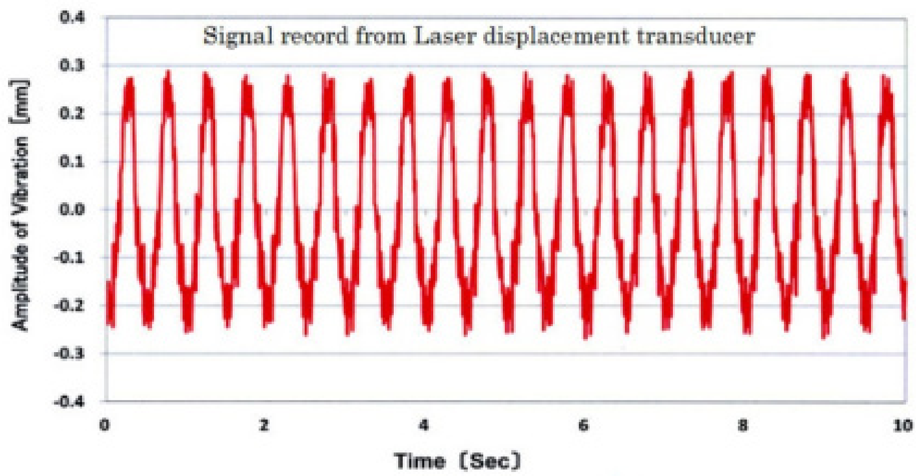
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 5: Vibrations
Due Friday March 24, 2017



1. The figure (from this publication) shows a vibration measurement from a displacement transducer attached to a bridge. Please calculate:

1.1 The amplitude of vibration

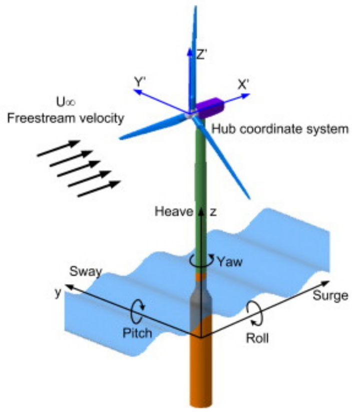
1.2 The period of the vibration

1.3 The frequency (in Hertz) and angular frequency (in rad/s)

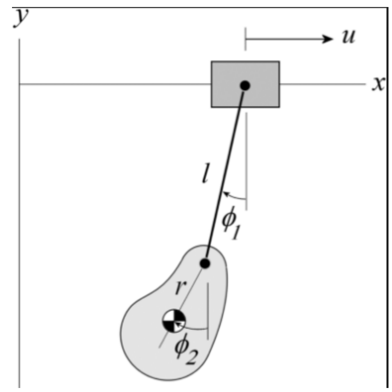
1.4 The amplitude of the velocity

1.5 The amplitude of the acceleration.

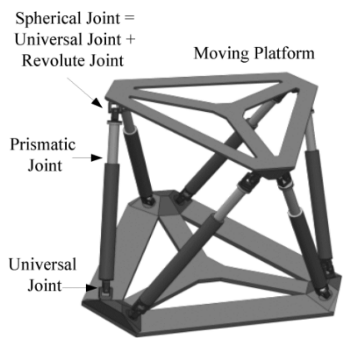
2. Find the number of degrees of freedom and vibration modes for each of the systems shown in the figures



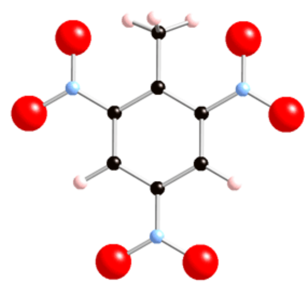
[Floating wind turbine]



[2D Idealization of an overhead crane]



[Stewart platform] (the base is fixed)



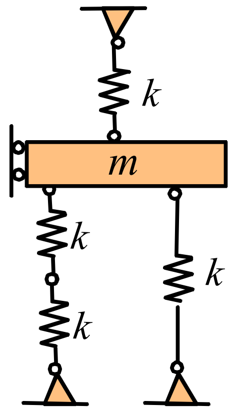
TNT Molecule

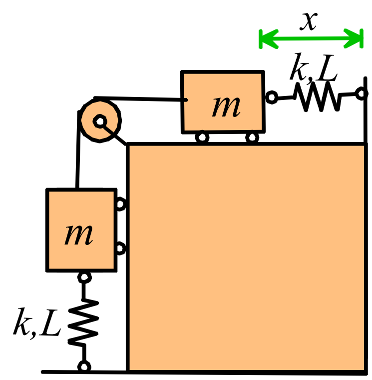
3. Use the tabulated solutions to solve the following differential equations

3.1 $\dfrac{d^2y}{dt^2} + 16y = 4 \qquad y = \dfrac{1}{4} \qquad \dfrac{dy}{dt} = -8 \qquad t = 0$

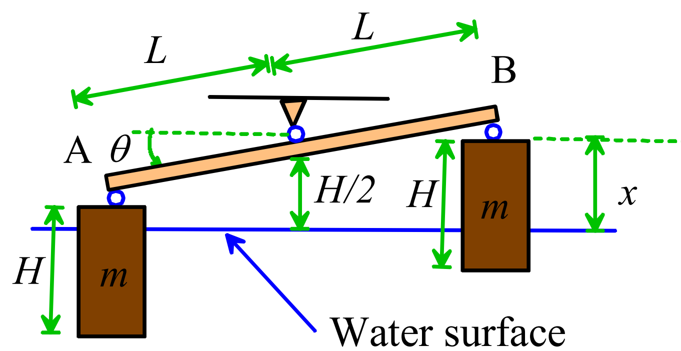
3.2 $\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} + 9y = 18\sin 3t + 36\cos 3t \qquad y = 0 \qquad \dfrac{dy}{dt} = 0 \qquad t = 0$ (Note that you can rearrange this into a 'Case 5' equation)

4. Find formulas for the natural frequency of vibration for the systems shown in the figure





5. The figure shows a schematic diagram of a wave energy harvesting device. It consists of two cylindrical buoys with cross-sectional area A, height H and mass m, attached to a bar with total length $2L$ and negligible mass. The buoys are immersed in water with mass density ρ.



During operation waves cause the bar AB to rock through a time-varying angle θ as shown. When $\theta = 0$ both buoys are half submerged $(x = H/2)$.

The goal of this problem is to analyze oscillation of the device in still water (no waves). No torque or friction acts at the pivot. Neglect the mass of AB and viscous drag of the water.

5.1 The buoyancy force acting (upwards) on one cylinder can be expressed in terms of its height x above the water surface as

$$F_B = \rho g A(H - x)$$

Show that (for $0 < x < H$) the potential energy of the buoyancy force is $V = -\rho g A\left(Hx - \frac{x^2}{2}\right) + C$ where C is a constant.

5.2 Write down a formula for the speed of the masses (which is equal to the speed of ends A and B of the bar) in terms of $L, d\theta/dt$

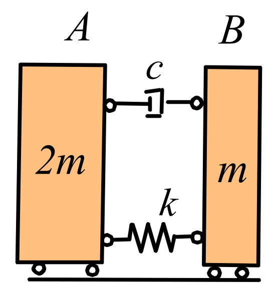
5.3 Hence, write down the total potential and kinetic energy of the system, in terms of ρ, A, L, m, g and θ and its time derivatives (assume that $0 < x < H$ at all times).

5.4 Hence, show that θ satisfies the equation of motion

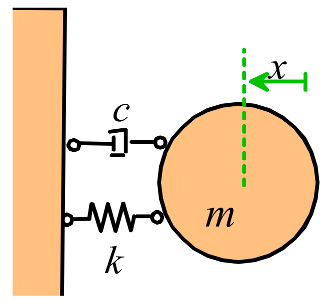
$$m\frac{d^2\theta}{dt^2} + \rho g A \sin\theta\cos\theta = 0$$

5.5 Linearize the equation of motion in 21.4 for small θ and hence find a formula for the natural frequency of vibration of the energy harvester, in terms of ρ, g, A, m.

6. When mass A is held fixed, and mass B vibrates, the system shown in the figure has a natural frequency ω_n and damping factor ζ. Find formulas for the new natural frequency and damping factor (in terms of ω_n ζ) when mass B is held fixed, and A vibrates.



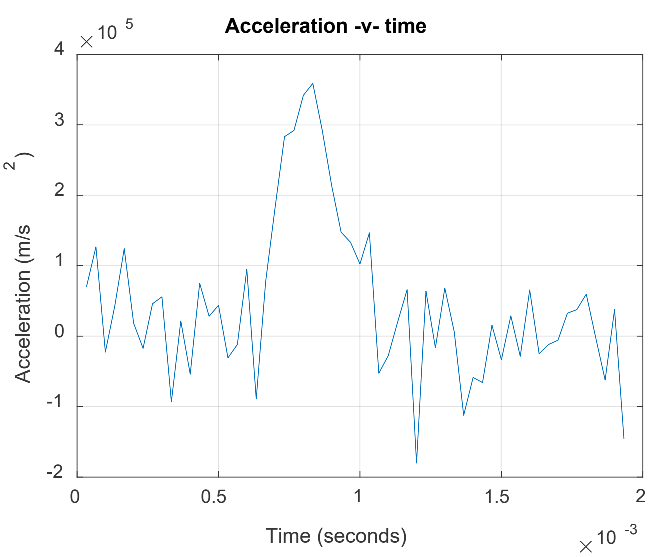
7. In this problem, we will estimate the contact stiffness and damping of the baseball that you analyzed in Homework 4. During impact, the baseball can be idealized as a spring-mass-damper system: the mass represents the interior core of the baseball; and the spring and damper represents the deformed part of the ball that is in contact with the steel plate. We can characterize the system using the natural frequency ω_n and damping factor ζ in the usual way.

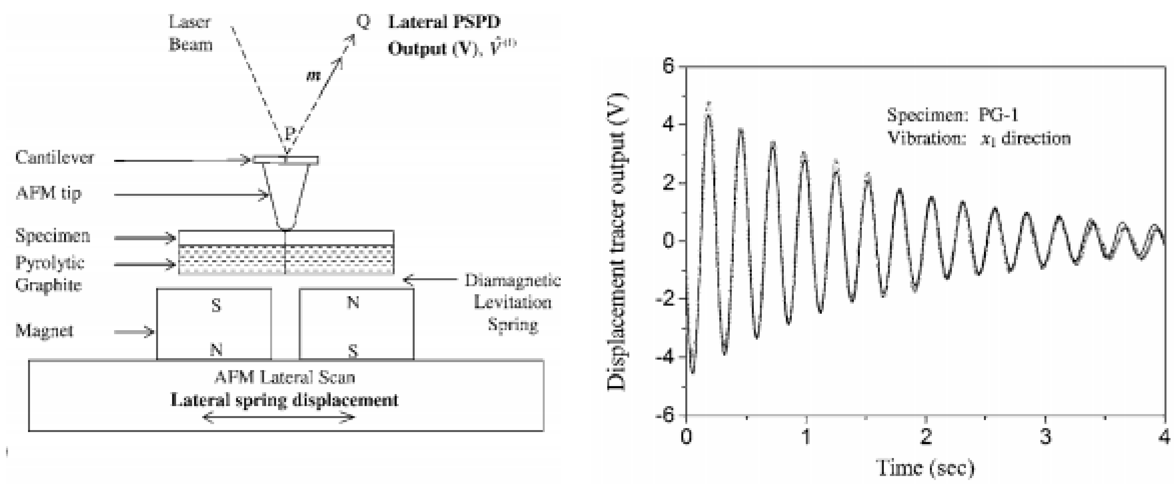


7.1 Assume that the center of the ball (i.e. the mass) is at position $x = 0$ and has velocity $dx/dt = -v_0$ just before impact. Write down an equation for the displacement *x(t)* and velocity $v(t)$ of the center of the ball (represented by the mass in the figure) during the impact, in terms of the natural frequency ω_n and damping factor ζ for the system.

7.2 If ζ is small, we can assume that the rebounding ball loses contact with the plate when $x(t) \approx 0$ after the first cycle of vibration. With this approximation, show that the restitution coefficient for the collision is $e \approx \exp(-\pi\zeta)$.

7.3. The graph shows the acceleration-v-time of the center of mass of the baseball calculated in HW4. Estimate values of ω_n, ζ (you can use the plot of acceleration –v- time to estimate the time that the ball is in contact with the plate) . Assuming the ball has mass 145 grams, calculate values for the effective spring stiffness k and dashpot coefficient c .





8. In this publication, professor K-S Kim's group at Brown describe a new approach to measure the stiffness of an atomic force microscope tip. Their approach is to push on a very soft spring with the AFM tip. If the spring stiffness is known, the force on the AFM tip can be calculated. Their 'spring' actually consists of a thin sheet of graphite suspended over four magnets (see the figure): the magnets exert restoring forces that keep the sheet centered over the magnets, and so act like a spring. The spring stiffness can't easily be calculated, so instead, they measured it by doing a vibration experiment. The figure shows the free vibration response of the graphite sheet (measured using a laser interferometer).

8.1 Estimate the period and the log decrement from the vibration response.

8.2 Hence, calculate the natural frequency ω_n and damping coefficient ζ for the system

8.3 The graphite sheet has mass m=15.052 milligrams. Calculate the effective spring stiffness of the magnetic levitation system.