



EN40: Dynamics and Vibrations

Homework 1:

Mupad and MATLAB practice

Due 12:00 noon Friday February 2 (online submission)

School of Engineering
Brown University

- Your solution to this homework should consist of two files:
 1. A commented MATLAB Live Script stored as a .mlx file
 2. A commented MATLAB function stored as a .m file
- Please submit the assignment electronically on the EN40 canvas website. You can log into canvas at <http://brown.edu/it/canvas/> (the login link is near the top right of the page). You can find some instructions on Canvas use (from CIT) at <https://ithelp.brown.edu/kb/articles/canvas-for-students>

Part 1: Write a Matlab ‘Live Script’ to solve the following problems.

1. Find all the solutions to the simultaneous equations

$$x^2 + y^3 = 4 \quad x + y - xy = 1$$

2. Plot the function $P(x, \gamma) = \gamma x^{\gamma-1} \exp(-x^\gamma)$ (exp denotes the exponential function, i.e. e^x) in the range $0.05 < x < 3$, for $\gamma = 0.5, \gamma = 2, \gamma = 5$ (on the same plot). ($P(t)$ is the ‘Weibull’ distribution function - it is used to predict failure probabilities in systems governed by ‘weakest link’ statistics). A few points of credit are awarded for making the plot look nice.

3. Evaluate the integral

$$\int_0^x P(\xi, \gamma) d\xi$$

4. Find the value of x that maximizes $P(x, \gamma = 5)$ (you can find potential maxima by differentiating P with respect to x ; then solving $dP/dx = 0$. MATLAB will give you a whole bunch of solutions. Your plots in problem 3 should help you see whether the solutions correspond to maxima, minima, or inflections).

NOTE: One of you found a bug in MATLAB 2017b that may crash matlab when you try to solve this problem (see Piazza announcement). It seems to be possible to fix this by using the ‘simplify’ command when you solve the equation, i.e. instead of using

```
... Calculate dPdx
xatmax = solve(dPdx==0,x)
use
xatmax = simplify(solve(dPdx==0,x))
```

5. Find the maximum value of $P(x, \gamma = 5)$ (use your answer to 4 and the ‘subs’ function)

6. (Challenge problem meant for people with a lot of math/matlab experience. It is worth only 2 points so if you get stuck on this or don't understand the matlab output skip it). Find the value of x that maximizes $P(x, \gamma)$ for an arbitrary value of γ . Work out how to tell MATLAB that γ is real, greater than zero, and find only the real valued solutions for x .

7. The differential equation

$$\frac{df}{dt} = rf(t) - d(f(t) + cf(t)^2) - H$$

is used to estimate the effects of harvesting on populations of fish. The variable f denotes the number of fish; r is a constant characterizing the rate at which they reproduce; d and c characterize the rate at which they die¹ and H is the rate of population depletion from fishing.

7.1 Use the 'dsolve' function to solve the differential equation for f . Use the initial condition $f(t) = f_0$ $t = 0$

7.2 Plot graphs (on the same figure) of $f(t)$ for $r=2$ per month; $d=0.05$ per month, $c=0.01$, $f_0 = 1000$ fish, for $0 < t < 6$ (months), with (i) $H=0$, (ii) $H=1300$ per month, and (iii) $H=1451$ per month. You will probably find that the plot fails because Matlab finds a negligibly small imaginary part of $f(t)$ after substituting values: you can fix this by plotting the real part of $f(t)$ (use the 'real()' function). You get a point for making the plot look nice...

Part 2: Please solve the remaining problems using MATLAB (write your code in a matlab .m file). You should make your MATLAB (.m) file a function, so that when the file is executed, it will solve all the homework problems. For example:

```
function EN40_Homework1_2018
    code that will solve problems 8-12
end
function B = update_matrix(A)
... Code
end
```

(You might find the solutions to homework 1, 2009-2017 helpful, if you get stuck – you could copy one of those homeworks and use it as a template, if you like.)

8. Using a loop, or dot notation, or the 'linspace' function, create a vector t of 500 equally spaced points between $-5\pi/12$ and 16π

9. Using the solution to problem 8, create two vectors x and y that contain values of the function

$$x = e^{-0.1t} \sin t$$

$$y = e^{-0.1t} \cos 2t$$

Hence, plot a graph of $y-v-x$. (You will get a few points for making your plot look nice!)

¹ (painlessly, after long, productive and happy lives)

10. Use the MATLAB 'magic()' function to create a 5x5 'Magic' matrix.

10.1 Calculate the sum of each row of the matrix (use a loop, or the 'sum' command)

10.2 Calculate the sum of each column of the matrix

10.3 Calculate the eigenvalues of the matrix (use the 'eig' function), and determine the sum of all the eigenvalues

11. Write a MATLAB function of the form B=update_matrix(A) that takes as input a matrix A with elements A(i,j) either zero or one; and returns a second matrix B (with the same size as A). B should be calculated inside your function as follows:

(i) Set B=A, then

(ii) Find the number of rows n and columns m of A (use the 'size' function)

(iii) For each $2 < i < n-1$ and $2 < j < m-1$ (use two nested loops over i and j and start the loops at 2 and end at $n-1$ and $m-1$) calculate $S = \left[\sum_{k=i-1}^{i+1} \sum_{l=j-1}^{j+1} A(k,l) \right]$ (The double sum denotes summing over both k and l –

you are adding up all the numbers in a small 3x3 matrix stored between $i-1, j-1$ and $i+1, j+1$. You could use the matlab 'sum' function to calculate S – but read the manual to be sure how it works! - or just type in all 9 terms) then

(iv) calculate B(i,j) from the following formula (use a conditional statement inside the loop from part (iii))

$$B(i, j) = \begin{cases} 1 & S = 3 \\ A(i, j) & S = 4 \\ 0 & \text{otherwise} \end{cases}$$

Test your code using the following lines of script (which should be in your main homework function)

```
A = zeros(50);
A(4:8, 23:27) = 1;
A(18:27, 23:27) = 1;
A(39:48, 24:26) = 1;
for k=1:182
    A = update_matrix(A);
    imshow(1-A, 'Initialmagnification', 500);
    pause(0.1);
end
```

If your code works, you should end up with a crude picture of the face of a bear². Go Bruno! You might recognize this code as a rather inefficient implementation of [Conway's 'Game of Life...'](#)

² If the picture does not look like a bear to you, try working on this homework really, really late at night.

12. The [‘Hindmarsh-Rose’ equations](#) are used physiology to describe how voltage spikes are emitted by neurons or cardiac cells³. The model predicts how the voltage V across a cell membrane varies with time, when an external current I crosses the cell membrane. The electrical response is mediated by the fast transport of sodium and potassium through ion-channels in the membrane, as well as by the slower transport of other chemicals. The equations have the form

$$\begin{aligned}\frac{dV}{dt} &= I + y + 3V^2 - V^3 - z \\ \frac{dy}{dt} &= 1 - 5V^2 - y \\ \frac{dz}{dt} &= r \left\{ 4V - z + \frac{32}{5} \right\}\end{aligned}$$

where $V(t), y(t)$ and $z(t)$ are time-dependent dimensionless variables representing the membrane potential, the ionic current of potassium, and a slower current of other chemicals, respectively; I is the applied current (a constant), r is a constant, and t is a (dimensionless) time.

12.1 Write a MATLAB function of the form `dwdt = neuron(t, w, I, r)` function that computes the vector of time derivatives $d\mathbf{w} / dt = [dV / dt; dy / dt; dz / dt]$ given a value of t and the current values of $\mathbf{w} = [V; y; z]$ and the values of the constants I and r .

12.2 Hence, use the MATLAB `ode45` function to calculate the variation of $V(t), y(t), z(t)$ with time, and plot the voltage V as a function of time. Use the following parameters:

- Initial conditions $V = -1, y = -12, z = 2$ at time $t=0$;
- Time interval $0 < t < 1500$
- Plot V as a function of time for $I=2$ and $r=0.001$. This produces ‘bursts’ of firing (it should look like Fig 6 in Hindmarsh and Rose). Hint: remember that `ode45` will give you a matrix that contains solutions for all three of V, y and z – if you just want to plot V and not the other two you can use `plot(times, sols(:, 1))`
- On a separate figure, plot V as a function of time for $I=3.25$ and $r=0.005$. Set the relative tolerance to 0.00001 for this simulation (use the ‘odeset’ command). This produces a series of bursts, but with a random number of spikes in each burst.

³ There are a large number of models of this kind – [Wikipedia](#) gives a list of references if you are curious. The ‘Hindmarsh-Rose’ equations were developed to predict periodic ‘bursts’ of voltage spikes, which had been observed experimentally, but were not predicted by earlier models.