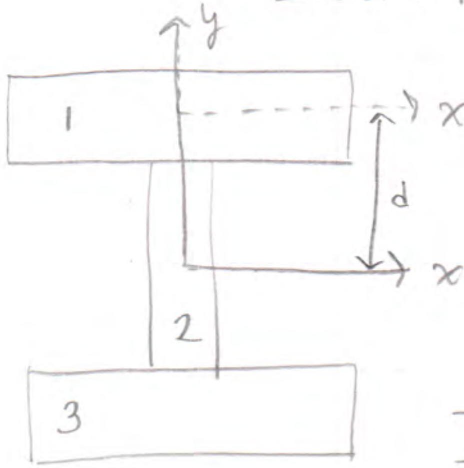
ENGN40 HW6 SOLNS 2016

1a)



$m = m_1 = m_2 = m_3 = \rho V$

$m = 2700(2)(.5)(.5)$

$m = 1350$ Kg

$I_{1x'} = \frac{1}{12}m(0.5^2 + 0.5^2) = 56.25$ Kg·m²

parallel axis theorem: $I_{1x} = I_{1x'} + md^2$

$I_{1x} = 2165$ Kg·m²

By symmetry $I_{1x} = I_{3x}$

$I_{2x} = \frac{1}{12}m(.50^2 + 2^2) = 478$ Kg·m²

$I_x = I_{1x} + I_{2x} + I_{3x} = \boxed{4808 \text{ Kg·m}^2}$

b) I_y: no parallel axis thereom needed

$I_y = I_{y1} + I_{y2} + I_{y3} = 2I_{y1} + I_{y2}$

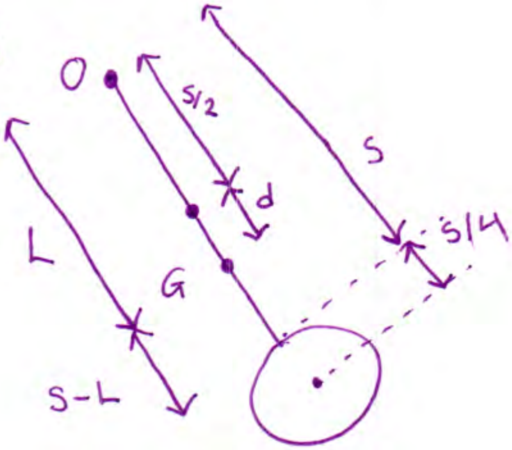
$I_y = 2\frac{m}{12}(2^2 + .5^2) + \frac{m}{12}(.5^2 + .5^2) = \boxed{1012.5 \text{ Kg·m}^2}$

c) $I_z = I_{z1} + I_{z2} + I_{z3} = I_{z2} + 2I_{z1}$

$I_z = \frac{1}{12}m(.5^2 + .5^2) + 2\left[\frac{m}{12}(.5^2 + 2^2) + m(1.25^2)\right]$

$\boxed{I_z = 5231 \text{ Kg·m}^2}$

2)



a) center of mass bar

$$= \frac{1}{2} s$$

center of mass disk

$$= 5/4 \, s$$

$$L = \frac{\frac{1}{2} s m_1 + \frac{5}{4} s m_2}{m_1 + m_2}$$

b) $$I_{G\,bar} = \frac{1}{12} m_1 s^2 + m_1 d^2$$

$$d = L - s/2$$

$$I_{G\,disk} = \frac{1}{2} m_2 \left(\frac{s}{4}\right)^2 + m_2 \left(s/4 + s - L \right)^2$$

$$I_{G\,bar} = m_1 \left(1/12 \, s^2 + (L - s/2)^2 \right) = m_1 \left(1/3 \, s^2 - \cancel{2} L s + L^2 \right)$$

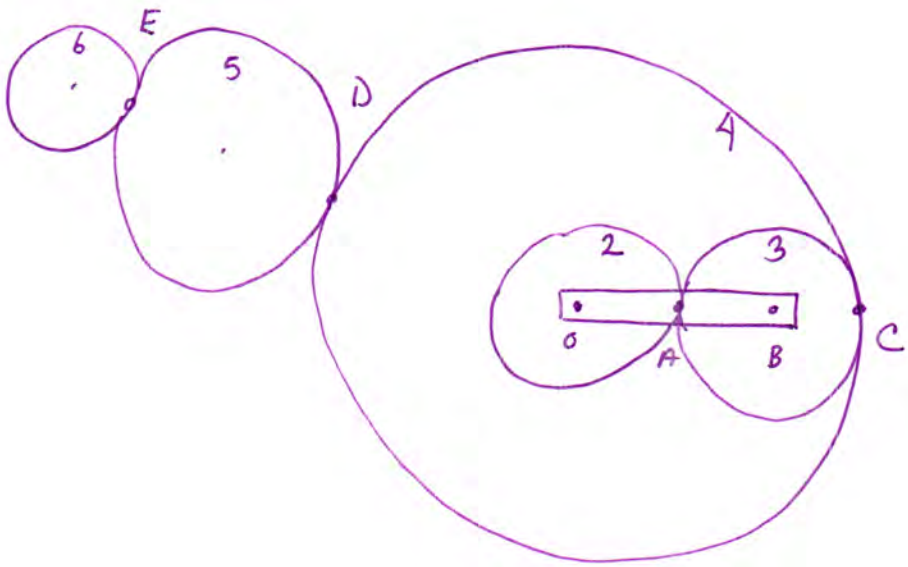
$$I_{G\,disk} = m_2 \left(s^2/32 + (5s/4 - L)^2 \right) = m_2 \left(51/32 \, s^2 - 5/2 \, L s + L^2 \right)$$

$$I_G = I_{G\,bar} + I_{G\,disk}$$

c) From class notes
$M \to m_1 + m_2$

$$\omega_n = \sqrt{\frac{L g (m_1 + m_2)}{I_G + L^2 (m_1 + m_2)}}$$

3)



a) $v_E = \omega_6 R_6 = \omega_5 R_5$, $\omega_5/\omega_6 = R_6/R_5$

$v_D = \omega_5 R_5 = \omega_4 R_4$, $\omega_4/\omega_5 = R_5/R_4$

$\omega_4/\omega_6 = \omega_4/\omega_5 \cdot \omega_5/\omega_6 = R_5/R_4 \cdot R_6/R_5 = R_6/R_4$

b) Motion of arm: $\underline{v}_B = \underline{\omega}_1 \times \underline{r}_{B/O} = -\omega_1 (R_2 + R_3)\hat{j}$

Motion of gear 3: $\underline{v}_C = \underline{v}_B + \underline{\omega}_3 \times \underline{r}_{C/B} = \underline{v}_B - R_3 \omega_3 \hat{j}$

$\underline{v}_C = \underline{v}_A + \underline{\omega}_3 \times \underline{r}_{C/A} = \underline{v}_A - 2R_3\omega_3\hat{j}$

Motion of gear 2: $\underline{v}_A = -\omega_2 R_2 \hat{j}$

eliminate $\underline{v}_C$: $\underline{v}_B - R_3\omega_3\hat{j} = \underline{v}_A - 2R_3\omega_3\hat{j}$

$+\omega_1(R_2 + R_3) + R_3\omega_3 = +\omega_2 R_2 + 2R_3\omega_3$

$$\boxed{\omega_1(R_2 + R_3) = \omega_2 R_2 + \omega_3 R_3}$$

c) E, D, C have zero velocity. gear 4 fixed.
(prius demo with MG2 = 0)

gear 3 $\circlearrowleft^{\omega_3}$

$\underline{V}_c = 0$ in equations above

$0 = -\omega_1 (R_2 + R_3)\hat{j} - R_3\omega_3\hat{j}$

$0 = -\omega_2 R_2\hat{j} - 2R_3\omega_3\hat{j}$

eliminate ω_3

$0 = 2\omega_1 (R_2 + R_3) - \omega_2 R_2$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{2(R_2 + R_3)}{R_2}}$$

d) prius demo $\rightarrow$ MG1 = 0, $\underline{V}_A = 0$

$\underline{V}_c = -\omega_4 R_4\hat{j} = \cancel{\underline{V}_A}^{0} + \underline{\omega}_3 \times \underline{r}_{c/A} = -2R_3\omega_3\hat{j}$

$-\omega_4 R_4 = -2R_3\omega_3$

The R1 below should be R2

$\underline{V}_c = -\omega_4 R_4\hat{j} = \underline{V}_B - R_3\omega_3\hat{j} = -\omega_1 (R_1 + R_3)\hat{j} - R_3\omega_3\hat{j}$

eliminate ω_3: $\frac{\omega_4 R_4}{2} = R_3\omega_3$

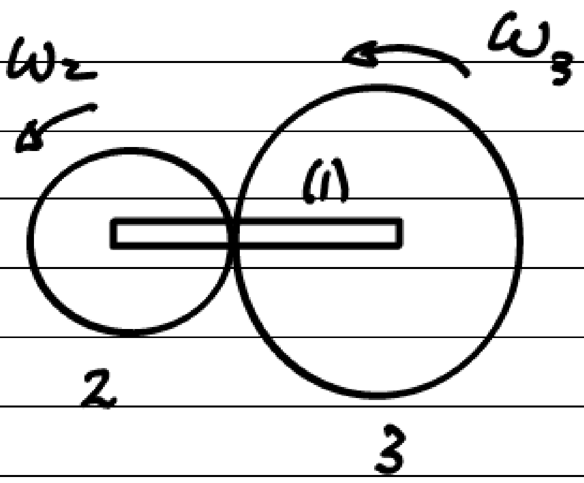
$+\omega_4 R_4 \cancel{+} - \omega_1 (R_1 + R_3) = R_3\omega_3$

$\omega_1 (R_1 + R_3) = \frac{\omega_4 R_4}{2}$

$$\boxed{\frac{\omega_1}{\omega_4} = \frac{R_4}{2(R_1 + R_3)}}$$

R1 should be R2

Alternative solution to 3(b)



First solve with bar (1) stationary

$$\omega_2 R_2 = -\omega_3 R_3$$

(standard gear formula)

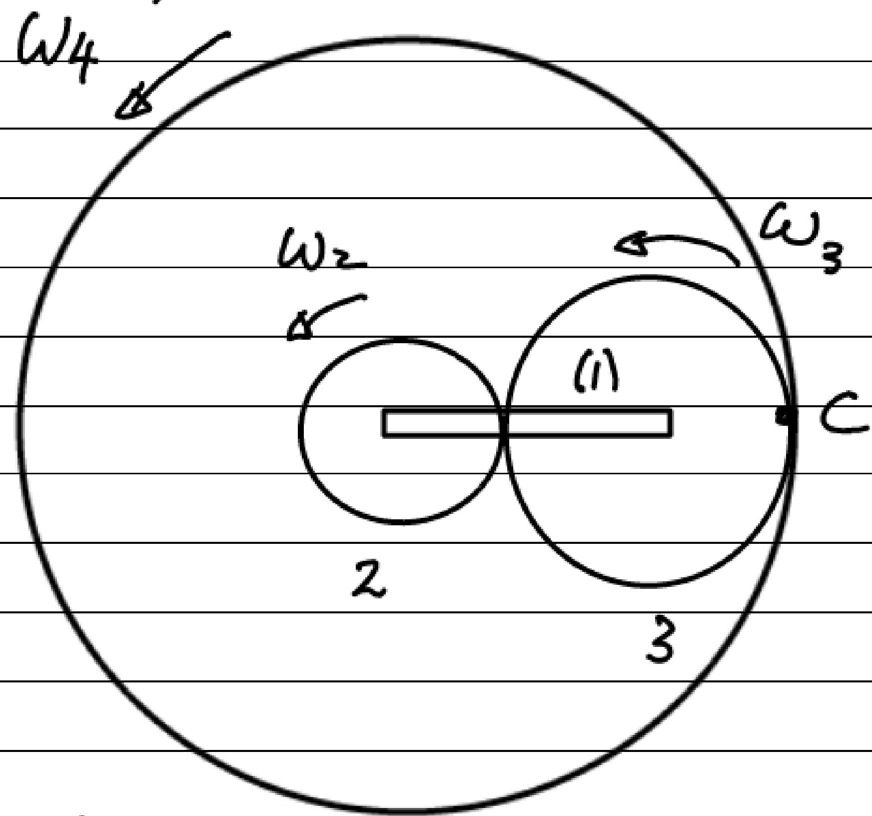
Now note if (1) rotates we can adopt a rotating reference frame that rotates @ same speed as bar (1)

In this frame (2) has angular speed $\omega_2 - \omega_1$, (3) has speed $\omega_3 - \omega_1$ and (1) is stationary

Thus $(\omega_2 - \omega_1) R_2 = -(\omega_3 - \omega_1) R_3$

$$\boxed{\Rightarrow \quad \omega_2 R_2 + \omega_3 R_3 - \omega_1 (R_2 + R_3) = 0}$$

Alternative solution to 3(c), 3(d



As before adopt a reference frame that rotates with bar (1)

In this frame the angular speeds are 0, $\omega_2-\omega_1$, $\omega_3-\omega_1$, $\omega_4-\omega_1$

We already know
$$(\omega_2-\omega_1)R_2 = -(\omega_3-\omega_1)R_3$$

Since (3)(4) touch @ C $(\omega_3-\omega_1)R_3 = (\omega_4-\omega_1)R_4$

Hence $\boxed{(\omega_2-\omega_1)R_2 = -(\omega_4-\omega_1)R_4 = -(\omega_4-\omega_1)(R_2+2R_3)}$

For (c) $\omega_4=0 \Rightarrow (\omega_2-\omega_1)R_2 = \omega_1(R_2+2R_3)$

Hence $\boxed{\dfrac{W_2}{W_1} = \dfrac{2(R_2+R_3)}{R_2} = \dfrac{R_4+R_2}{R_2}}$

For (d) $W_2 = 0 \Rightarrow -W_1 R_2 = -(W_4 - W_1) R_4$

$\Rightarrow \boxed{\dfrac{W_1}{W_4} = \dfrac{R_4}{R_2+R_4} = \dfrac{R_4}{2(R_2+R_3)} = \dfrac{R_2+2R_3}{2(R_2+R_3)}}$

4) $\underline{r}_{A/O} = .1\hat{\imath} + .25\hat{\jmath}$

$\underline{r}_{B/A} = .5\hat{\imath} - .25\hat{\jmath}$

$\underline{V}_A = \underline{\omega}_{OA} \times \underline{r}_{A/O} = -.25\omega_{OA}\hat{\imath} + .1\omega_{OA}\hat{\jmath}$

$\underline{V}_B = \underline{V}_A + \underline{\omega}_{AB} \times \underline{r}_{B/A} = -.25\omega_{OA}\hat{\imath} + .1\omega_{OA}\hat{\jmath} + .25\omega_{AB}\hat{\imath} + .5\omega_{AB}\hat{\jmath}$

Constraint on B $\Rightarrow V_{By} = 0$

$0 = .1\omega_{OA}\hat{\jmath} + .5\omega_{AB}\hat{\jmath} \Rightarrow \omega_{OA} = 4 \text{ rad/s}$

$\hat{\imath}$ eqn: $\omega_{AB} = -.8 \text{ rad/s}$

$-.25\omega_{OA}\hat{\imath} + .25\omega_{AB}\hat{\imath} = -1.2 \text{ m/s}$

$\boxed{\underline{V}_B = -1.2 \text{ m/s } \hat{\imath}}$

$\underline{a}_A = \underline{\alpha}_{OA} \times \underline{r}_{A/O} + \underline{\omega}_{OA} \times \underline{\omega}_{OA} \times \underline{r}_{A/O} = -.25\alpha_{OA}\hat{\imath} + .1\alpha_{OA}\hat{\jmath} - .1\omega_{OA}^2\hat{\imath} - .25\omega_{OA}^2\hat{\jmath}$

$\boxed{\underline{a}_A = -2.1\hat{\imath} - 3.8\hat{\jmath}}$

$\underline{a}_B = \underline{a}_A + \underline{\alpha}_{AB} \times \underline{r}_{B/A} + \underline{\omega}_{AB} \times \underline{\omega}_{AB} \times \underline{r}_{B/A}$

$\underline{a}_B = \underline{a}_A + .25\alpha_{AB}\hat{\imath} + .5\alpha_{AB}\hat{\jmath} - .5\omega_{AB}^2\hat{\imath} + .25\omega_{AB}^2\hat{\jmath}$

$\hat{\jmath}$: $0 = -3.8 + .5\alpha_{AB} + .25(-.8)^2 \Rightarrow \alpha_{AB} = +7.28 \text{ rad/s}^2$

$\hat{\imath}$: $a_{Bx} = -2.1 + .25\alpha_{AB} - .5\omega_{AB}^2 \Rightarrow \boxed{a_{Bx} = -0.6 \text{ m/s}^2}$

5. $\omega = \text{constant}$ $\quad \theta = \int \omega\, dt = \cancel{\theta_0}^{\,0} + \omega t$, $\theta = \omega t$

a) $\underline{V}_B = \underline{\omega} \times \underline{r}_{B/A}$, $\underline{r}_{B/A} = R\sin\theta\,\hat{\imath} - R\cos\theta\,\hat{\jmath}$

$\underline{V}_B = R\cos(\omega t)\cdot\omega\,\hat{\imath} + R\sin(\omega t)\cdot\omega\,\hat{\jmath}$

b) $\underline{V}_C = \underline{V}_B + \underline{\omega}_{BC} \times \underline{r}_{C/B}$, $\underline{r}_{C/B} = -L_1\sin\beta\,\hat{\imath} - L_1\cos\beta\,\hat{\jmath}$

$\underline{V}_C = \underline{V}_B + \omega_{BC}L_1\cos\beta\,\hat{\imath} - \omega_{BC}L_1\sin\beta\,\hat{\jmath}$

$\underline{V}_C = (R\omega\cdot\cos\omega t + \omega_{BC}L_1\cos\beta)\hat{\imath} + (R\omega\cdot\sin(\omega t) - \omega_{BC}L_1\sin\beta)\hat{\jmath}$

c) $\underline{V}_C = \cancel{\underline{V}_D}^{\,0} + \underline{\omega}_{CD} \times \underline{r}_{C/D}$, $\underline{r}_{C/D} = -L_2\cos\phi\,\hat{\imath} - L_2\sin\phi\,\hat{\jmath}$

$\underline{V}_C = \omega_{CD}L_2\sin\phi\,\hat{\imath} - \omega_{CD}L_2\cos\phi\,\hat{\jmath}$

d) set b) & c) equal

$\hat{\imath}$) $R\omega\cdot\cos\omega t + \omega_{BC}L_1\cos\beta = \omega_{CD}L_2\sin\phi$

$\hat{\jmath}$) $R\omega\sin\omega t - \omega_{BC}L_1\sin\beta = -\omega_{CD}L_2\cos\phi$

R, ω, L_1, L_2 known

$\omega_{BC}, \omega_{CD}, \beta, \phi$ unknown

e) Geometry : $\underline{r}_{C/A} + \underline{r}_{D/C} = \underline{r}_{D/A} = \underbrace{D_x\hat{\imath} - D_y\hat{\jmath}}_{\text{fixed distance}} = \text{constant}$

$\underline{r}_{C/A} = \underline{r}_{C/B} + \underline{r}_{B/A}$

$D_x\hat{\imath} - D_y\hat{\jmath} = \underline{r}_{C/B} + \underline{r}_{B/A} + \underline{r}_{D/C}$

$\hat{\imath}$) $D_x = -L_1\sin\beta + R\sin\omega t + L_2\cos\phi$

$\hat{\jmath}$) $-D_y = -L_1\cos\beta - R\cos\omega t + L_2\sin\phi$