School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 2: Kinematics and Dynamics of Particles
Due Friday Feb 9, 2018

1. ‘Bloodhound’ is an experimental vehicle designed to break the land speed record (with a goal of eventually achieving 1000mph). The vehicle has a mass of 7500kg. It is accelerated to its maximum speed by a combined jet and rocket engine; and then slowed by aerodynamic braking. The average acceleration is expected to be about 1g; the average deceleration is about 0.8g.

1.1 How long will the vehicle take to reach 1000mph?

1.2 How much distance is required for the test (including both acceleration and deceleration)?

2. In this article, a group from Brown led by Prof Mathiowitz tracked the motions of magnetic pills inside the stomachs of various animals (including humans) to estimate the forces acting on the pills. Your goal in this problem is to repeat their calculations. The raw data consists of (x,y,z) coordinates (in mm) of the position of the pill at 0.1 second intervals. You can download the data from the links below (store the files with a .csv extension) and you can look at the file with MS excel.

- Position data for a fed rat
- Position data for a fasted rat

The pills used in rats have mass 4.5 grams.

Write a MATLAB script that accomplishes the following tasks (Your MATLAB code should be uploaded to CANVAS as a submission to this problem):

2.1 Read the data into a matrix using the MATLAB ‘csvread’ command. Note that since the data has a header row you will need to use (eg, assuming you saved your data in a file called Fed_Rat.csv)

```
data_Fed = csvread('Fed_Rat.csv',1,0);
```

to skip the first row.

2.2 Plot the trajectory (as a 3D curve) for both files (on separate plots)

2.3 Calculate the velocity vector of the pill as a function of time. You can do this by calculating the change in position between two successive readings, and dividing by the time difference between them, e.g. if $x(i)$ denotes the ith value of x, then the **i** component of acceleration is

$$v_x(i) = \left(x(i+1) - x(i) \right) / \left(t(i+1) - t(i) \right)$$

You can construct the vector v_x using a loop in MATLAB, e.g. to find the x component of velocity

```
for i=1:length(time)-1
    vx(i) = (x(i+1)-x(i))/(time(i+1)-time(i));
end
```

Here x is a vector containing the *x* component of position and vx is a vector containing x components of velocity (note that vx contains one point fewer than the vector x). Alternatively, you could use the built-in MATLAB 'diff' function (see the MATLAB manuals for details).

2.4 Use the solution to 2.3 to calculate the acceleration vector of the pill as a function of time. You will need to do this with a second loop (or use the 'diff' function again).

Use your data to calculate and plot the magnitude of the force acting on the pill as a function of time (you will have to calculate the magnitudes with a loop). Plot the data for fed and fasted rats on separate figures.

2.5 Use the MATLAB 'trapz' function (this is slightly different from the 'cumtrapz' function used in class) to integrate the magnitude of the force with respect to time (note: integrate magnitude of the force, do not integrate the force and then take its magnitude. The time integral of the force must be approximately zero, or the pill would fly out of the rats stomach!). Hence, compute and compare the average force magnitudes acting on a pill inside a fasted and a fed rat.

3. The goal of this problem is to estimate more accurately the variations of acceleration, speed and distance traveled by the Bloodhound as functions of time. An attempt at the speed record will have 3 phases (i) the car will accelerate under jet engine power to 150 m/s; (ii) acceleration will continue for 20 secs under combined jet engine and rocket power; (iii) The jet engine and rocket will be shut off, and air brakes and a parachute will be deployed to slow the vehicle. Its designers provide the following data

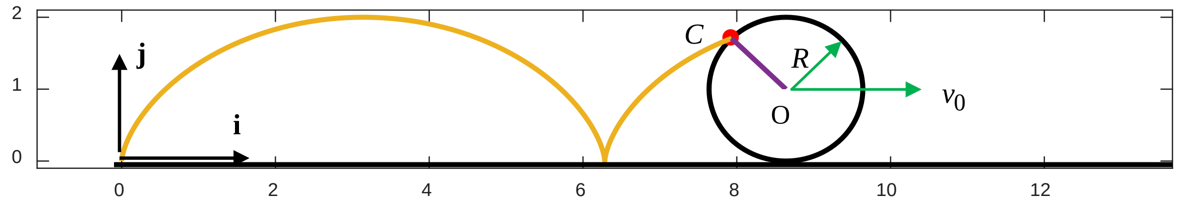
- Vehicle mass 7500kg
- Total thrust during acceleration: 90kN for velocities between 0 and 150 m/s (under jet engine only) and 200kN for velocities above 150 m/s (under combined jet engine and rocket propulsion)
- The rocket fires for 20 seconds
- Drag during acceleration $F_D = cV^2$, where *V* is the speed and *c=0.7* Ns^2/m (estimated by us)
- Drag (aerodynamic braking) during deceleration $F_D = cV^2$ where *c*=1.1 (estimated by us) (there is no engine thrust and wheel brakes are not applied until low speeds)

3.1 To make the calculus simple, neglect the drag during acceleration (phases i and ii). Use Newton's laws to obtain formulas (or in some cases just values) for the acceleration of the vehicle during the following three phases of the run: (i) under jet engine power only; (ii) under combined rocket and jet propulsion; and (iii) under aerodynamic braking (this will be a formula in terms of *V*).

3.2 Hence, integrate the accelerations to calculate the speed and distance traveled as a function of time (**Optional –** if you can figure out how to do it, account for drag during phases (i) and (ii) - MATLAB can do the rather messy calculus, as long as you substitute numbers into the formulas before doing them**).** There is no need to submit Matlab scripts for this problem (and only the optional problem really needs matlab at all).

3.3 Use MATLAB to plot a graph of the velocity as a function of time. You can compare your prediction with the designer's data here.

3.4 What do you think might cause the largest errors in this estimate? How would you go about estimating the magnitudes of the errors? What would you have to do to correct them?



4. A point C on the circumference of a wheel that rolls without slip on a flat surface follows the parametric curve

$$x = v_0 t - R\sin\left(\frac{v_0 t}{R}\right) \qquad y = R - R\cos\left(\frac{v_0 t}{R}\right)$$

where R is the radius of the wheel, v_0 is the speed of its center (at O), and t is time. **Assume that v_0 is constant throughout this problem.** You might find the animation here helpful to visualize the motion of C.

4.1 Find formulas for the velocity and acceleration vectors of the point C on the wheel in {**i**,**j**} components, in terms of v_0, t, and R.

4.2 What is the magnitude of the acceleration of point C, in terms of v_0 and R?[1]

4.3 Find a formula for the time(s) when C touches the ground (i.e. $y=0$) in terms of v_0 and R. What is the speed of C at these instants?

4.4 Find a formula for the time(s) when C is at its greatest height above the ground (i.e. $y=2R$). What is the speed of C at these instants? What is the direction of the acceleration (i.e. give a unit vector parallel to the acceleration in **i**,**j** components)

4.5 Use 4.4 and 4.2 to calculate the radius of curvature of the path of C (in terms of R) at the instants when C is at its greatest height above the surface.

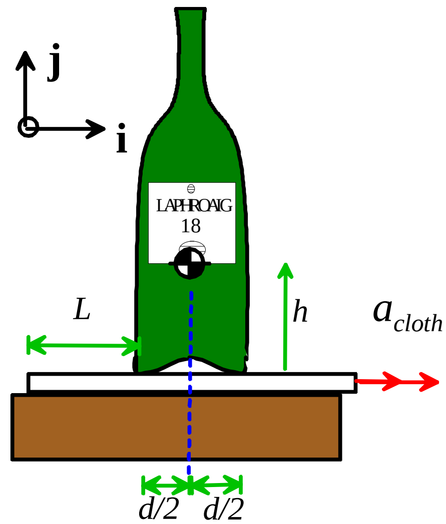
4.6 Find formulas for unit vectors normal and tangent to the path of C at time $t = \pi R/(3v_0)$ (express the unit vector in **i**,**j** components) (hint: think about the direction of the velocity vector, and note that **n** must be perpendicular to the tangent).

4.7 Use 4.6 and 4.1 to calculate the normal and tangential components of acceleration at time $t = \pi R/(3v_0)$

4.8 What is the radius of curvature of the path of C at time $t = \pi R/(3v_0)$?

[1] (For discussion: notice that you get something that looks like the circular motion formula, but V is the speed of O, instead of C; and R is the radius of the wheel, not the radius of curvature of the path. Can you explain the result in language your parents would understand?)

5. A bottle with mass m rests on a table-cloth. The contact between them has friction coefficient μ. At time $t=0$ the object and cloth are both stationary. For time $t>0$, the cloth is pulled with constant acceleration $a_{cloth} = 5\mu g$ the right. Note that since $a_{cloth} > \mu g$, slip must occur at the contact just after $t=0$.



5.1 Draw a free body diagram showing the forces acting on the bottle.

5.2 Find a formula for the horizontal acceleration of the bottle. Assume that the bottle does not tip over.

5.3 What is the maximum possible value of the friction coefficient for the bottle not to tip (express your answer in terms of h,d?

5.4 Find a formula for the horizontal distance moved by the bottle as a function of time.

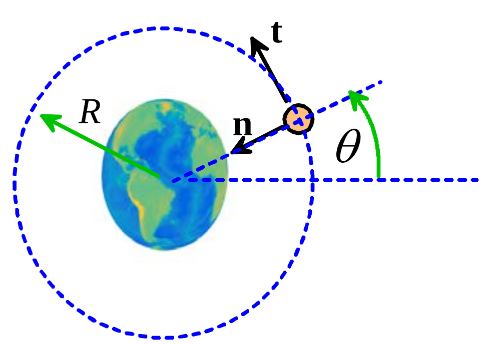
5.5 Find a formula for the distance moved by the cloth as a function of time

5.6 At time $t=0$ the bottle is a distance L from the edge of the cloth. As the cloth is pulled, L will decrease, but the bottle will move to the right (relative to a stationary observer). Calculate the total distance that the bottle has moved in the **i** direction (again, relative to a stationary observer) when $L=0$. Express your answer in terms of L and any other parameters you think are relevant. (this tells you how far from the edge of the table you need to put the bottle to make sure it wont fall off!)

6. The figure shows a satellite with mass m in a circular orbit around the earth. The satellite is subjected to a radial force

$$\mathbf{F} = \frac{\mu m}{R^2}\mathbf{n}$$

where $\mu = 3.986012 \times 10^5 \text{ km}^3\text{s}^{-2}$. The satellite takes a time T to complete a full orbit.



6.1 Find a formula for the velocity vector of the satellite, in terms of R and T. Express your answer in the normal-tangential basis $\mathbf{n},\mathbf{t}$ shown.

6.2 Find a formula for the acceleration vector of the satellite (in **n-t** coordinates), in terms of R and T.

6.3 Use Newton's law to find a formula for R in terms of T and other relevant variables

6.4 Find the radius (in km) and speed (in km/s) of (i) a satellite in low earth orbit with T=100 minutes; (ii) the moon (its orbit is very close to circular).