



School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 2: Kinematics and Dynamics of Particles Due Friday Feb 8, 2019

1. Straight Line Motion with constant acceleration. Virgin Hyperloop One is a prototype rapid transportation system. It consists of an electric-powered vehicle that travels through a sealed tube in which the air pressure has been reduced to minimize air drag. Its developers anticipate that the vehicle will cruise at 670mph (1080 km/hr).



1.1 If the acceleration of the vehicle may not exceed 2.5 m/s^2 (a [typical discomfort threshold](#)), how long will it take the vehicle to reach cruise speed?

1.2 What distance is traveled during acceleration?

1.3 How long would it take for the vehicle to travel between Providence and Boston (41.25 miles)? (Assume that both acceleration and deceleration will be at 2.5 m/s^2). You can compare your estimate with one made by the company [here](#)).

2. Calculus review (apologies for inflicting this on you but hopefully it will be helpful. We suggest doing the problems by hand rather than MATLAB; the problems are meant to help you brush up on calculus that sometimes appears on exams): A particle at position $x = 0$ and has speed V_0 at time $t=0$. For each case below, please find formulas for the speed v and position x of the particle as functions of time.

2.1 The acceleration of the particle depends on time $a(t) = A_0 \exp(-t/t_0)$, where A_0, t_0 are constants

2.2 The acceleration of the particle depends on its speed $a(v) = -cv^2$ (e.g. air drag), where c is a constant

2.3 The acceleration of the particle depends on its position $a(x) = -\omega^2 x$ where ω is a constant (e.g. the acceleration of a mass on a spring). (You could solve this problem by finding v as a function of x first, then find x as a function of t , and finally use your answer to find v as a function of t . There are other ways too).

3. Using MATLAB to calculate position and acceleration from velocity measurements [CRS-8](#) was a 2016 Space-X cargo resupply mission to the international space station. [This file](#) contains telemetry data from the launch (in csv format). Your goal in this problem is to calculate the acceleration and altitude of the launch vehicle from the data.

Write a MATLAB script that accomplishes the following tasks (Your MATLAB code should be uploaded to CANVAS as a submission to this problem):

3.1 Read the data into a matrix using the MATLAB ‘csvread’ command. When you read the file you will need to skip the header row. Check the matlab documentation to find out how to do this.

3.2 Plot the speed (in m/s) as a function of time (in sec).

3.3 Calculate the acceleration of the vehicle as a function of time. You can do this by calculating the change in speed between two successive readings, and dividing by the time difference between them, e.g. if $v(i)$ denotes the i th value of x , then the i component of acceleration is

$$a(i) = (v(i+1) - v(i)) / (t(i+1) - t(i))$$

You can construct the vector $a(i)$ using a loop (note that it will have one fewer rows than the vector v storing speed). Alternatively, you could use the built-in MATLAB ‘diff’ function (see the MATLAB manuals for details).

3.4 You will find the results of 3.3 are very noisy. You can smooth the data using a simple first-order filter, as follows. Using a loop, create a new vector $as(i)$ with

$$as(1) = a(1)$$

$$as(i) = z as(i-1) + (1-z) a(i), \quad i=2,3,4\dots$$

where $0 < z < 1$ is a number you can use to control how much smoothing is applied. You could try $z=0.8$. Plot the smoothed acceleration as a function of time.

3.5 Use the MATLAB ‘cumtrapz’ function to calculate the distance traveled by the rocket and plot the results as a function of time.

4. Simple circular motion problem The rotors on the fields point wind turbines have a diameter of 92m. Measure their rotation rate (radians/sec) on a suitably windy day (you can see them out of the windows on the 7th floor of Barus-Holley). Hence, calculate the speed and the magnitude of the acceleration of the rotor tips. Be sure to explain your working, since everyone will get different numbers....

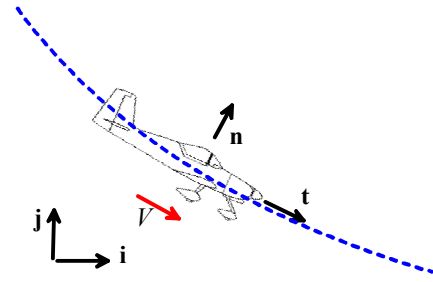


5. Normal-Tangential Coordinates. An aircraft in a dive follows a trajectory

$$x = -1000 + 60t - (t - 100)^3 / 1000$$

$$y = 20(0.005t^2 - t + 50)$$

with x and y in meters, and t in seconds. Use a MATLAB live script to:



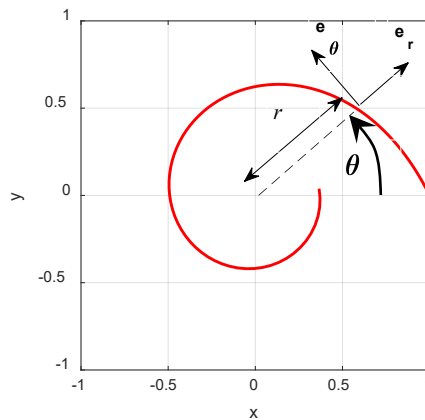
5.1 Plot the trajectory (y vs x) for a time interval $0 < t < 200$ s

5.2 Plot the speed as a function of time, for a time interval $0 < t < 200$ s

5.3 Plot the normal and tangential components of acceleration (on the same plot), for a time interval $0 < t < 200$ s. (This is quite tricky – if you can figure it out that’s great; but we’ll discuss in section if necessary).

5.4 The lift force on the aircraft acts in the \mathbf{n} direction. The load factor is defined as the ratio of the lift force to the aircraft weight. Plot the load factor as a function of time for $0 < t < 200$ s. (You will need to (i) draw an FBD for the aircraft; and (ii) use Newton’s law and the calculations in 5.3 to find the lift force).

There is no need to submit matlab code for this problem, just include your graphs in your pdf submission and explain briefly how you did the calculation.



6. Polar Coordinates: The figure shows the trajectory of a particle with mass m in an electric field. The polar coordinates of the particle vary with time t according to the formulas

$$r = e^{-t} \quad \theta = e^{2t} - 1$$

6.1 Find a formula for the velocity vector of the particle as a function of time, as components in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis

6.2 Find a formula for the acceleration vector of the particle as a function of time, as components in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis

6.3 Show that the force on the particle acts in the \mathbf{e}_r direction, and find a formula for the magnitude of the force in terms of m and r .