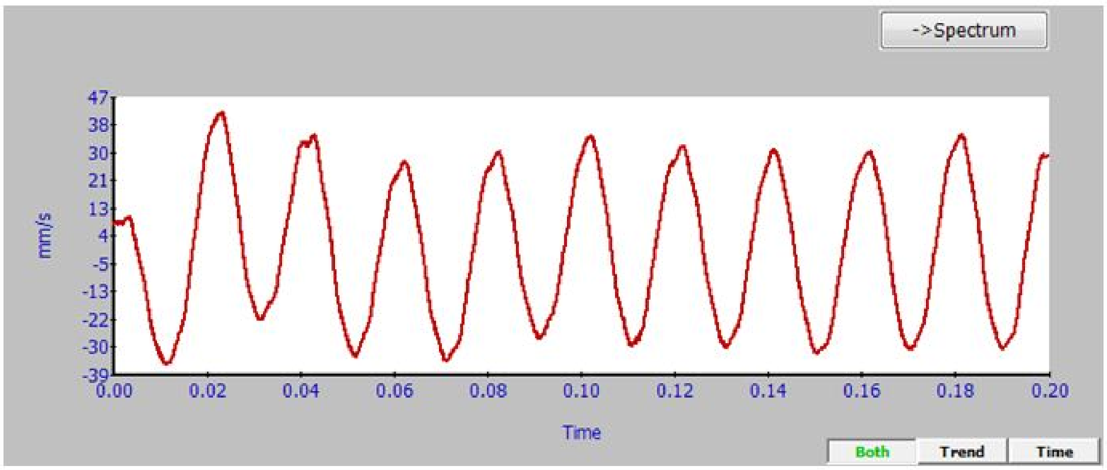
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 5: Free Vibrations - Solutions
Due Friday March 25th 2015
55 POINTS TOTAL



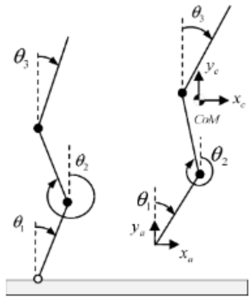
1. The figure (from this reference) shows the measured velocity of lateral vibration of an out-of-balance rotor. Calculate

1.1 The amplitude, the period, and frequency of the vibration (give the frequency both in Hertz and in radians per second)

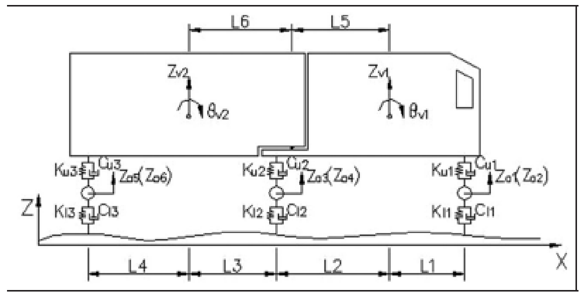
1.2 The amplitude of the acceleration.

1.3 The amplitude of the displacement.

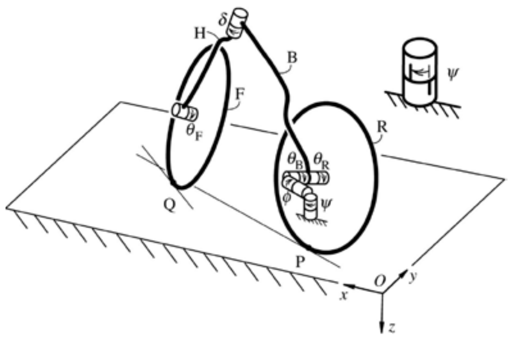
2. State the number of degrees of freedo m for each of the systems shown below. For systems (a), (b) and (d) state the number of vibration modes



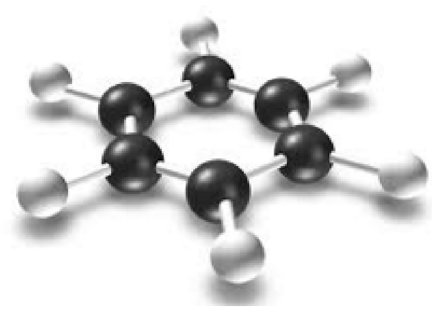
(a) Model of a hopping robot (motion is confined to the *x*,*y* plane. Consider the robot in the air only)



(b) 2D Model of an articulated truck (the model idealizes the wheels as particles, which can only move perpendicular to the truck body. Assume that the connection between the two parts of the body is a pin joint.



(c) Model of a bicycle The wheels and both links B and H are rigid bodies. (A riderless bicycle is usually unstable and so has no vibration modes. It is stable for a range of speeds, in which case there is one vibration mode. A simple explanation has not been found for this behavior).

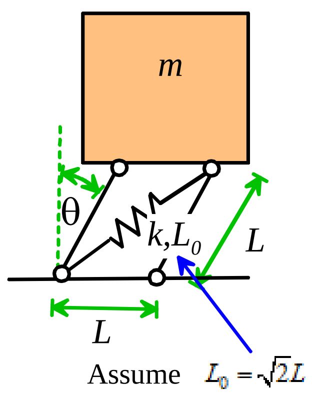


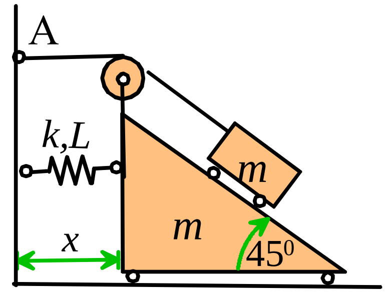
(d) Benzene molecule (the spheres are particles, the rods are springs)

3. Solve the following differential equations (use the Solutions to Differential Equations)

3.1 $\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+4y=0 \qquad y=0 \quad \dfrac{dy}{dt}=2 \quad t=0$

3.2 $\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+4y=16\sin(2t) \qquad y=-2 \quad \dfrac{dy}{dt}=1 \quad t=0$





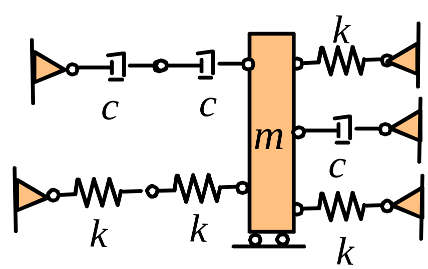
4. For the two conservative single-degree of freedom systems shown in the figure (note that in (a) the unstretched spring length is $\sqrt{2}L$) :

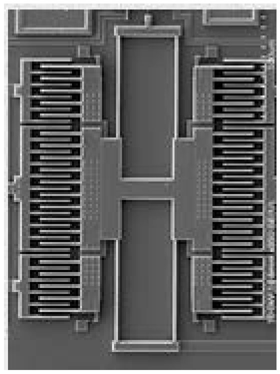
4.1 Derive the equation of motion (use energy methods, and include gravity. The pulley and cable are massless). State whether the equation of motion is linear or nonlinear.

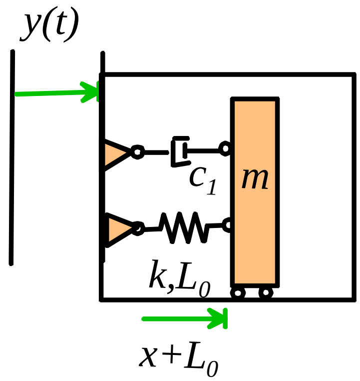
4.2 If appropriate, linearize the equation of motion for small amplitude vibrations (that means doing that Taylor series stuff discussed in class. "Linearizing" means replacing the nonlinear function of the variable with an approximate linear function)

4.3 Arrange the (linearized) equation of motion into standard form, and find an expression for the natural frequency of vibration.

5. Replace the system shown in the figure with an equivalent spring-mass system consisting of a mass with only one spring and dashpot. Hence, determine a formula for the undamped natural frequency and the damping factor for the system.







6. The figure shows a MEMS accelerometer (the figure on the left is from this company). It consists of of a proof mass m inside a sealed casing. The mass is suspended on springs and its motion is damped electrostatically. If the accelerometer accelerates to the right, the spring is compressed. The capacitative combs provide an electrical signal that senses the position x and hence provides a signal proportional to the acceleration of the device.

6.1 Show that the equation of motion for the distance x shown in the figure has the form

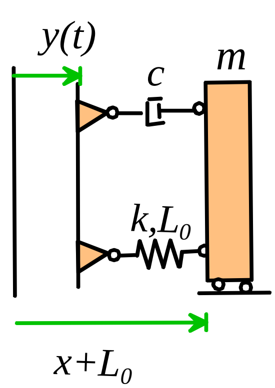
$$\frac{1}{\omega_n^2}\frac{d^2x}{dt^2}+\frac{2\zeta}{\omega_n}\frac{dx}{dt}+x=-\frac{K}{\omega_n^2}\frac{d^2y}{dt^2}$$

Give formulas for ω_n, ζ, K in terms of m,k,c shown in the figure.

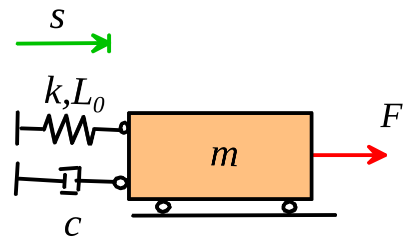
6.3 Suppose that the box accelerates with constant acceleration, so that $y=\frac{1}{2}at^2$. Assume that the system starts with $x=dx/dt=0$. Show that $x(t)\rightarrow -a/\omega_n^2$ as $t\rightarrow\infty$, so (once the transient has died out) x is proportional to a. This means that, once the transient motion has stopped, the signal will correctly measure acceleration.

6.4 The accelerometer designed in this publication has a resonant frequency of 41 kHz (don't forget the dreaded 2π factor between frequency and ω_n) and a damping factor of order $\zeta\approx 0.05$. Plot $x(t)$ with $a=1$g for this accelerometer (you can do the plot from $t=0$ to $t=0.4$ milliseconds). Use the graph to estimate how long it takes for the accelerometer reading to settle to within 5% of the correct value.

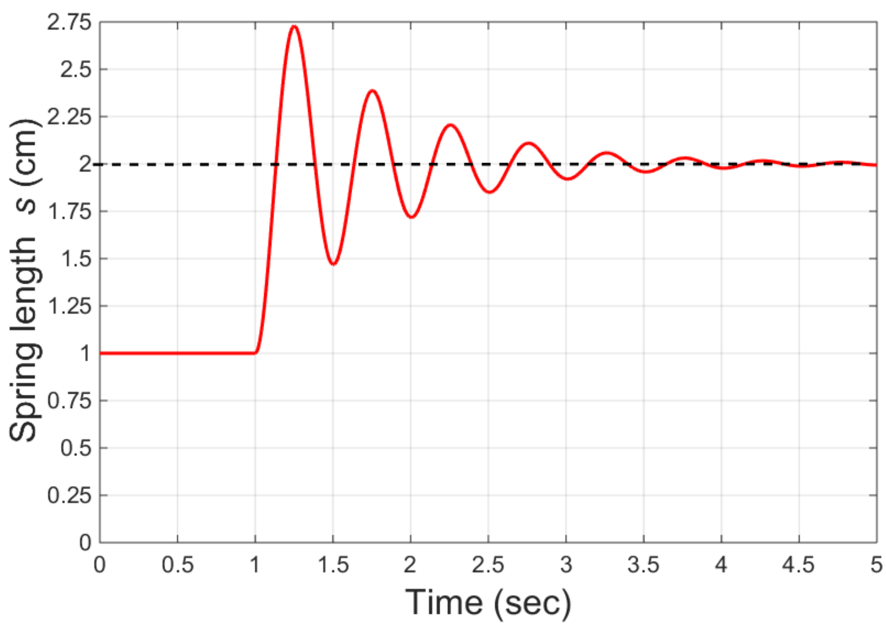
7. Determine the steady-state amplitude of vibration for the base excited spring-mass systems shown in the figure (you don't need to derive the equations of motion – this is a standard textbook systems and you can just use the standard formulas). The mass $m=20$kg, the stiffness $k=2000N/m$ $c=20Ns/m$. The base motion is $y(t)=0.01\sin 20t$ N.



8 The figure shows a simple idealization of a force sensor. Its purpose is to measure the force F, by providing an electrical signal that is proportional to the length s of the spring.



At time $t=0$ the system is at rest, and $F=0$. At time $t=1s$ a constant force of $F=100N$ is applied to the mass. The figure below shows the variation of s with time for $0<t<5$s.



8.1 Using the graph provided, calculate values for the following quantities.

(a) The period of vibration

(b) The damped natural frequency ω_d

(c) The log decrement of the vibration δ (be careful to use the correct origin)

(d) The damping factor of the system ζ

(e) The undamped natural frequency of the system ω_n

(f) The un-stretched length of the spring L_0

(g) The spring stiffness k

(h) The mass m

(i) The dashpot coefficient c.

8.2 The sensor is now used to measure a force that vibrates harmonically $F(t) = F_0 \sin \omega t$. The figure below shows the steady-state variation of the spring length s with time. Calculate the amplitude of the force F_0 .