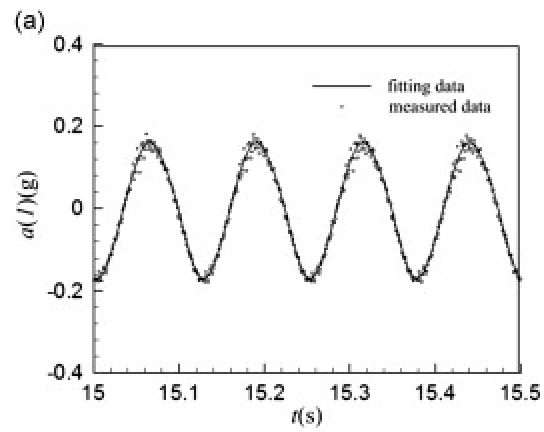
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 5: Vibrations
Due Friday March 23, 2018

1. The figure (from this publication) shows a vibration measurement from a vibration isolation system. Use the figure to estimate



1.1 The amplitude of the acceleration

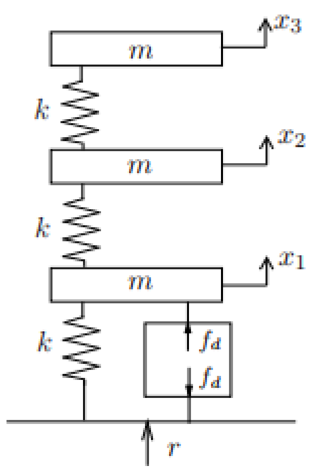
1.2 The period of the vibration

1.3 The frequency (in Hertz) and angular frequency (in rad/s)

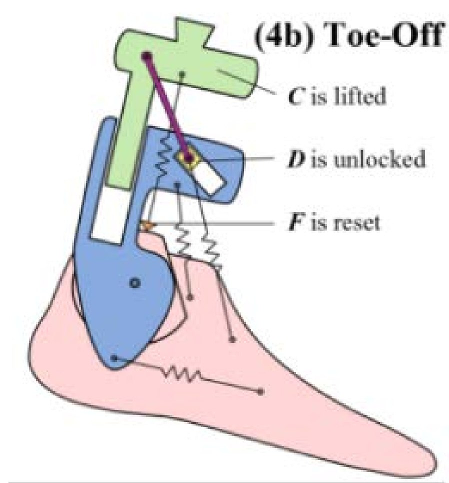
1.4 The amplitude of the velocity

1.5 The amplitude of the displacement

2. Find the number of degrees of freedom and vibration modes for each of the systems shown in the figures

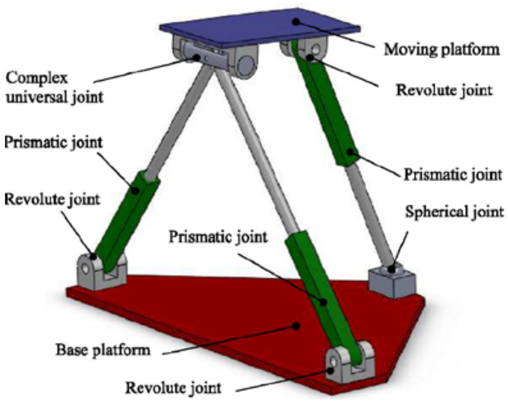


(b) Model of a 3 storey building with vibration suppression

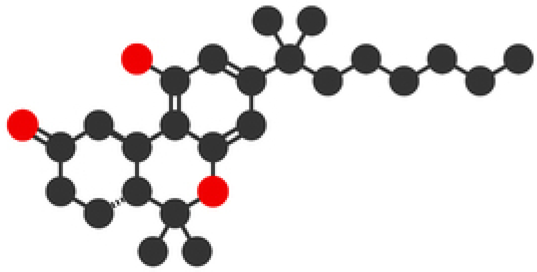


(a) 2D Model of an artificial joint Assume C is fixed, and count only the foot (pink) and ankle (blue)

2 (continued) . Find the number of degrees of freedom and vibration modes for each of the systems shown in the figures



(d) Model of an articulated platform



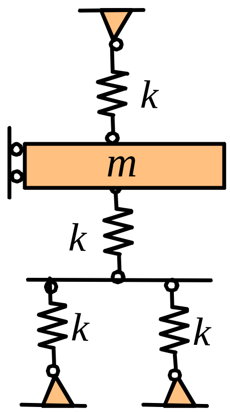
(c) THC Molecule

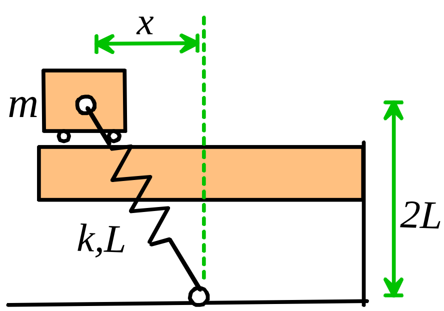
3. Solve the following differential equations

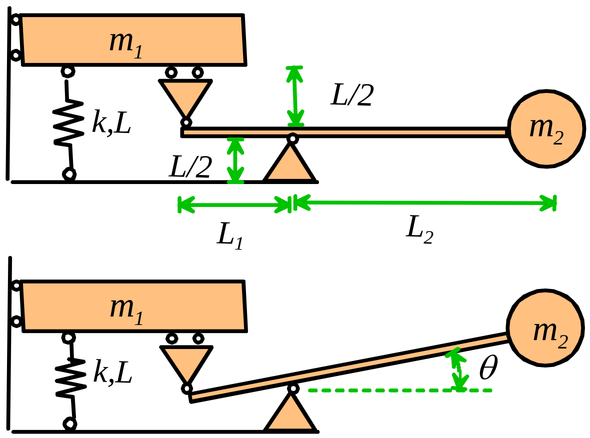
3.1 $\dfrac{d^2y}{dt^2} + 25y = 100 \qquad y = 0 \qquad \dfrac{dy}{dt} = 0 \qquad t = 0$

3.2 $\dfrac{d^2y}{dt^2} + 100\dfrac{dy}{dt} + 25y = 50\sin(t) \qquad y = 0 \qquad \dfrac{dy}{dt} = 0 \qquad t = 0$

4. Find formulas for the natural frequency of vibration for the systems shown in the figure







5. The figure shows a design for an ‘anti-resonant’ vibration isolation system (see here for the patent; this reference analyzes the system in detail).

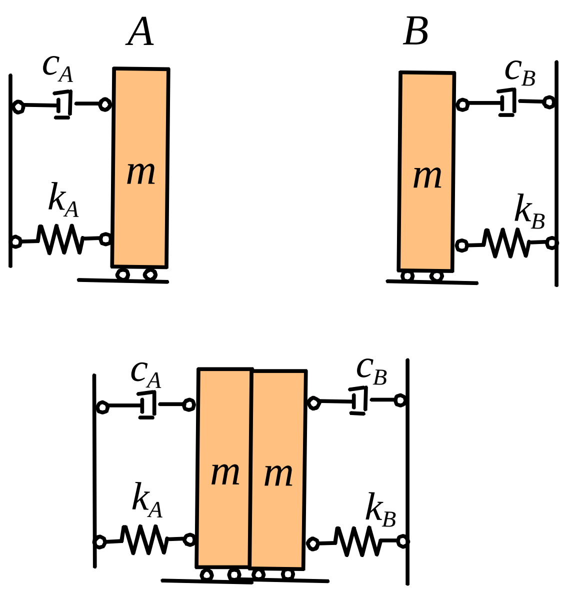
5.1 Find formulas for the kinetic and potential energy of the system (**neglect gravity**) in terms of θ and other relevant variables.

5.2 Find the equation of motion for the angle θ

5.3 Hence, calculate a formula for its natural frequency.

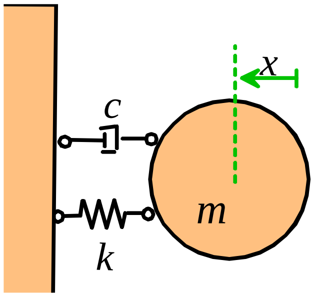
6. When the two masses shown in the figure are un-coupled, systems A and B have natural frequency and damping factor ζ_A, ω_{nA} ζ_B, ω_{nB}

If the two masses are connected together, what are the natural frequency and damping factor of the new system?



7. The impact of a baseball with a flat rigid wall is idealized as a spring-mass system (this high-speed movie might help visualize the impact)

7.1 Assume that the center of the ball (i.e. the mass) is at position $x = 0$ and has velocity $dx/dt = v_0$ just before impact. Write down an equation for the displacement *x(t)* and velocity $v(t)$ of the center of the ball (represented by the mass in the figure) during the impact, in terms of the natural frequency ω_n and damping factor ζ for the system (use the solution for the value of ζ that you think makes most sense, based on the behavior you see in the video).



7.2 In a homework problem from 2017, the class estimated the stiffness, damping coefficient and mass of the baseball as $m=$ 0.145kg, $k = 8.94MN/m$ $c = 364Ns/m$. Estimate the maximum value of x during the impact. Assume an impact velocity of 50 m/s.