School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 8: Rigid Body Dynamics
Due Friday April 27, 2018

1. The Fields Point wind turbines have the following specifications:

- Rated power: 1.5MW
- Rotor diameter: 92m
- Rated wind speed 13 m/s

1.1 Find a formula for the mass moment of inertia of the rotor, in terms of the total mass *M* of the three blades and the blade length *L* (approximate the blades as slender rods)

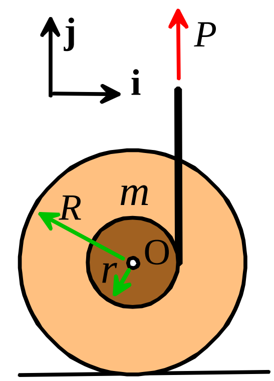
1.2 The mass of a MW class rotor can be estimated from the empirical relation $M = 0.486d^{2.6}$ where *M* is the mass in kg and *d* is the diameter in m. Estimate the angular speed of the turbine on a day that they are operating (you can just look out the windows on the 7th floor of B&H) and hence determine the kinetic energy of the rotor. Also estimate the tip speed ratio (the ratio of the speed of a blade tip divided by the wind speed – just google the wind speed).

1.3 Assume that the turbine operates at a constant tip speed ratio of 7. Calculate the angular speed at the rated wind speed of 13 m/s.

1.4 Calculate the rated torque (i.e. the torque on the wind turbine when it is producing the rated power of 1.5MW at the angular speed in 1.3)

1.5 The 'natural time constant' of a wind turbine is defined as the time required to spin up the rotor from rest to its rated speed under the rated torque (assumed to be constant). Estimate the natural time constant for the Fields Point turbines.

2. The figure shows a spool (e.g. a yo-yo) with outer radius *R*, mass *m* and (2D) mass moment of inertia $I_{Gzz} = mR^2/2$ resting on a table. The hub has radius *r*. A constant vertical force *P* is applied to the yo-yo string. The goal of this problem is to (i) find a formula for the (horizontal) acceleration of the spool, and (ii) find a formula for the critical value of *P* that will cause slip at the contact between the spool and the table



2.1 Draw a free body diagram showing the forces acting on the spool. Assume that the spool remains in contact with the surface, and that no slip occurs at the contact.

2.2 Write down the equations of linear (Newton's law) and rotational (the moment-angular acceleration relation) motion. Your equation should include forces from 2.1, and the linear and angular acceleration of the spool. Please state which point you are taking moments about for the moment equation.

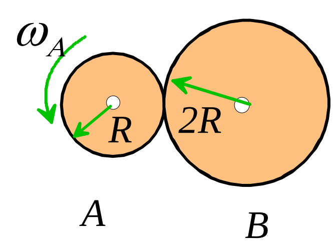
2.3 Write down a relationship between the angular acceleration α_z and linear acceleration $\mathbf{a}_G$ of the center of mass of the spool

2.4 Use 2.2 and 2.3 to find formulas for (a) the angular acceleration and (b) the linear acceleration of the spool in terms of *P*, and other relevant variables.

2.5 Find formulas for the reaction forces at the contact, in terms of *P*, *m*, *g*, *R* and *r*

2.6 The contact has a friction coefficient μ . Find a formula for the critical value of *P* at the point where the contact begins to slip

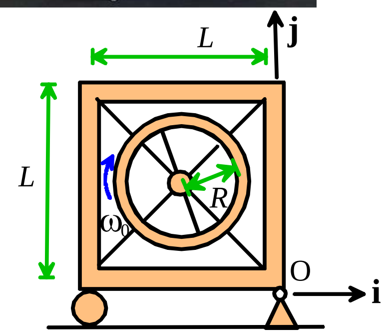
3. The two gears A and B in the figure have radii *R* and *2R*, and mass *m*. Their centers are stationary. Gear A rotates at angular speed ω_A . Find a formula for the total kinetic energy of the two gears in terms of *m*, *R* and ω_A



4. The 'Cubli' is used to develop control algorithms used to stabilize aircraft and spacecraft. It consists of a cube whose attitude can be controlled by spinning a set of reaction wheels inside the cube.

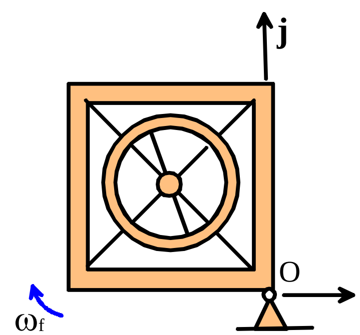
This simplified 1-D version of the device is used to test the algorithm that stands the cube up on one edge. The goal of this problem is to do the preliminary design calculations needed to set up the system.

Idealize the rectangular frame as four rods with length L and combined mass M and the spinning wheel as a ring with radius R and mass m. The corner at O is supported by a frictionless bearing.

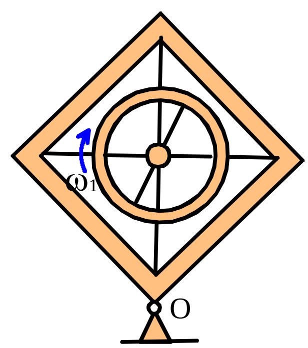


4.1 Find formulas for the mass moments of inertia of the frame and the wheel (about the center of the wheel), in terms of m, M, R and L.

4.2 The frame is at rest and the wheel is spun up (clockwise) to an angular speed ω_0 . Find a formula for the total angular momentum of the system about the corner at O, in terms of ω_0 and other relevant variables.



4.3 The wheel is then braked quickly, which causes the frame to rotate about the corner O at angular speed ω_f , while the motor driving the ring spins at (clockwise) angular speed ω_1 (note that this is relative to the frame). Write down the angular momentum of the system about O. in terms of ω_f and ω_1, and other relevant variables.

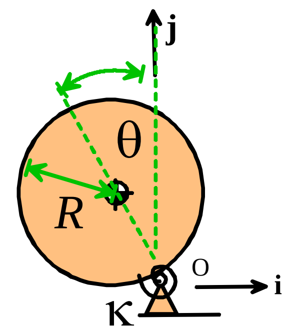


4.4 Explain why angular momentum is conserved about O during the braking. Use angular momentum conservation to find an equation relating ω_f to $(\omega_1 - \omega_0)$

4.5 For the special case $\omega_1 = 0$ (i.e. the ring and frame have the same angular speed) show that the critical value of ω_0 required to flip the frame (and ring) into the stationary vertical configuration is

$$\omega_0 = \sqrt{\left(\frac{5}{6}ML^2 + m(R^2 + \frac{L^2}{2})\right)(m+M)}\,\frac{\sqrt{gL}\sqrt{\left(\sqrt{2}-1\right)}}{mR^2}$$

5. The figure shows a simple idealization of a human head on a neck. The head is a sphere with radius R and mass m; the neck is a pivot with a torsional spring with (torsional) stiffness κ .



5.1 Write down the total kinetic and potential energy of the system, in terms of θ and $d\theta / dt$

5.2 Use the energy method to derive an equation of motion for θ

5.3 Find a formula for the natural frequency of vibration

5.4 Measurements report a natural frequency of 1.5 Hertz. Take the mass of a representative head to be 5kg and radius 11cm (detailed data is published here). Estimate the torsional stiffness of the neck.