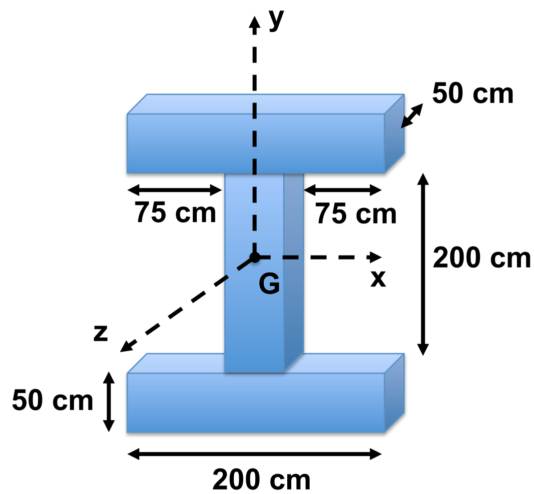
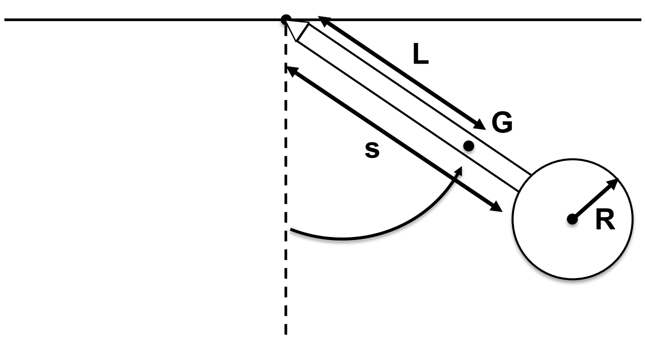
ENGN 0040: Dynamics and Vibrations, Spring 2016

Homework 6: Rigid Body Kinematics
Due Friday April 22nd

Problem 1 [6 pts]: For the I-beam shape of constant density ρ shown below, calculate a) I_{Gx} b) I_{Gy} and c) I_{Gz} [2 pts each]



Problem 2 [6 pts]: The rigid body pendulum shown is composed of a slender bar of length s and mass m_1 and a thin disk of radius $R = s/4$ and mass m_2.

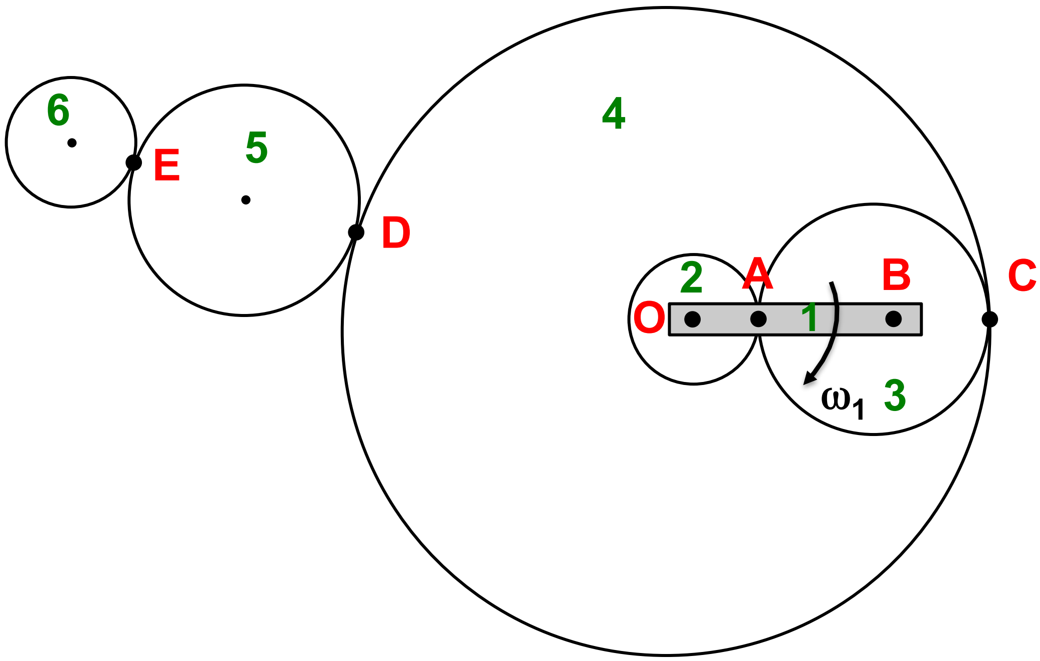


a) [1 pt] Calculate an expression for L, the distance from O to G.

b) [3 pts] Calculate an expression for I_G in terms of s, L, m_1 and m_2.

c) [2 pts] Assuming small oscillations, what is the natural frequency of the rigid body pendulum? Write your answer in terms of L, I_G, m_1 and m_2.

Problem 3 [10 pts]: In the planetary gear set below the rigid arm 1 (attached at points O and B) rotates at an angular velocity ω_1 and each of the gears rotates at an angular velocity of ω_2 through ω_6, with a corresponding radius R_2 through R_6.



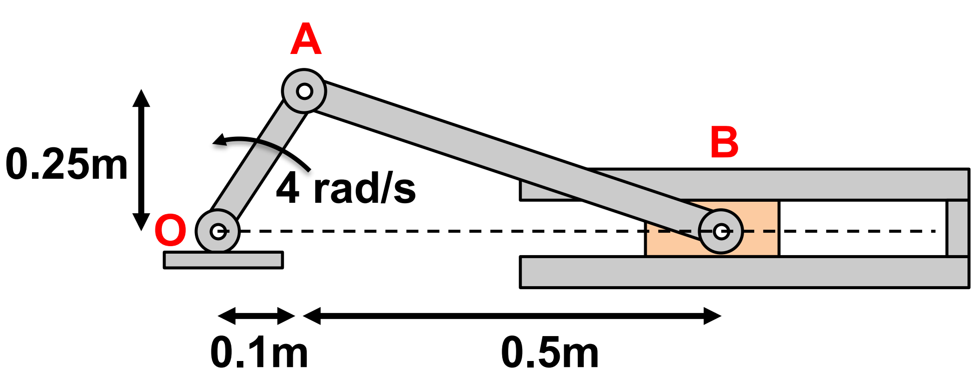
a) [1 pt] What is the gear ratio ω_4/ω_6?

b) [3 pts] Assuming all gears are rotating, what is the relationship between ω_1, ω_2 and ω_3?

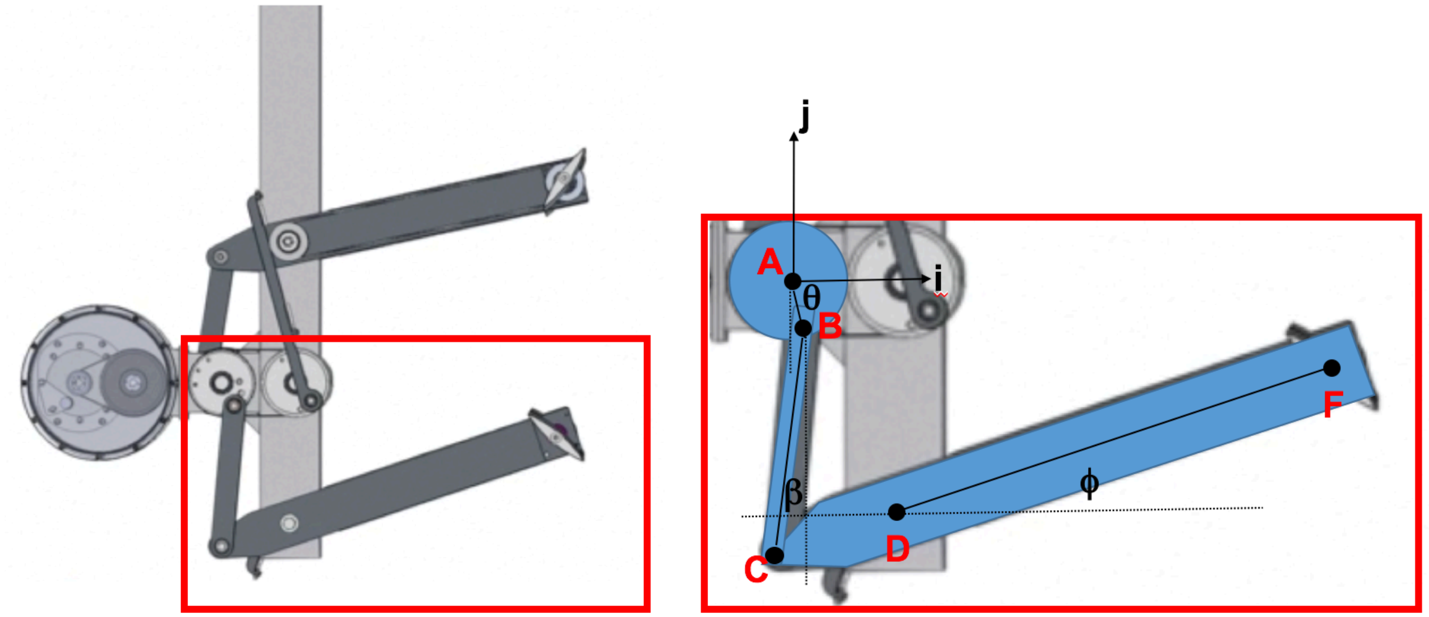
c) [3 pts] Assuming gear 6 is held fixed, what is the gear ratio ω_1/ω_2? (which direction is gear 3 rotating?)

d) [3 pts] Assuming gear 2 is held fixed (gear 6 is rotating), what is the gear ratio ω_1/ω_4? (which direction is gear 3 rotating?)

Problem 4 [6 pts]: In the slider crank mechanism below, arm OA has an angular velocity of 4 rad/s and an angular acceleration of 2 rad/s^2. What is the velocity and acceleration of the piston [3 pts each]?



Problem 5 [10 points]: A hydrofoil can be used as a tidal hydrokinetic turbine to generate power by heaving up and down and rotating the foil about its center with specific kinematics. An example is shown in the video: `https://drive.google.com/file/d/0B__lKXx8i5W7OUtfYzNfajc4ZkE/view?usp=sharing`. This problem will begin to analyze a portion of the kinematics shown in the diagram below. Point B rotates at a constant angular velocity ω, with $\theta = 0$ at $t = 0$. The position of fixed point D with respect to fixed point A is $Dx\mathbf{i} - Dy\mathbf{j}$. The length of AB is denoted by R, the length of BC is denoted by L_1, the length of CD is denoted by L_2, and the length of DF is denoted by L_3. The angles θ and β are defined relative to the vertical, and ϕ is defined relative to the horizontal.



a) [2 pts] Write an expression for $\mathbf{v_B}$ in terms of R, ω, and t

b) [2 pts] Write an expression for $\mathbf{v_C}$ in terms of $\mathbf{v_B}$, ω_{BC}, β and L_1

c) [2 pts] Write an expression for $\mathbf{v_C}$ in terms of ω_{CD}, ϕ and L_2

d) [2 pts] Write a vector equation consisting of unknown quantities ω_{BC}, ω_{CD}, β, and ϕ. Note you will have 2 equations (1 vector equation) and 4 unknowns.

e) [2 pts] Write a vector equation relating the geometric variables β, and ϕ in order to close the system of equations. (Note you do not have to solve).