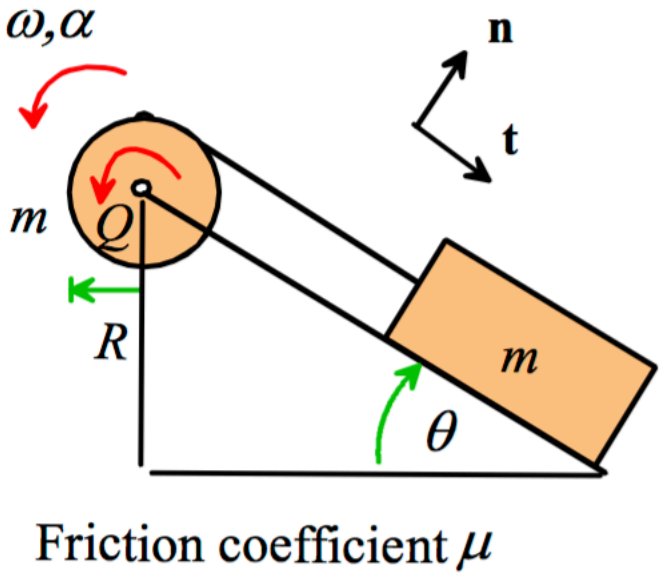
Homework 7: Rigid Body Dynamics
Due Friday April 29nd

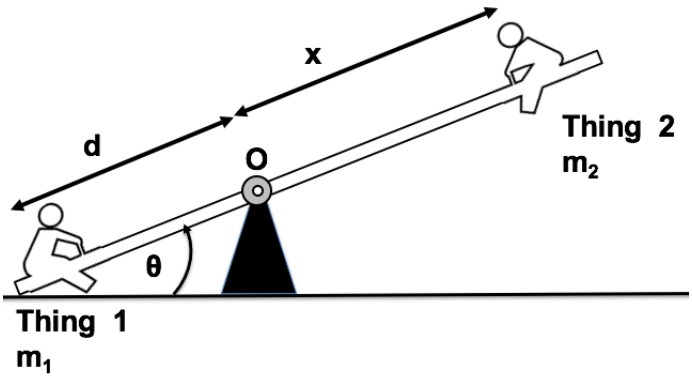
Problem 1 [10 pts]: A crate of mass m is pulled up a slope with angle θ by an inextensible cable that is wrapped around a pulley. The contact between the crate and slope has friction coefficient μ. The pulley has mass m and radius R and mass moment of inertia $mR^2/2$. It is rotated counterclockwise by a motor attached to an axle at its center, which exerts a moment (torque) with magnitude Q on the pulley. The bearings supporting the axle of the pulley are frictionless. The goal of this problem is to find a formula for the angular acceleration α of the pulley.



a) [1 pt] Draw the FBD of the pulley.
b) [1 pt] Draw the FBD of the crate.
c) [2 pts] Write out suitable expressions for $\mathbf{F} = m\mathbf{a}$ for the crate, in the ($\mathbf{n}$, $\mathbf{t}$) coordinate basis.
d) [2 pts] Write out suitable expressions for $M = I_G\alpha$ for the pulley.
e) [2 pts] Write the kinematics relationship between α and the acceleration of the crate.
f) [2 pts] Show that $\alpha = \frac{2}{3}(\frac{Q}{mR^2} - \frac{g}{R}(sin\theta + \mu cos\theta))$

Problem 2 [8 pts]: Professor Franck's children are playing on a see-saw in the backyard. To protect their identity we will call them "Thing 1" and "Thing 2". Thing 1 is older and larger, with a mass of m_1, whereas Thing 2 has a mass of m_2. The mass ratio is $m_1/m_2 = 6/5$. We will assume the see-saw bar is massless, that there is no friction in the bearings or elsewhere, and approximate that Thing 1 and Thing 2 are particles. Thing 1 has positioned herself on the seesaw a distance d from the pivot point and refuses to move.

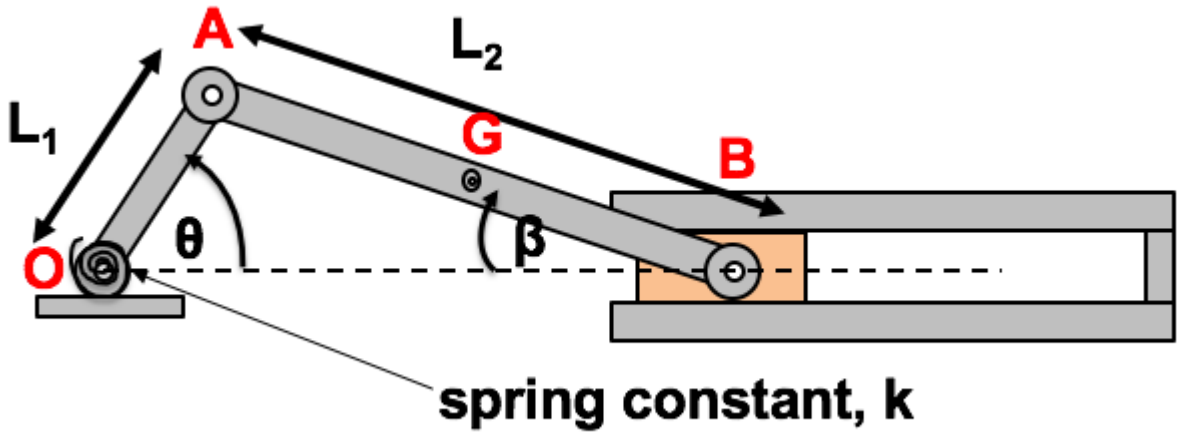
a) [1 pt] Draw a FBD of the see-saw
b) [1 pt] Find an expression for the distance x at which Thing 2 must position herself for the see-saw to begin moving.
c) [2 pts] What is the mass moment of Inertia about the pivot point, I_o?
d) [4 pts] Thing 2 suddenly moves a distance $2d$ from the pivot point. What is the initial linear acceleration experienced by Thing 1?



Problem 3 [8 pts]: The mass of a homogeneous cylindrical disk is m and its radius R. The disk is stationary when a constant clockwise moment (torque) M is applied to it. The disk rolls without slipping. Determine the velocity after it has travelled a horizontal distance b by:

a) [4 pts] Drawing a FBD, writing out 3 equations of motion, and calculate the velocity using kinematics.
b) [4 pts] Using energy methods.

Problem 4 [12 pts]: Consider the slider crank mechanism below, that has a torsional spring that produces a moment $M = -k\theta$ at the fixed point O. Slender bar OA has a mass m_1 and length L_1, and slender bar AB has a mass m_2 and a length L_2. The system is released from rest from a position θ_0.



a) [2 pts] Using position vectors or geometric arguments, calculate an expression for β in terms of θ, L_1 and L_2.
b) [4 pts] Write an energy balance between the initial release position and the position in which $\theta = 0$. Keep your answer in terms of g, k, ω_{OA}, ω_{AB}, L_1, L_2, m_1, m_2, V_G and θ.
c) [2 pts] Write an expression for the vector V_B as a function of ω_{OA}, ω_{AB}, L_1, L_2 and θ. What is the constraint on V_B? Find a relationship between ω_{OA} and ω_{AB}.
d) [2 pts] Write a vector equation relating the V_G (the velocity of the center of bar AB) to V_B. What is V_G when $\theta = 0$?
e) [2 pts] Let $m_2 = 2m_1$ and $L_2 = 2L_1$. Find an expression for the angular velocity ω_{OA} in terms of the initial angle θ_0, g, k, L_1, m_1.

Problem 5 [6 points]: A flywheel attached to an electric motor is initially at rest. At $t = 0$ the motor exerts a torque $M = 200e^{-0.1t}\ Nm$ on the flywheel. The moment of inertia of the flywheel is 10 kgm^2.

a) [1 pt] Plot or sketch the torque as a function of time.

b) [2 pts] What is the flywheel's angular velocity at $t = 10$ seconds.

c) [2 pts] Plot or sketch the angular velocity as a function of time (over a time interval of 100 seconds).

d) [1 pt] What is the maximum angular velocity the flywheel will attain?