

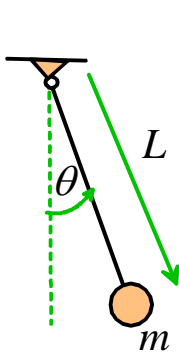


School of Engineering
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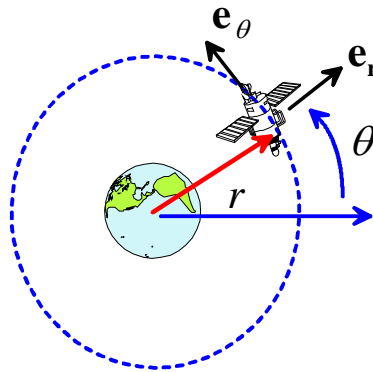
EN40: Dynamics and Vibrations Homework 4: Work, Energy and Linear Momentum Due Friday March 4th

Note that linear momentum and collisions will be covered in class on Tues Feb 1

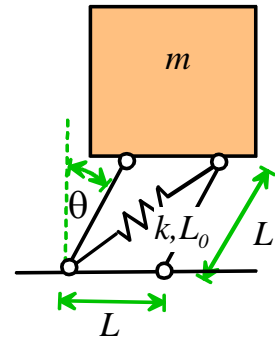
1. For each of the systems shown in the figure, write down formulas for (i) the total potential energy and (ii) the total kinetic energy of the system, in terms of variables (and their time derivatives) provided in the figures (e.g. r , θ , etc).



(a) Pendulum



(b) Orbiting satellite (do not assume r is constant)



(c) Vibration isolator

Hint: the length of the spring for (c) is $2L\sin(\pi/4 + \theta/2)$. Can you show this?

2. The [Airbus E-Fan 1.0](#) is an experimental electrically powered aircraft, with the following specifications

- Empty Weight: 500kg.
- Total engine power 60kW.
- Endurance 1 hr
- Maximum speed 220 km/hr
- Lift/drag ratio (optimal) 16
- Cruise speed 160 km/hr

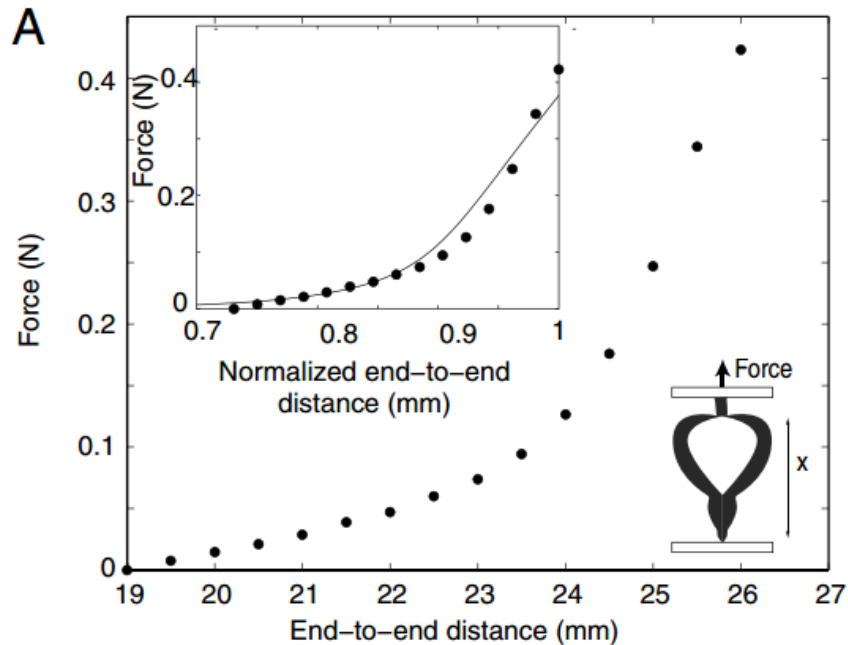


Assume the pilot and her/his baggage adds 100kg.

2.1 Calculate the power consumption during level cruise (assume that the aircraft is flying at the optimal Lift/drag ratio)

2.2 Calculate the total battery capacity (the total energy stored in the batteries) required to achieve the stated endurance, plus a 30 min reserve. Give your answer in kilowatt-hours.

2.3 Calculate the maximum rate of climb of the aircraft at cruise speed (you can assume the horizontal component of velocity is equal to the cruise speed. Give your answer in ft/min)



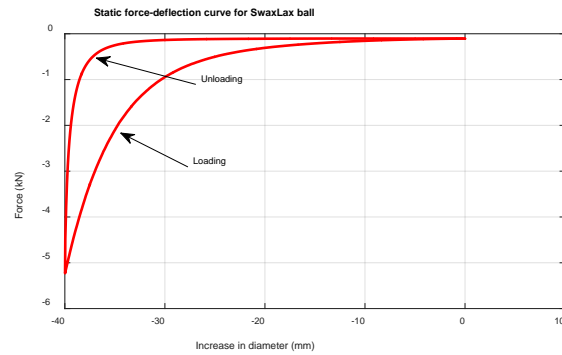
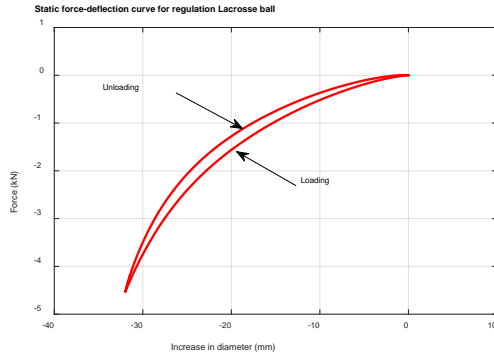
3. ‘[Jewelweed](#)’ is an example of a plant that uses a biological spring mechanism to disperse its seeds (a process known as ‘dehiscence’) (you can see videos of the process [here](#)). The goal of this problem is to estimate the efficiency of the plants launch mechanism (the ratio of the kinetic energy of the seeds to the stored energy in the seed-pod). Use the following data.

- The measured force-v-extension curve for a seed-pod (from [this reference](#)) is shown in the figure
- The mass of a typical seed is 10.7 mg ([Hayashi et al, 2009](#))
- A pod contains on average 3 seeds
- Seeds are launched with an average velocity of 1.25 m/s.

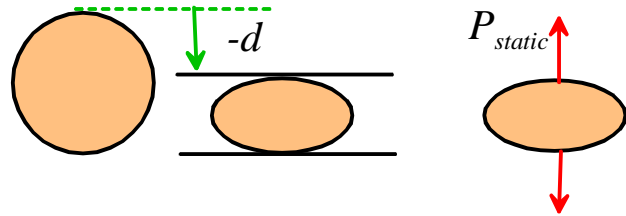
3.1 Estimate the energy stored in the seed-pod (i.e. the maximum amount of work that can be done by the force exerted by the seed pod on its seeds. You don’t need to do a very accurate calculation – approximating the force-displacement curve with two straight lines is sufficient).

3.2 Calculate the total kinetic energy of the launched seeds

3.3 Hence, estimate the dynamic efficiency.

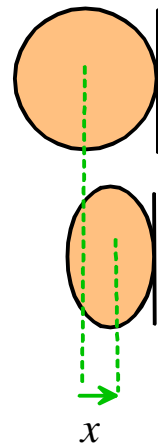


4. [Swaxlax](#) manufactures lacrosse ‘training’ balls that are intended to reduce the risks of head injury. To obtain a rough estimate of the restitution coefficients for both regulation and Swaxlax lacrosse balls, their force- v -deflection curves were measured using a static compression test. The data from these tests is on the Homework page of the course website.



In the test, a ball is compressed between two rigid platens, as shown in the figure. The force $P_{static}(d)$ is measured. The test was conducted in an [Instron](#) in Prince Lab, which is set up to stretch, rather than compress, specimens by default, and so records both d and P_{static} as negative numbers. The figures show the force as a function of the change in diameter of the balls. Note that the $P_{static}(d)$ curve differs during loading and unloading, because the ball is permanently deformed. The goal of this problem is to estimate the coefficient of restitution for the two balls from this data.

For this purpose, we assume that the impact between a ball and a rigid surface can be idealized as a particle that is subjected to an external force, and estimate the impact force from the static force – deflection curve. Specifically, we assume that if the particle moves a distance x towards the wall after impact it is subjected to a force $P_{static}(x/2)$. The factor of 2 is needed because during the static test the ball is compressed between two rigid surfaces, whereas the collision occurs with only one surface. In the static test, both contacts experience the same force, and the change in diameter is the sum of the two deflections at the contacts.



4.1 Assume that just before contacting the wall the ball has a speed V_0 . Using the work-energy relation, write down a formula relating the speed of the ball $v(x)$ to P_{static} .

4.2 Write a MATLAB script that will read the csv files of data provided, and plot the graphs of force- v -displacement shown in the figure. You can open the files with Microsoft excel to see the format of the data. To read them use

```
data = csvread('filename', 2, 0);
```

This will tell MATLAB to start reading data from the second row.

4.3 Use the data in the files provided to calculate the impact speed V_0 that would be expected to subject the balls to the maximum force in the file. Note that

- The regulation Lax ball had a mass of 149 grams;
- the SwaxLax ball had a mass of 145 grams.

You will need to find the row in the matrix that stores the csv data that has the largest force. You can do this using the matlab function

```
[value,index] = min(data(:,3))
```

Here the variable 'index' will return the row containing the minimum value of the 3rd column of the matrix 'data' – we use min because the data are negative. You can use the 'trapz' function evaluate the necessary integrals.

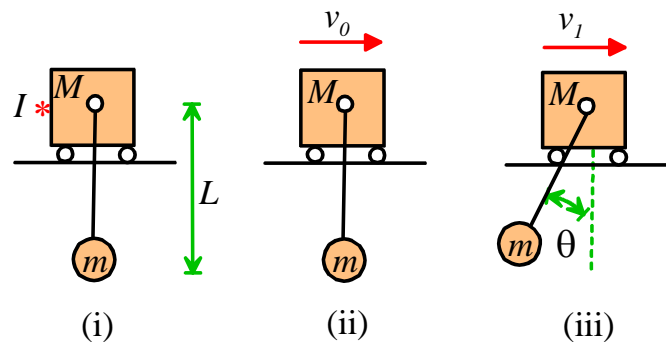
4.4 Note that the formula in part 4.1 can also be used to calculate the rebound velocity, by using the force-displacement curve during the unloading portion of the graph. Calculate the expected rebound velocity for both regular and SwaxLax balls

4.5 Hence, calculate a value for the restitution coefficients for both the regular and SwaxLax ball.

4.6 Using the restitution coefficient calculated in 4.4, calculate the impulse exerted on a head by a collision with both a regulation and a SwaxLax ball. You will have to do this by (i) calculating the change in velocity of the ball/head during the collision, and then (ii) using the impulse/momentum formula for a single particle to calculate the impulse. Use the following data:

- Take the [mass of a human head](#) to be 4.5kg, treat the head as particle ('particle-head' is a useful insult),
- Assume that both balls travel at 40 m/s (about 90 mph)
- Assume the head is stationary before the collision
- Ignore the impulse exerted by the player's neck during the collision.

5. The figure shows a proposed design for a ballistic pendulum. At time $t=0$ (fig i) the mass M is subjected to an impulse I . This causes the mass to translate to the right (fig ii); which in turn will cause the pendulum to swing to the left of the mass (iii). The goal of this problem is to calculate a formula relating the maximum angle of swing of the pendulum to the impulse, and other relevant parameters.

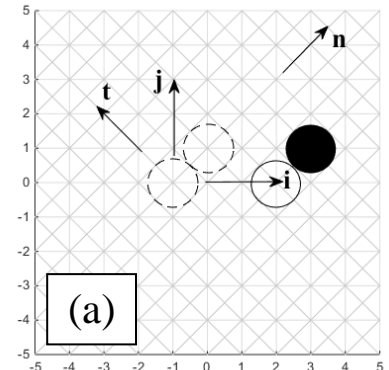
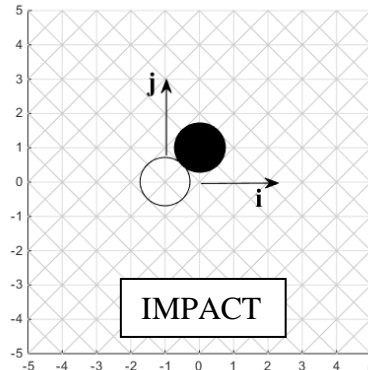
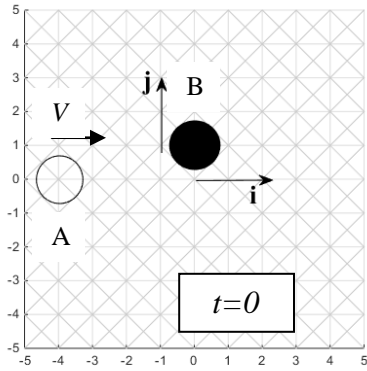


5.1 Use the impulse-momentum formula to find a formula for the velocity of mass M just after it is struck.

5.2 Now consider the system at the instant when the pendulum reaches the maximum angle of swing. Use momentum and energy conservation to show that

$$I = \sqrt{2M(M+m)gL(1 - \cos \theta)}$$

6. Two spheres with identical mass and restitution coefficient $e=0$ have initial positions shown in the figure below. Before impact sphere B is stationary and sphere A has velocity $V\mathbf{i}$. The collision is frictionless. By answering the true/false questions below, identify which of the figures (a-d) shows the correct position of the spheres after collision.



CIRCLE ONE RESPONSE TO EACH STATEMENT BELOW

(a) Total Momentum is conserved in the \mathbf{j} direction T F
 Momentum of B is conserved in the \mathbf{t} direction T F
 The restitution formula is satisfied in the \mathbf{n} direction T F

(b) Total Momentum is conserved in the \mathbf{j} direction T F
 Momentum of B is conserved in the \mathbf{t} direction T F
 The restitution formula is satisfied in the \mathbf{n} direction T F

(c) Total Momentum is conserved in the \mathbf{j} direction T F
 Momentum of B is conserved in the \mathbf{t} direction T F
 The restitution formula is satisfied in the \mathbf{n} direction T F

(d) Total Momentum is conserved in the \mathbf{j} direction T F
 Momentum of B is conserved in the \mathbf{t} direction T F
 The restitution formula is satisfied in the \mathbf{n} direction T F

Correct figure: a b c d

