (when B hits floor)

$V_f = m_{AB}\, g\, h_G + 0 = 5/2\, g$

$T_f = \frac{1}{2} m_{AB} V_G^2 + \frac{1}{2} I_G \omega_{AB}^2 + \frac{1}{2} m_{BC} V_H^2 + 1/2\, I_H \omega_{BC}^2$

Kinematics

$\underline{V}_A = 0$

$\underline{V}_B = \cancel{\underline{V}_A}^{0} + (\underline{\omega}_{AB} \times \underline{r}_{B/A}) = \omega_{AB}\hat{i} + \sqrt{3}\,\omega_{AB}\hat{j}$

$\underline{r}_{B/A} = (+\sqrt{3}\,\hat{i} - 1\hat{j})\,m$

$\underline{V}_C = \underline{V}_B + (\underline{\omega}_{BC} \times \underline{r}_{C/B}) = \omega_{AB}\hat{i} + (\sqrt{3}\,\omega_{AB} + \sqrt{2}\,\omega_{BC})\hat{j}$

$\underline{r}_{C/B} = \sqrt{2}\,\hat{i}$

Constraint: $V_{Cy} = 0 \Rightarrow \sqrt{3}\,\omega_{AB} + \sqrt{2}\,\omega_{BC} = 0 \Rightarrow \boxed{\omega_{BC} = -\sqrt{3}/\sqrt{2}\;\omega_{AB}}$

$\underline{V}_H = \underline{V}_B + (\underline{\omega}_{BC} \times \underline{r}_{H/B}) = \omega_{AB}\hat{i} + (\sqrt{3}\,\omega_{AB} + \sqrt{2}/2\;\omega_{BC})\hat{j}$

$\underline{r}_{H/B} = \sqrt{2}/2\;\hat{i}$

$\underline{V}_H = \omega_{AB}\hat{i} + \frac{\sqrt{3}}{2}\omega_{AB}\hat{j}$

$\underline{V}_G = \cancel{\underline{V}_A}^{0} + \underline{\omega}_{AB} \times \underline{r}_{G/A} = .5\,\omega_{AB}\hat{i} + \frac{\sqrt{3}}{2}\omega_{AB}\hat{j}$

$$|V_H|^2 = \left(\omega_{AB}^2 + 3/4\,\omega_{AB}^2\right) = 7/4\,\omega_{AB}^2.$$

$$|V_G|^2 = \left(\tfrac{1}{2}\omega_{AB}\right)^2 + \left(\sqrt{3}/2\right)\omega_{AB}^2 = \omega_{AB}^2$$

substituting back into Energy

$$I_G = 1/12\, m_{AB} L_{AB}^2 = \frac{1}{12} m_{AB} 2^2 = \frac{1}{3} m_{AB}$$

$$I_H = 1/12\, m_{BC} L_{BC}^2 = \frac{1}{12} m_{BC} \cdot 2 = \frac{1}{6} m_{BC}$$

$$T_f = \frac{1}{2} m_{AB}\omega_{AB}^2 + \frac{1}{2}\left(\frac{1}{3} m_{AB}\right)\omega_{AB}^2 + \frac{1}{2} m_{BC}\frac{7}{4}\omega_{AB}^2 + \frac{1}{2}\left(\frac{1}{6} m_{BC}\right)\frac{3}{2}\omega_{AB}^2$$

$$T_f = \frac{1}{2}\omega_{AB}^2\left(m_{AB} + 1/3\, m_{AB} + 7/4\, m_{BC} + 1/4\, m_{BC}\right) = 19/3\,\omega_{AB}^2$$

$$V_i = V_f + T_f$$

$$\frac{13}{2}g = 5/2\, g + 19/3\,\omega_{AB}^2$$

$$\omega_{AB}^2 = 4g\frac{3}{19} = 6.19$$

$$\boxed{\begin{aligned}\omega_{AB} &= -2.5 \text{ rad/s}\\ \omega_{BC} &= 3.0 \text{ rad/s}\end{aligned}}$$

5. (5pts)

$$W = \cancel{T_f}^{\,0} + \cancel{V_f}^{\,0} - T_i - \cancel{V_i}^{\,0}$$

$$W = 1/2\, I_G\, \omega^2 = 1/2\left(1/2 \cdot 6 \cdot (.05)^2\right) \cdot 10^2$$

$$\boxed{W = 0.375\text{ J}}$$