



EN40: Dynamics and Vibrations

Homework 1:

Mupad and MATLAB practice

Due 12:00 noon Friday February 5 (online submission)

School of Engineering
Brown University

- Your solution to this homework should consist of two files:
 - A commented MUPAD .mw file
 - A commented MATLAB .m file
- Please submit the assignment electronically on the EN40 canvas website. You can log into canvas at <http://brown.edu/it/canvas/> (the login link is near the top right of the page). Instructions for uploading are at <https://sites.google.com/a/brown.edu/teaching-with-technology/canvas/student-help-site/assignments>

Part 1: Use Mupad to solve the following math problems. Be sure to save your work frequently!

1. Use Mupad to find all the solutions to the simultaneous equations

$$x^4 + y = 1 \quad x + y = 1$$

(Use IgnoreSpecialCases to simplify the results).

2. Plot the function $P(t, \nu) = \left(1 + \frac{t^2}{\nu}\right)^{-(1+\nu)/2}$ in the range $-3 < t < 3$, for $\nu = 1$ and $\nu = 11$ (on the same plot). ($P(t)$ is the 'student' distribution function). It is helpful to create a function of two variables as

$$P := (t, x) \rightarrow (1 + t^2/x)^{-(x+1)/2}$$

You can then use `plot({P(t, 1), P(t, 11)}, t=-3..3)` (or one of the other Mupad plot functions)

3. Compute the first 2 nonzero terms of the Taylor series expansion of $P(t)$ about $t=0$, in terms of ν .

4. Evaluate the integral

$$\int_{-\infty}^{\infty} P(t) dt$$

'infinity' is mupad's convention for ∞ . To simplify the answer, insert the line

`assume(nu>0)` :

just before you do the integral. The answer will contain the 'Gamma' function – google will tell you what this is.

5. Use Mupad to calculate the maximum value of the slope dP/dt . This will require several steps: (1) Calculate the slope $S = dP/dt$; (2) Solve the equation $dS/dt = 0$ to find the value(s) of t that maximize S ; and finally (3) substitute one of the values of t into the formula for S .

6. The differential equation

$$\frac{dv}{dt} = g - \frac{c}{m}v$$

Describes the speed of an object in free fall through the earth's atmosphere (g is the acceleration of gravity, c is the air drag coefficient, and m is the object's mass). Use MUPAD to solve the equation with initial condition $v=0$ (use IgnoreSpecialCases, and use 'simplify' to simplify the result).

Part 2: Please solve the remaining problems using MATLAB (write your code in a matlab .m file). You should make your MATLAB (.m) file a function, so that when the file is executed, it will solve all the homework problems. For example:

```
function domyhomeworkplease
    Solutions to problems 8-11
    Functions for the differential equations in probs 13 and 14
end
function z = filter(y,alpha)
... Code
end
```

(You might find the solutions to homework 1, 2009-2015 helpful, if you get stuck – you could copy one of those homeworks and use it as a template, if you like.)

7. Create a vector x of equally spaced points between 0 and 8192, with a spacing of 0.5.

8. Create five vectors y_1 , y_2 , y_3 , y_4 and y_5 , which contain elements

$$y_i = \sin(4\pi n x_i / 8192)$$

where x_i are the elements of the vector created in problem 7, and $n=440$ for y_1 ; $n=550$ for y_2 ; $n=660$ for y_3 $n=880$ for y_4 , and $n=1760$ for y_5 . To see what your first four vectors sound like, type

```
sound(y1)
sound(y2)
sound(y3)
sound(y4)
pause(2)
```

into your MATLAB script just below the lines where you created the four vectors (make sure your computer has the sound turned on, or you have a headset plugged in). To understand what you just did, note that MATLAB plays back a vector by sampling its elements in turn, at a rate of 8192 samples per second. For $n=440$, your vector therefore becomes a sin wave with frequency 440 cycles per second – the standard frequency for middle ‘A.’ The other frequencies of 550, 660 and 880 produce a third; a fifth and the octave above middle A, and produce an approximate chord¹. The numbers were chosen to make the notes last for 2 seconds, hence the 2 second pause.

9. Plot an x-y graph of the first 200 points of y_1+y_5 (on the vertical axis) as a function of $x/16.384$ (on the horizontal axis – the scale factor is to convert the scale to milliseconds). Also, play the sound of the vector y_1+y_5 (it sounds bad – after you have tried it you might want to comment out this line!)

10. You can (sometimes!) remove noise from a signal using filters. In this problem you will write MATLAB function that acts as a simple low-pass digital filter. Let y be a vector (a list of numbers) that needs to be filtered. A vector z containing the filtered signal (a second list of numbers) can be constructed as follows:

$$z_1 = y_1$$

$$z_i = \alpha y_i + (1 - \alpha) z_{i-1} \quad i = 2, 3, 4, \dots, n$$

where $0 < \alpha < 1$ is a parameter that controls the cutoff frequency of the filter, and y_i , z_i denotes the i th value of y and z , respectively, and n is the length of the vector y .

¹ The frequencies are those of the mathematical harmonic series; the tempered scale (which is used to tune instruments with a fixed pitch, such as a piano) uses a slightly higher frequency for the C# and a slightly lower one for the E, and people playing instruments with a variable pitch (or singing) usually adjust the pitch of the notes according to the chord being played rather than using a fixed frequency for each note.

Write a MATLAB function of the form

```
function z = filter(y, alpha)
    ...code...
end
```

which will take as input a vector \mathbf{y} and will return the filtered vector \mathbf{z} . (you can do the calculation using a loop, for example). Use your function to filter the vector $\mathbf{y1+y5}$ with $\alpha = 0.2$ (in other words use the vector $\mathbf{y1+y5}$ as input, and produce a filtered vector \mathbf{z}). Plot the first two hundred elements of \mathbf{z} against those of $\mathbf{x}/16.384$ on the same graph as problem 10 (use the MATLAB 'hold on' command). Also play the sound of \mathbf{z} . Note that your ear can still pick up the two frequencies in the signal – the human ear and brain are very good at Fourier transforms!)

11, Optional (no credit but mildly entertaining). Write a MATLAB function that takes as input a one-letter character string, and will play the corresponding note of a scale (eg `sing('C')` could play middle C – a Google search will give you the frequencies of the notes). For a more sophisticated version you could add extra arguments to specify sharps and flats and the octave, as well as the length and amplitude of the note. Make your MATLAB script play a happy tune that will put the graders in a good mood.

12. By using a loop and a conditional statement, construct a vector \mathbf{r} that contains values of the function

$$y = \begin{cases} \sin(880\pi x / 8192) & \sin(880\pi x / 8192) > 0 \\ 0 & \sin(880\pi x / 8192) < 0 \end{cases}$$

for each value of x in the vector created in Problem 7. In addition, use the function you wrote for Problem 10 to create a vector \mathbf{w} that contains values of the vector created by filtering \mathbf{r} with a value of $\alpha = 0.01$. Plot the first 800 points of \mathbf{r} and \mathbf{w} as a function of $\mathbf{x}/16.384$.

13. The differential equation

$$\frac{ds}{dt} = Ka(t)(1 - s(t) / S_{\max}) - Ds(t)$$

is called the [Vidale and Wolfe](#) model to predict the effects of advertising on sales. The variables $s(t)$ and $a(t)$ represent the rate of revenue accumulation from sales, and the rate of spending on advertising, respectively (in \$ per month, e.g.). K , S and D are constants².

Suppose that a new product is introduced, and spending on advertising will be a constant ($a(t)=A$). Write a matlab function that will compute $[ds/dt]$ given values for (t,s) . Hence, use the `ode45` function to calculate a numerical solution for $s(t)$. Use the following values for parameters:

- $K= 1 \text{ month}^{-1}$, $S_{\max} = 200 \text{ 000\$ / month}$ $D=0.3 \text{ months}^{-1}$.
- Assume $s=0$ at time $t=0$,
- Plot solutions for $A=10 \text{ 000\$ / month}$, $A=20000 \text{ \$ / month}$ and $A=60000 \text{ \$ / month}$

² (D controls how quickly sales drop if the product is not advertised; K controls the rate at which sales increase when advertised; and S determines the maximum possible income from sales if an infinite amount is spent on advertising).

14. In this problem you will solve the equations that describe cyclic biochemical reactions in a simple genetically engineered system that make E-coli bacteria repeatedly fluoresce green, then fade again. The details can be found in [this publication](#). The system consists of three repressor-proteins, which have time dependent concentrations $p_1(t), p_2(t), p_3(t)$ (each p represents the number of proteins per bacterium); together with the concentrations $m_1(t), m_2(t), m_3(t)$ of messenger RNA corresponding to each protein. The concentrations are governed by the following six differential equations:

$$\begin{aligned} \frac{dm_1}{dt} &= \frac{1}{T} \left[\frac{\alpha}{(1+p_3^2)} - m_1 \right] & \frac{dm_2}{dt} &= \frac{1}{T} \left[\frac{\alpha}{(1+p_1^2)} - m_2 \right] & \frac{dm_3}{dt} &= \frac{1}{T} \left[\frac{\alpha}{(1+p_2^2)} - m_3 \right] \\ \frac{dp_1}{dt} &= -\beta(p_1 - m_1) & \frac{dp_2}{dt} &= -\beta(p_2 - m_2) & \frac{dp_3}{dt} &= -\beta(p_3 - m_3) \end{aligned}$$

where α, β and T are three constants. Write a function that computes the vector of time derivatives $d\mathbf{w} / dt = [dm_1 / dt; dp_1 / dt; dm_2 / dt; dp_2 / dt; dm_3 / dt; dp_3 / dt]$ given a value of t and the current values of $\mathbf{w} = [m_1; p_1; m_2; p_2; m_3; p_3]$.

Hence, use the ode45 function in MATLAB to integrate (approximately) the system of equations with respect to time, and plot a graph of $p_1(t), p_2(t), p_3(t)$ and $m_1(t), m_2(t), m_3(t)$ as a function of time (you can plot both variables on the same graphs if you like, but it is easier to see them if they are plotted on two graphs. Use the following values for parameters: $T = 5 \text{ sec}$, $\alpha = 4000$, $\beta = 0.1$. Use initial conditions $\mathbf{w} = [m_1; p_1; m_2; p_2; m_3; p_3] = [10, 15, 20, 25, 30, 30]$ at time $t=0$. Run the simulation for 800 sec.

As shown in the publication, the reactions give rise to steady oscillations in the concentration of each protein.