

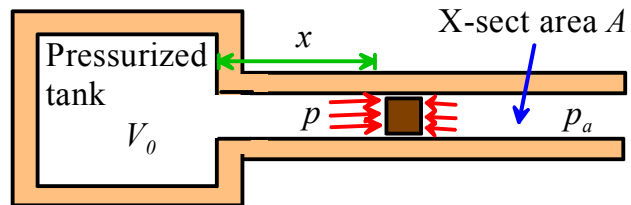


School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 2: Kinematics and Dynamics of Particles Due Friday Feb 12, 2016

1. The figure illustrates an idealized model of a gas gun (used, e.g. in Professor Clifton's [high-stain rate impact experiments](#) at Brown)). The goal of this problem is to derive an approximate formula for the exit velocity of the projectile.



The projectile is propelled by a pressurized gas chamber. If the gas expands isothermally the pressure p in the tank (together with the portion of the barrel behind the projectile) is related to its volume V by Boyle's law $pV = \text{constant}$. The external air has pressure p_a . Friction can be neglected.

1.1. Show that the force acting on the projectile is related to its position by

$$F = \frac{Ap_0V_0}{V_0 + Ax} - p_aA$$

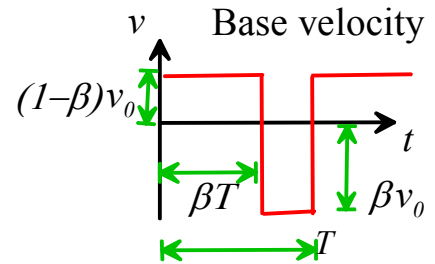
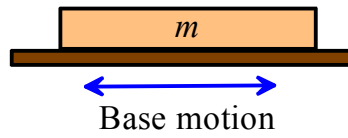
where p_0 is the initial pressure in the tank when the projectile is at position $x=0$ and V_0 is the volume of the tank (the gas volume when $x=0$)

1.2 Hence, show that the velocity of the projectile is

$$v = \sqrt{[2p_0V_0 \log(1 + Ax/V_0) - 2p_aAx] / m}$$

(you will need to use $a = vdv/dx$ and separate variables to do the integral. Be careful when substituting the limits of integration)

2. Piezoelectric inertia drives are used as [high-precision actuators](#) and as a propulsion mechanism in [micro-robots](#). They are based on the physics that makes the ‘table cloth’ trick demonstrated in class possible.



The figure illustrates a simple idealization that can be used to illustrate how an inertia drive works. It consists of a mass m on a vibrating base (the mass represents the platform and payload, while the base is the piezoelectric actuator). The contact between the mass and the base has friction coefficient μ . The base oscillates left and right with a velocity

$$v(t) = \begin{cases} v_0(1-\beta) & 0 < t < \beta T \\ -v_0\beta & \beta T < t < T \end{cases}$$

where v_0 and $0.5 < \beta < 1$ are constants, and T is the time for one cycle. The base moves faster while moving to the left than while it is moving to the right. The slip/stick behavior at the contact will cause the mass to gradually move to the right (you can demonstrate this by putting something on a piece of paper and pulling the paper back-and-forth). The goal of this problem is to derive an expression for the average velocity of the mass in terms of v_0, β . To keep things simple, assume steady-state conditions, so the platform moves at the same speed at time $t=0$ and time T . Our goal is to find formulas for the speed of the mass between $0 < t < T$, using Newton’s laws of motion and straight-line motion formulas.

2.1 Start by considering the motion of the platform and mass during the time interval $\beta T < t < T$. During this phase of the cycle the mass is moving to the right, and the base is moving left, so slip must occur at the contact between them. Draw a free body diagram showing the forces acting on the mass during this phase of the motion, and use Newton’s law to write down a formula for its acceleration, in terms of μ and g .

2.2 Assume that that at time $t = \beta T$ the mass has the same velocity as the base, i.e. $v = v_0(1 - \beta)$. Hence, use the result of (2.1) to calculate the speed of the mass at time T .

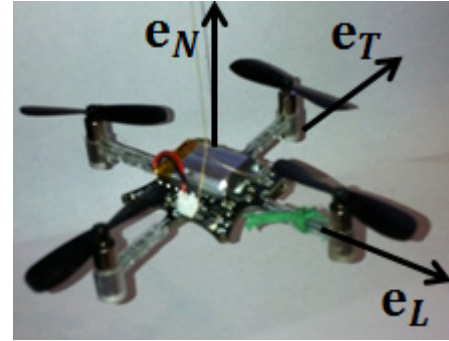
2.3 Next, consider the motion of the platform and mass during the time interval $0 < t < \beta T$. During this phase of the motion, both the mass and the base move to the right. At time $t=0$ the mass is moving more slowly than the base, so slip must occur at the contact between them (slip will stop when the mass reaches the same speed as the base). Draw a free body diagram showing the forces acting on the mass during the period of slip, and use Newton’s law to write down a formula for its acceleration.

2.4. Using the results of 2.2 and 2.3, calculate how long it takes the mass to reach the same speed as the base (recall that we assume that during steady-state slip-stick the speed of the mass at time $t=0$ and $t=T$ are equal, so you can find the speed at time $t=0$ using the solution to part 2.2).

2.5 Hence, sketch a graph showing the speed of the mass as a function of time. Use the graph to deduce the distance traveled by the mass during one cycle, and hence find a formula for its average speed.

3. The homework website provides a datafile that contains measurements of the position and acceleration of a quadcopter during flight.

The seven columns in the file specify the time, three components of position (x,y,z) (measured by the Kinect sensor) and the accelerations (a_x, a_y, a_z) determined from the accelerometers and gyros on board the quadcopter. The coordinate system is such that x is the perpendicular distance of the quadcopter from the Kinect sensor, z is vertically upwards, and y is lateral. The data is in SI units (m and s).



Write a MATLAB code that will do the following:

- (i) Read the datafile using the MATLAB `data=csvread('filename')` command. Note that the variable data will be a matrix: the first column of the matrix will be time values, the second column the x component of position, and so on.
- (ii) Estimate the velocity by integrating the accelerations using the MATLAB 'cumtrapz' function, and plot the three velocity components as a function of time. Note that the accelerometers sense gravity, in addition to acceleration. You will need to subtract the gravitational acceleration (9.8m/s^2) from a_z .
- (iii) Estimate the velocity by differentiating the position vector, and plot this estimate as a function of time. You can do this by calculating the change in position between two successive readings, and dividing by the time difference between them, e.g. if $x(i)$ denotes the i th value of x , then

$$v_x(i) = (x(i) - x(i-1)) / (t(i) - t(i-1))$$

You can construct the vector v_x using a loop in MATLAB, e.g. to find the x component of velocity

```
vx=zeros(length(time),1)
for i=2:length(time)
    vx(i) = (x(i)-x(i-1))/(time(i)-time(i-1));
end
```

Here x is a vector containing the x component of position.

- (iv) Why do the two estimates of velocity differ? Which is likely to be more accurate?

Please submit your MATLAB code on Canvas. Please include a separate printed copy of your graphs with your homework

4. The motion of a particle is described using polar coordinates as $r = 1 + t - (\pi/4)$ $\theta = t$ where t is time. The figure shows the resulting trajectory. At the instant when $\theta = \pi/4$ calculate:

4.1 The position vector of the particle in the polar coordinate basis

4.2 The position vector of the particle in the \mathbf{i}, \mathbf{j} basis

4.3 The velocity vector of the particle in the polar coordinate basis

4.4 The velocity vector of the particle in the \mathbf{i}, \mathbf{j} basis

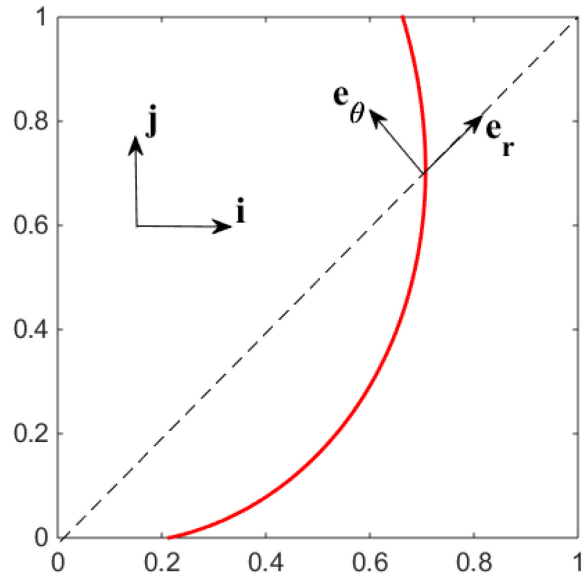
4.5 A unit vector parallel to the direction of motion of the particle in the \mathbf{i}, \mathbf{j} basis

4.6 A unit vector in the perpendicular to the direction of the motion of the particle, in \mathbf{i}, \mathbf{j} basis

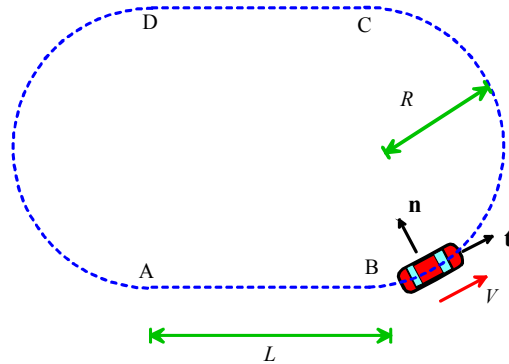
4.7 The velocity vector in normal-tangential coordinates

4.8 The acceleration vector in polar coordinates

4.9 The acceleration vector in \mathbf{n}, \mathbf{t} coordinates (this is quite tricky – if you can work this out from 4.8 and 4.5 or 4.3 without a lot of calculus you really understand vectors and the meaning of acceleration and velocity components!)



5. A race-car travels around the track shown in the figure. The magnitude of the acceleration of the vehicle cannot exceed μg (μ is the friction coefficient and g is the gravitational acceleration – if $|\mathbf{a}| > \mu g$ the car will skid). The goal of this problem is to find a formula for the shortest possible time for the vehicle to complete a lap around the course, in terms of μ, g, R, L .

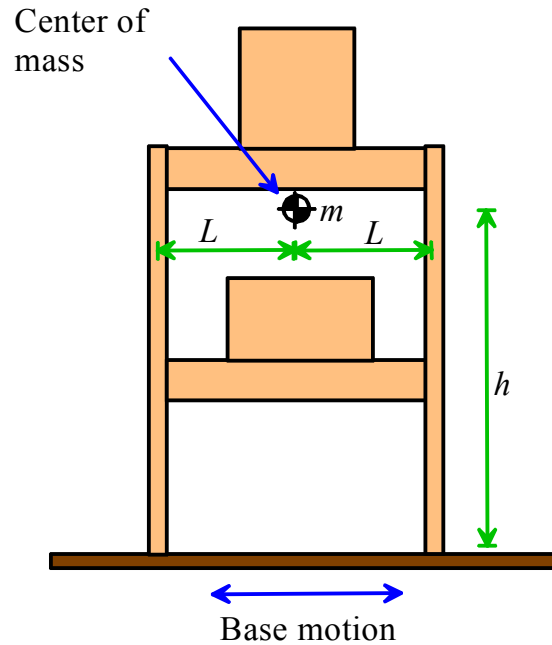


5.1 Assume the vehicle travels at constant speed from B to C, and hence write down a formula for the acceleration vector in \mathbf{n}, \mathbf{t} coordinates in terms of speed and R . Hence find a formula for the shortest time to travel from B to C.

5.2 Assume that while traveling from C to D the car first speeds up with the maximum possible acceleration and then slows down again (with maximum deceleration). Using 5.1, find a formula for the time to travel from C to D.

5.3 Hence, calculate the total time for one lap of the course.

6. The figure shows a shelving rack during an earthquake. The unit has width $2L$ and the center of mass of the shelf and its contents is a height h above the ground. Assume that the ground vibrates with a horizontal motion $x(t) = X_0 \sin 2\pi t / T$ where X_0 is the amplitude and T is the time for one cycle. The goal of this problem is to find a formula for the amplitude at which the rack will tip over.



6.1 Draw a free body diagram showing the forces acting on the shelf and its contents together.

6.2 Use Newton's law $\mathbf{F} = m\mathbf{a}$ and the equation of rotational motion $\mathbf{M} = \mathbf{0}$ to calculate formulas for the vertical reaction forces at the base. Assume there is enough friction to prevent slip at the contacts

6.3 Hence, find a formula for the critical value of X_0 that will cause the shelf to start to tip.

(Designing earthquake resistant storage racks is a non-trivial process [as this detailed report](#) shows)