

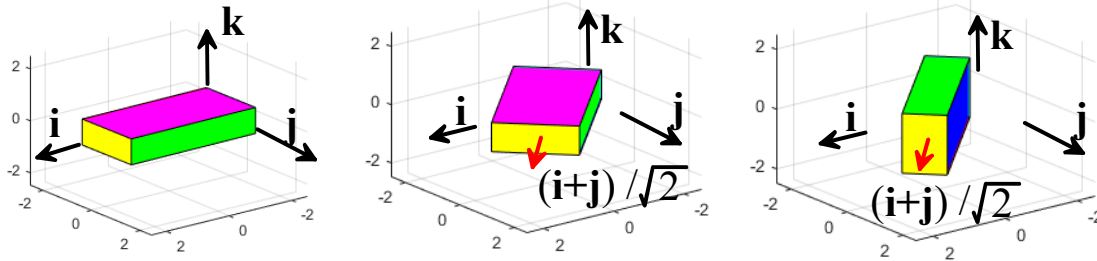
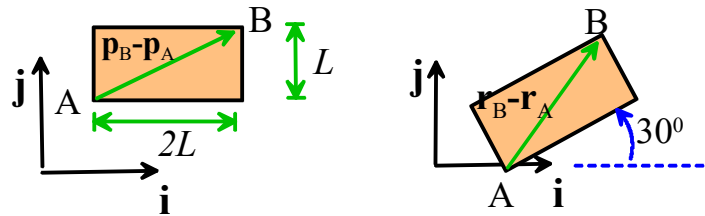


EN40: Dynamics and Vibrations

Homework 7: Rigid Body Kinematics, Inertial properties of rigid bodies
Due Friday April 21, 2017

School of Engineering
Brown University

1. Find the rotation tensor (matrix) that rotates the rectangle from its initial to its final position shown in the figure. Hence, express the rotated vector $\mathbf{r}_B - \mathbf{r}_A$ in (\mathbf{i}, \mathbf{j}) components.



2. The rectangular prism shown in the figure is subjected to two sequential rotations:

- (1) A 45 degree rotation about the \mathbf{k} axis
- (2) A 90 degree rotation about a line parallel to $\mathbf{n} = (\mathbf{i} + \mathbf{j}) / \sqrt{2}$

2.1 Write down the rotation tensor (matrix) for each rotation

2.2 Find the rotation matrix $\mathbf{R} = \mathbf{R}^{(2)}\mathbf{R}^{(1)}$ that describes the combined effects of both rotations (you can use Mupad or matlab do do the matrix multiplication, but there is no need to submit the code or script).

2.3 Find the axis \mathbf{n} and rotation angle θ that will complete the rotation \mathbf{R} directly.

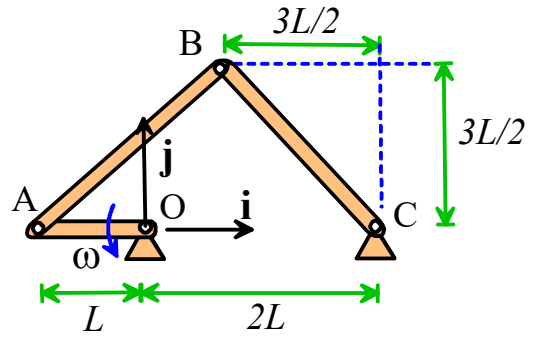
Optional: You can check your answer by downloading a matlab script from the homework page of the course website that will animate a rigid rotation through an angle θ about an axis parallel to a unit vector \mathbf{n} . You can use the code by navigating to the directory storing the file in the Matlab command window, and then typing

```
Animate_rotation(angle,[n_x,n_y,n_z ])
```

Where *angle* is your solution for the rotation angle θ (in radians), and n_x, n_y, n_z are the components of your solution for the unit vector \mathbf{n} .

3. The figure shows a four-bar chain mechanism. Member AO rotates counterclockwise at constant angular speed ω .

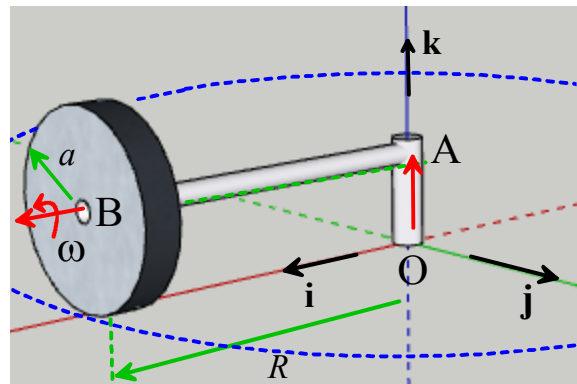
At the instant shown, calculate the angular velocity and acceleration of links AB and BC.



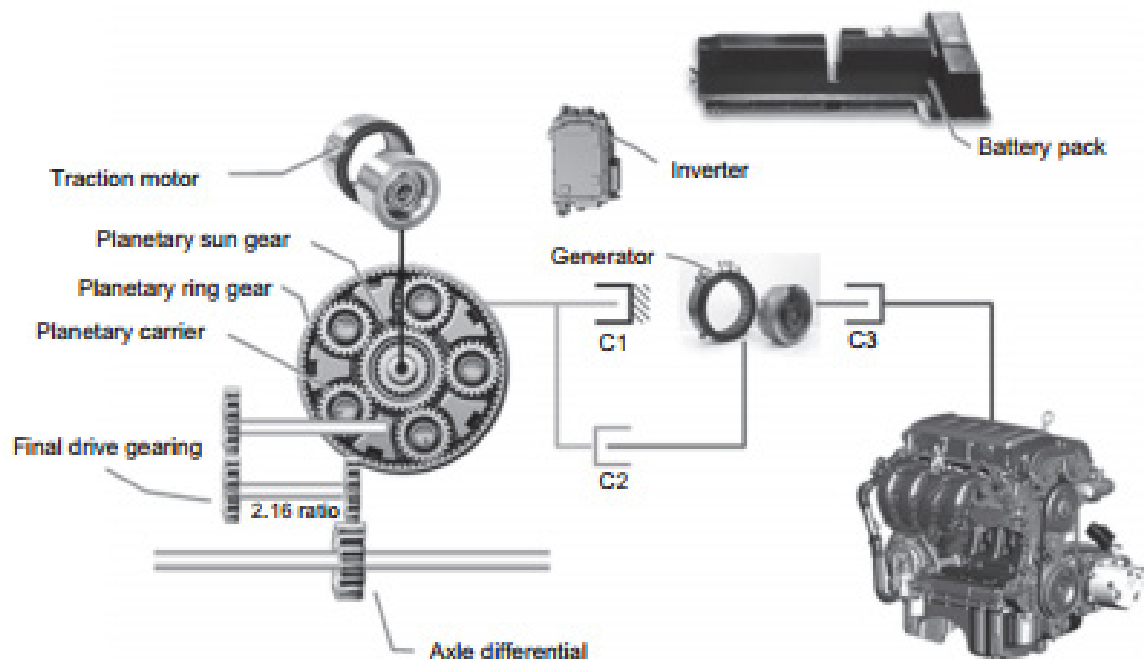
4. The wheel shown in the figure rotates at angular speed ω about its axle, and the wheel rolls without slip. Calculate

4.1 The velocity vector of the center of the wheel B at the instant shown

4.2 Calculate the angular velocity of the axle AB (assume that there is a bearing at B that allows the wheel to rotate on the axle and a bearing at O that allows OA to rotate. The connection at A is rigid).



Hence, determine the angular velocity vector of the wheel



5. The figure ([from this publication](#)) shows a schematic of the split-power transmission system for the first generation Chevy Volt ([the 2016 version is a lot more sophisticated](#)). The wheels are always connected to the planet carrier through the ‘final drive gearing’. The sun gear is driven by the electric ‘traction motor’, while the ring gear can either be locked (close clutch C1 and open C2), to make the car an electric vehicle, or connected to the internal combustion engine (open C1 and close C2,C3) to make the car a hybrid. We can find the following specifications for the vehicle:

- [Tire diameter 16.5 inches](#)
- The gear ratio between the planet carrier and wheels is $\omega_{PC} / \omega_W = 2.16$
- With the ring gear locked, the sun gear: planet carrier ratio is $\omega_S / \omega_{PC} = 3.24$

5.1 Design the planetary gear system necessary to achieve the required gear ratio (i.e. specify the smallest integer numbers of teeth N_S, N_R, N_P on the sun, ring and planet gears that will achieve the required ratios). You could start by finding the number of teeth on the sun and ring gears, using the general epicyclic gear ratio formula from class and the given $\omega_S / \omega_{PC} = 3.24$. Once you have N_S, N_R you can find N_P from geometry (recall that the number of teeth on each gear is proportional to its radius)

5.2 If the car drives at 30 mph in EV mode (ring gear locked), calculate (i) the angular speed of the wheels; (ii) the angular speed of the planet carrier, and hence (iii) calculate the required speed of the electric motor.

6. The figure shows three particles with equal mass m connected by rigid massless links.

6.1 Calculate the position of the center of mass of the assembly

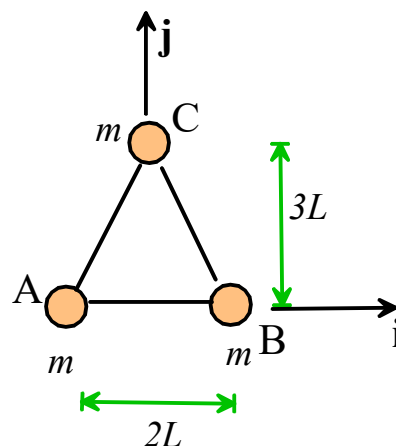
6.2 Calculate the 2D mass moment of inertia of the system about the center of mass

$$I_{Gzz} = \sum_i m_i (d_{xi}^2 + d_{yi}^2)$$

where $\mathbf{d}_i = d_{xi}\mathbf{i} + d_{yi}\mathbf{j} = \mathbf{r}_i - \mathbf{r}_G$ is the position vector of the i th particle with respect to the center of mass.

6.3 Suppose that the assembly rotates about its center of mass with angular velocity $\omega\mathbf{k}$ (the center of mass is stationary). What are the speeds of the particles A,B and C?

6.4 Calculate the total kinetic energy of the system (a) using your answer to 6.2; and (b) using your answer to 6.3



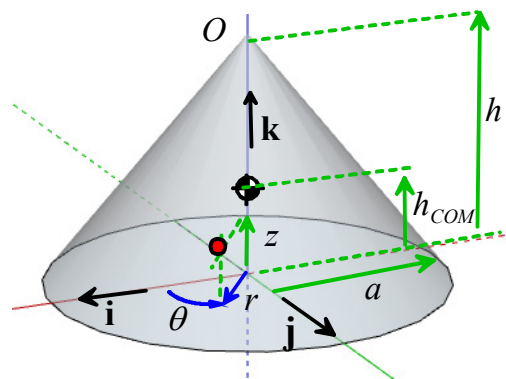
7. The figure shows a cone with height h and base radius a and uniform mass density ρ . Using Mupad, calculate

7.1 The total mass M

7.2 The height of the center of mass h_{COM} above the base

7.3 The inertia tensor (matrix) about the center of mass, in the basis shown

7.4 Using the parallel axis theorem, calculate the mass moment of inertia about the tip O.



Please upload your Mupad script to Canvas as a solution this problem

8. Flywheel energy storage systems are an alternative to batteries. The [Beacon](#) BP400 flywheel is used in several [experimental grid-scale energy storage facilities](#). It has the following specifications:

- Cylinder height: approx. 7 ft
- Cylinder diameter: 3ft
- Cylinder mass: 2500lb
- Angular speed: 15500 rpm

Calculate

8.1 The total energy stored in the flywheel (in Joules – please convert the info provided to SI)

8.2 The energy density (energy per kg) For comparison the energy density of an Li ion battery is about 150 watt-hours/kg, which is $150 \times 3600 \text{ J/kg} = 540 \text{ kJ/kg}$

8.3 The magnitude of the angular momentum vector of the cylinder, when spinning at full speed

