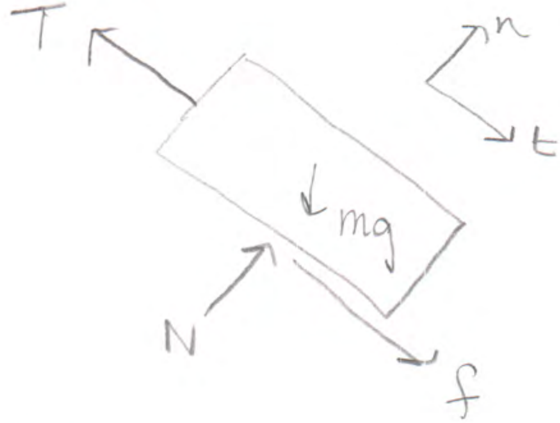
ENGN 40 HW 7 SOLNS 2016

problem 1

a)

b)



c) $\sum F_n = m a_n \Rightarrow N - mg\cos\theta = m a_n = 0$

$\sum F_t = m a_t \Rightarrow f + mg\sin\theta - T = m a_t$

$f = \mu N$

d) $Q - TR = \frac{1}{2} m R^2 \alpha$

e) $a_t = -R\alpha$

f) substituting $N = mg\cos\theta$

$T = \frac{Q}{R} - \frac{1}{2} m R\alpha$

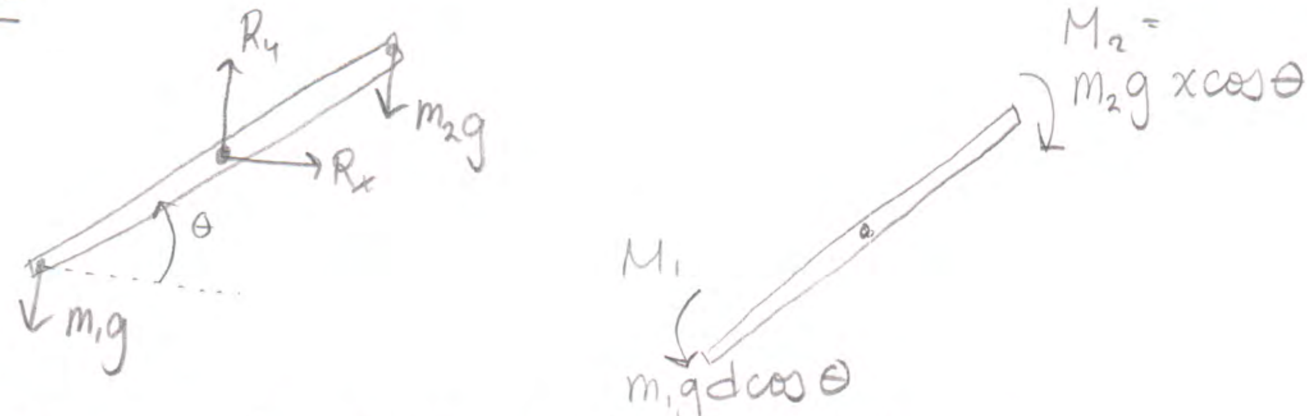
$\mu \cancel{m} g\cos\theta + \cancel{m} g\sin\theta - \frac{Q}{mR} + \frac{1}{2}\cancel{m} R\alpha = -\cancel{m} R\alpha$

$g(\mu\cos\theta + \sin\theta) - Q/mR = -\frac{3}{2} R\alpha$

$\alpha = \frac{2}{3}\left\{ Q/mR^2 - g/R(\sin\theta + \mu\cos\theta)\right\}$

problem 2

a)



b) Balance moments about "O" for equilibrium

$$m_1 g d\cos\theta = m_2 g x\cos\theta$$

$$x = d\frac{m_1}{m_2} = \frac{6}{5}d \qquad x > 6/5\,d$$

for motion!

c) 2 point masses, massless bar (can accept $x = 6/5\,d$)

$$I_o = m_1 d^2 + m_2\left(6/5\,d\right)^2$$

$$I_o = m_1 d^2 + 36/25\, m_2 d^2$$

d) Now: $I_o = m_1 d^2 + m_2 4d^2 = d^2(m_1 + 4m_2)$

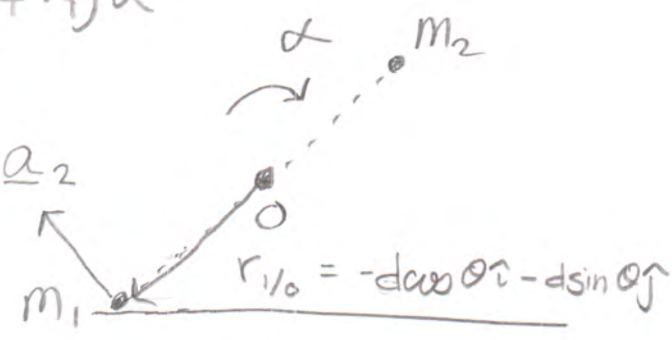
$$\Sigma M_o = I_o\alpha \implies m_1 g d\cos\theta - m_2 g 2d\cos\theta = I_o\alpha$$

$$\frac{1}{m_2} g \cancel{d}\cos\theta(m_1 - 2m_2) = \frac{1}{m_2} d^{\cancel{2}}(m_1 + 4m_2)\alpha$$

$$g\cos\theta\left(\frac{m_1}{m_2} - 2\right) = d\left(\frac{m_1}{m_2} + 4\right)\alpha$$

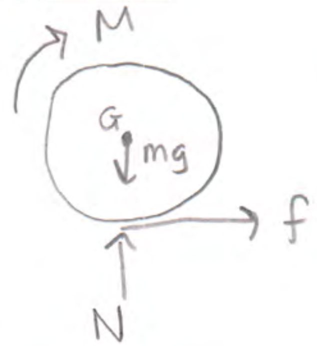
$$\alpha = \frac{g\cos\theta\,(m_1/m_2 - 2)}{d(m_1/m_2 + 4)}$$

$$\alpha = -\frac{2g\cos\theta}{13d} \quad \& \quad \omega = 0$$



$$\underline{a}_2 = \underline{\alpha} \times \underline{r}_{1/0} = d\alpha\sin\theta\,\hat{\imath} - d\alpha\cos\theta\,\hat{\jmath} = -\frac{2g\cos\theta\sin\theta}{13}\hat{\imath} + 2g\cos^2\theta/13\,\hat{\jmath}$$

problem 3



FBD (1 pt)

EOM (1 pt)

$$f = m a_{Gx}$$
$$N - mg = 0$$
$$fR - M = I_G \alpha = \tfrac{1}{2} m R^2 \alpha$$
$$I_G = \tfrac{1}{2} m R^2$$

Kinematics (1 pt) $a_{Gx} = -\alpha R$

Solve

$$m a_{Gx} R - M = \tfrac{1}{2} m R^2 \left(\frac{-a_{Gx}}{R}\right)$$
$$m a_{Gx} R - M = -\tfrac{1}{2} m R a_{Gx}$$
$$\tfrac{3}{2} m a_{Gx} R = M$$
$$a_{Gx} = \frac{2}{3}\frac{M}{mR}$$

constant acceleration formula:

$$v_{Gx} = \sqrt{2 a_x b} = \sqrt{\frac{4Mb}{3mR}}$$

Energy Work done $= \int_{\theta_0 = 0}^{\theta_f} M \, d\theta = M \Delta\theta$

1 rotation $= 2\pi R$ distance $= 2\pi$ radians

a distance b is $\frac{b}{2\pi R} \cdot 2\pi = b/R$ radians

(2 pts) $\frac{Mb}{R}$ = Work done

$$\text{Work done} = T_f - \cancel{T_i}^{\to 0}$$

$$\frac{Mb}{R} = \frac{1}{2} m V_{Gx}^2 + \frac{1}{2} I_G \omega^2 \qquad I_G = \tfrac{1}{2} m R^2$$

(1pt) $$\frac{Mb}{R} = \frac{1}{2} m V_{Gx}^2 + \frac{1}{4} m R^2 \omega^2$$

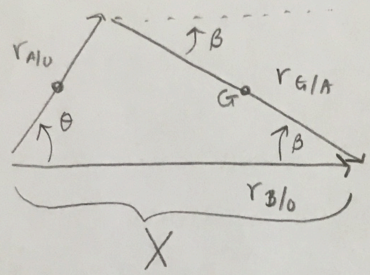
Kinematics $\quad V_{Gx} = -R\omega \;, \; \omega = -V_{Gx}/R$

$$\frac{Mb}{R} = \frac{1}{2} m V_{Gx}^2 + \frac{1}{4} m \cancel{R^2} \frac{V_{Gx}^2}{\cancel{R^2}} = \frac{3}{4} m V_{Gx}^2$$

(1pt) Solve

$$V_{Gx} = \sqrt{\frac{4Mb}{3mR}}$$

a)



$$r_{A/O} + r_{G/A} = r_{B/O}$$

$$L_1\cos\theta\,\hat{\imath} + L_1\sin\theta\,\hat{\jmath} + L_2\cos\beta\,\hat{\imath} - L_2\sin\beta\,\hat{\jmath} = X\hat{\imath}$$

$$L_1\sin\theta - L_2\sin\beta = 0$$

$$\beta = \text{asin}\left(\frac{L_1\sin\theta}{L_2}\right)$$

b) $V_i + \cancel{T_i}^{\,0} = \cancel{V_f}^{\,0} + T_f$

$$\frac{1}{2}k\theta^2 + m_1 g L_1/2 \sin\theta + m_2 g L_2/2 \sin\beta = V_i$$

$$\frac{1}{2}I_{O1}\omega_{OA}^2 + \frac{1}{2}m_2 V_G^2 + \frac{1}{2}I_{G2}\omega_{AB}^2 = T_f$$

$$I_{O1} = I_{G1} + m_1\left(\frac{L_1}{2}\right)^2 = \frac{1}{12}m_1L_1^2 + \frac{m_1L_1^2}{4} = \frac{1}{3}m_1L_1^2$$

$$I_{G2} = 1/12\, m_2 L_2^2$$

$$1/2\, k\theta^2 + m_1 g\frac{L_1}{2}\sin\theta + m_2 g\frac{L_1}{2}\sin\theta = \frac{1}{6}m_1L_1^2\omega_{OA}^2 + \frac{1}{2}m_2V_G^2 + \frac{1}{24}m_2 L$$

$$\frac{1}{2}k\theta^2 + g\frac{L_1}{2}\sin\theta(m_1+m_2) = \frac{1}{6}m_1L_1^2\omega_{OA}^2 + \frac{1}{2}m_2V_G^2 + \frac{1}{24}m_2 L$$

c) $\underline{V}_A = -L_1\omega_{OA}\sin\theta\,\hat{\imath} + L_1\omega_{OA}\cos\theta\,\hat{\jmath}$

$$\underline{V}_B = \underline{V}_A + \underline{\omega}_{AB}\times\underline{r}_{B/A}$$

$$\underline{V}_B = V_{Bx}\hat{\imath} + 0\hat{\jmath} = (-L_1\omega_{OA}\sin\theta + L_2\omega_{AB}\sin\beta)\hat{\imath} + \underbrace{(L_1\omega_{OA}\cos\theta + L_2\omega_{AB}\cos\beta)}_{=0}\hat{\jmath}$$

Constraint

$$\boxed{L_1\omega_{OA}\cos\theta = -L_2\omega_{AB}\cos\beta}$$

4c)

$$L_1 \omega_{OA} \cos\theta = -L_2 \omega_{AB} \cos\beta$$

$$\boxed{\omega_{OA} = -\omega_{AB} \frac{L_2}{L_1} \frac{\cos\beta}{\cos\theta}}$$

can sub.

$$\left[\beta = \operatorname{asin} \frac{L_1 \sin\theta}{L_2}\right]$$

4d)

$$\underline{V}_G = \underline{V}_B + \underline{\omega}_{AB} \times \underline{r}_{G/B}$$

$$\underline{r}_{G/B} = -\frac{L_2}{2}\cos\beta\,\hat{\imath} + \frac{L_2}{2}\sin\beta\,\hat{\jmath}$$

$$\underline{V}_G = \left(-L_1\omega_{OA}\sin\theta + \frac{L_2}{2}\omega_{AB}\sin\beta\right)\hat{\imath} + \left(L_1\omega_{OA}\cos\theta + L_2/_2\,\omega_{AB}\cos\beta\right)\hat{\jmath}$$

when $\theta = 0$, $\beta = 0$

$$V_G = 0\hat{\imath} + \left(L_1\omega_{OA} + L_2/_2\,\omega_{AB}\right)\hat{\jmath}$$

Substitute $\omega_{OA} = -\omega_{AB}\frac{L_2}{L_1}$

$$V_G = -\omega_{AB}L_2 + \frac{L_2}{2}\omega_{AB} = \boxed{-\frac{L_2}{2}\omega_{AB}\,\hat{\jmath}}$$

OR

$$\boxed{V_G = \frac{L_1}{2}\omega_{OA}\,\hat{\jmath}}$$

4e) $|v_G|^2 = \frac{L_1^2}{4}\omega_{OA}^2$, $m_2 = 2m_1$, $L_2 = 2L_1$

$$\frac{1}{2}K\theta_0^2 + \frac{gL_1}{2}\sin\theta_0 \cdot (3m_1) = \frac{1}{6}m_1L_1^2\omega_{OA}^2 + \frac{1}{2}2m_1 \cdot \frac{L_1^2}{4}\omega_{OA}^2$$

$$+ \frac{1}{24} \cdot 2m_1 4L_1^2 \frac{L_1^2}{4L_1^2}\omega_{OA}^2$$

$$= \frac{1}{6}m_1L_1^2\omega_{OA}^2 + m_1L_1^2\omega_{OA}^2$$

$$+ \frac{1}{12}m_1L_1^2\omega_{OA}^2$$

$$\frac{1}{2}K\theta_0^2 + \frac{3}{2}gL_1m_1\sin\theta_0 = \left(\frac{5}{4}m_1L_1^2\right)\omega_{OA}^2$$

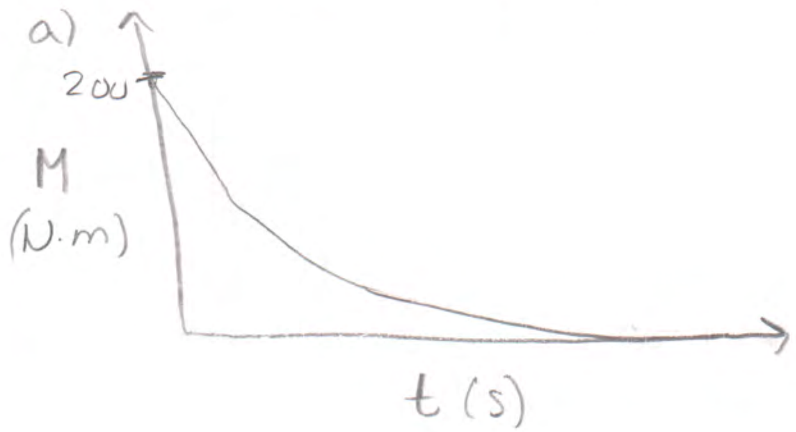
$$\boxed{\omega_{OA} = \sqrt{\frac{2K\theta_0 + 6/5\, gL_1m_1\sin\theta_0}{m_1L_1^2}}}$$

$$\boxed{\omega_{BC} = -\frac{L_1}{L_2}\omega_{OA}}$$

problem 5

$$M = 200e^{-.1t} \ N\cdot m$$

a)



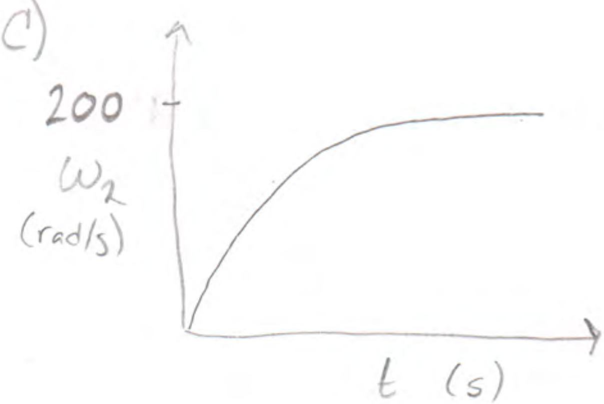
b)

$$\int_0^{10} M\,dt = H_{G2} - H_{G1} = I_G(\omega_2 - \cancel{\omega_1}^{\,0}) = I_G\omega_2$$

$$-200/(0.1)\, e^{-0.1t}\Big|_0^{10} = -2000e^{-1} + 2000e^{0} = 10\,\omega_2$$

$$\omega_2 = -200/e^1 + 200 = 126.4 \text{ rad/s}$$

c)



$$\omega_2(t) = -200\left[e^{-.1t} - 1\right] \frac{\text{rad}}{\text{s}}$$

d) as $t \to \infty$ $\omega_2 = 200$ rad/s