



School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 2: Kinematics and Dynamics of Particles Due Friday Feb 8, 2019

1. Straight Line Motion with constant acceleration. Virgin Hyperloop One is a prototype rapid transportation system. It consists of an electric-powered vehicle that travels through a sealed tube in which the air pressure has been reduced to minimize air drag. Its developers anticipate that the vehicle will cruise at 670mph (1080 km/hr).



1.1 If the acceleration of the vehicle may not exceed 2.5 m/s^2 (a [typical discomfort threshold](#)), how long will it take the vehicle to reach cruise speed?

Using the straight-line motion formula $v = v_0 + at \Rightarrow t = \frac{1}{2.5} \times \frac{1080 \times 10^3}{3600} = 120 \text{ sec}$

[1 POINT]

1.2 What distance is traveled during acceleration?

The straight line motion formula gives $x = x_0 + v_0t + \frac{1}{2}at^2 \Rightarrow x = \frac{1}{2} \times 2.5 \times 120^2 = 18 \text{ km}$

[1 POINT]

1.3 How long would it take for the vehicle to travel between Providence and Boston (41.25 miles)? (Assume that both acceleration and deceleration will be at 2.5 m/s^2). You can compare your estimate with one made by the company [here](#)).

41.25 miles is 66.4 km.

It will take 2 mins to accelerate, and another 2 min to decelerate.

The total distance traveled during acceleration and deceleration is 36km, leaving 30.4km at cruise speed.

1080 km/hr is 300 m/s, so it will take $30400/300 = 101 \text{ sec}$ to over the remaining 30.4km.

The total time is $240 + 101 = 341 \text{ sec} = 5.7 \text{ mins}$.

Virgin hyperloop estimates 10 mins – perhaps they plan to accelerate less aggressively; and don't anticipate being able to travel in a straight line...

[3 POINTS]

2. Calculus review (apologies for inflicting this on you but hopefully it will be helpful. We suggest doing the problems by hand rather than MATLAB; the problems are meant to help you brush up on calculus that sometimes appears on exams): A particle at position $x = 0$ and has speed V_0 at time $t=0$. For each case below, please find formulas for the speed v and position x of the particle as functions of time.

2.1 The acceleration of the particle depends on time $a(t) = A_0 \exp(-t/t_0)$, where A_0, t_0 are constants

$$\begin{aligned} a = \frac{dv}{dt} = A_0 \exp(-t/t_0) &\Rightarrow \int_{V_0}^v dv = \int_0^t A_0 \exp(-t/t_0) dt \\ \Rightarrow v - V_0 = A_0 t_0 [1 - \exp(-t/t_0)] &\Rightarrow v = V_0 + A_0 t_0 [1 - \exp(-t/t_0)] \\ \Rightarrow \frac{dx}{dt} = V_0 + A_0 t_0 [1 - \exp(-t/t_0)] &\Rightarrow \int_0^x dx = \int_0^t (V_0 + A_0 t_0 [1 - \exp(-t/t_0)]) dt \\ \Rightarrow x = (V_0 + A_0 t_0)t + A_0 t_0^2 [\exp(-t/t_0) - 1] \end{aligned}$$

[3 POINTS]

2.2 The acceleration of the particle depends on its speed $a(v) = -cv^2$ (e.g. air drag), where c is a constant

$$\begin{aligned} a = \frac{dv}{dt} = -cv^2 &\Rightarrow \int_{V_0}^v \frac{dv}{v^2} = \int_0^t -c dt \\ \Rightarrow -\frac{1}{v} + \frac{1}{V_0} = -ct &\Rightarrow v = \frac{V_0}{1 + cV_0 t} \\ \Rightarrow \frac{dx}{dt} = \frac{V_0}{1 + cV_0 t} &\Rightarrow \int_0^x dx = \int_0^t \frac{V_0}{1 + cV_0 t} dt \\ \Rightarrow x = \frac{1}{c} \log(1 + cV_0 t) \end{aligned}$$

[3 POINTS]

2.3 The acceleration of the particle depends on its position $a(x) = -\omega^2 x$ where ω is a constant (e.g. the acceleration of a mass on a spring). (You could solve this problem by finding v as a function of x first, then find x as a function of t , and finally use your answer to find v as a function of t . There are other ways too).

$$\begin{aligned} a = v \frac{dv}{dx} = -\omega^2 x &\Rightarrow \int_{V_0}^v v dv = \int_0^x -\omega^2 x dx \\ \Rightarrow \frac{1}{2}(v^2 - V_0^2) = -\frac{1}{2}\omega^2 x^2 &\Rightarrow v = \sqrt{V_0^2 - \omega^2 x^2} \\ \Rightarrow \frac{dx}{dt} = \sqrt{V_0^2 - \omega^2 x^2} &\Rightarrow \int_0^x \frac{1}{\sqrt{V_0^2 - \omega^2 x^2}} dx = \int_0^t dt \Rightarrow \int_0^{\arcsin(\omega x/V_0)} \frac{\cos \theta}{\omega \sqrt{1 - \sin^2 \theta}} d\theta = \int_0^t dt \\ \Rightarrow \arcsin(\omega x/V_0) = \omega t & \\ \Rightarrow x = (V_0/\omega) \sin \omega t & \quad v = V_0 \cos \omega t \end{aligned}$$

[4 POINTS]

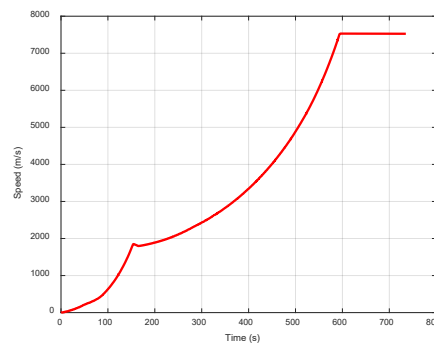
3. Using MATLAB to calculate position and acceleration from velocity measurements [CRS-8](#) was a 2016 Space-X cargo resupply mission to the international space station. This file contains telemetry data from the launch (in csv format). Your goal in this problem is to calculate the acceleration and altitude of the launch vehicle from the data, and hence to estimate the rocket motor thrust.

Write a MATLAB script that accomplishes the following tasks (Your MATLAB code should be uploaded to CANVAS as a submission to this problem):

3.1 Read the data into a matrix using the MATLAB ‘csvread’ command.

[0 POINTS]

3.2 Plot the speed (in m/s) as a function of time (in sec).

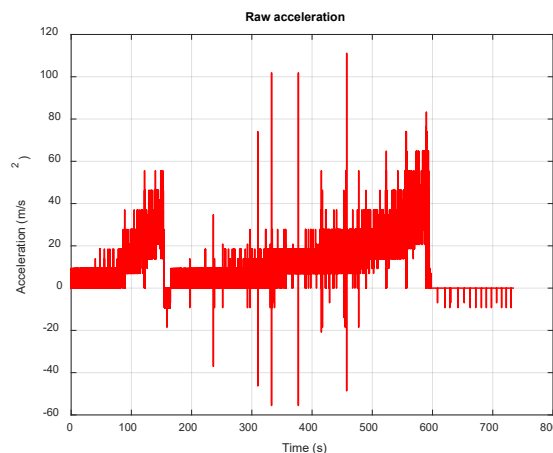


[1 POINT]

3.3 Calculate the acceleration of the vehicle as a function of time. You can do this by calculating the change in speed between two successive readings, and dividing by the time difference between them, e.g. if $v(i)$ denotes the i th value of x , then the i component of acceleration is

$$a(i) = (v(i + 1) - v(i)) / (t(i + 1) - t(i))$$

You can construct the vector $a(i)$ using a loop (note that it will have one fewer rows than the vector v storing speed). Alternatively, you could use the built-in MATLAB ‘diff’ function (see the MATLAB manuals for details).



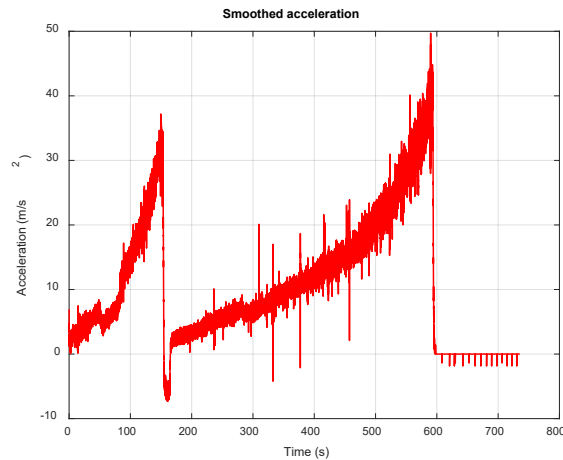
[2 POINTS]

3.4 You will find the results of 3.3 are very noisy. You can smooth the data using a simple first-order filter, as follows. Using a loop, create a new vector $as(i)$ with

$$as(1) = a(1)$$

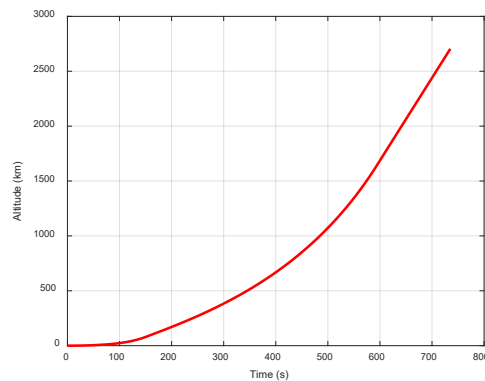
$$as(i) = z as(i-1) + (1-z) a(i), \quad i=2,3,4\dots$$

where $0 < z < 1$ is a number you can use to control how much smoothing is applied. You could try $z=0.8$. Plot the smoothed acceleration as a function of time.



[2 POINTS]

3.5 Use the MATLAB 'cumtrapz' function to integrate the speed with respect to time and plot the results.



[2 POINTS]

4. Simple circular motion problem The rotors on the fields point wind turbines have a diameter of 92m. Measure their rotation rate (radians/sec) on a suitably windy day (you can see them out of the windows on the 7th floor of Barus-Holley). Hence, calculate the speed and the magnitude of the acceleration of the rotor tips. Be sure to explain your working, since everyone will get different numbers....

The circular motion formula gives $V = R\omega$. A typical angular speed is about 2 rad/sec, which gives a tip speed of 92m/s. The acceleration of the rotor tips is $R\omega^2 \approx 184ms^{-2}$ (this is large!)



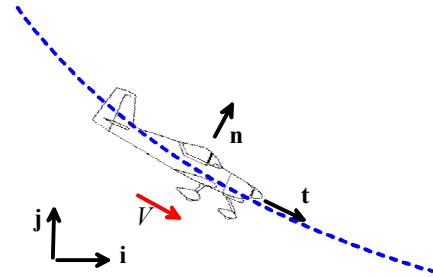
[3 POINTS]

5. Normal-Tangential Coordinates. An aircraft in a dive follows a trajectory

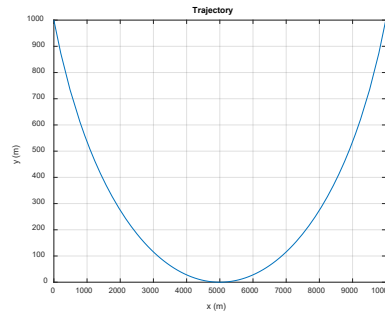
$$x = -1000 + 60t - (t - 100)^3 / 1000$$

$$y = 20(0.005t^2 - t + 50)$$

with x and y in meters, and t in seconds. Use a MATLAB live script to:



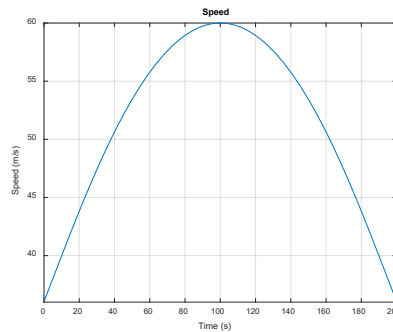
5.1 Plot the trajectory (y vs x) for a time interval $0 < t < 200$ s



(This is just a simple parametric plot; no explanation required)

[2 POINTS]

5.2 Plot the speed as a function of time, for a time interval $0 < t < 200$ s



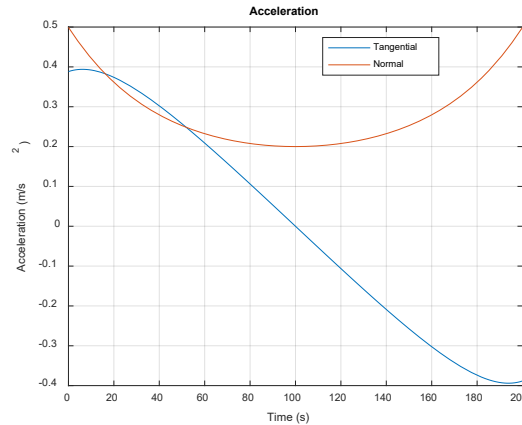
The plot is generated using $V = \sqrt{(dx/dt)^2 + (dy/dt)^2}$

[2 POINTS]

5.3 Plot the normal and tangential components of acceleration (on the same plot), for a time interval $0 < t < 200$ s.

To calculate the normal-tangential components, you can (i) find the acceleration in \mathbf{i}, \mathbf{j} components (differentiate the velocity with respect to time); (ii) find a unit vector tangent to the path – you could use the velocity vector, eg, and set $V\mathbf{t} = \mathbf{v} \Rightarrow \mathbf{t} = \mathbf{v}/V$; (iii) Find a second unit vector normal to the path (use the fact that the normal and tangent are mutually perpendicular which means $\mathbf{n} \cdot \mathbf{t} = 0$ or you can also use $\mathbf{n} = \mathbf{k} \times \mathbf{t}$, both of these will tell you that $\mathbf{n} = (-dy/dt\mathbf{i} + dx/dt\mathbf{j})/V$); and (iv) recall that you can use a dot product to find the component of a vector parallel to a unit vector, so the tangential and normal accelerations are $a_t = \mathbf{a} \cdot \mathbf{t}$ $a_n = \mathbf{a} \cdot \mathbf{n}$ respectively. Another way to find the tangential acceleration is to use the formula $a_t = dV/dt$ and have matlab crank through the time derivative of V from 5.2. There are various other exotic formulas for normal acceleration that you could use if you happen to know them, for example $a_n = |\mathbf{t} \times \mathbf{a}|$ (can you prove this?), or you could use V^2/R if you

know how to calculate the radius of curvature. But these exotic methods weren't covered in ENGN40 (you might see them in MA0200/MA0180) and we expected the dot product approach.



[3 POINTS]

5.4 The lift force on the aircraft acts in the \mathbf{n} direction. The load factor is defined as the ratio of the lift force to the aircraft weight. Plot the load factor as a function of time for $0 < t < 200$ s. (You will need to (i) draw an FBD for the aircraft; and (ii) use Newton's law and the calculations in 5.3 to find the lift force).

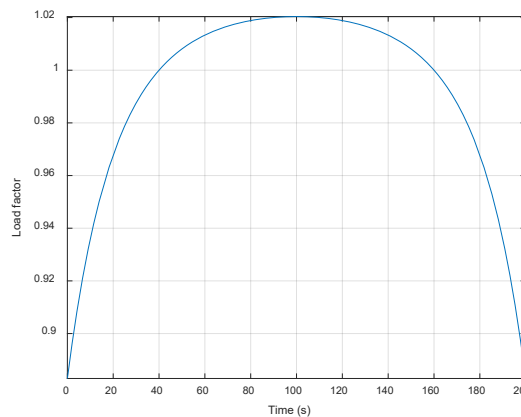
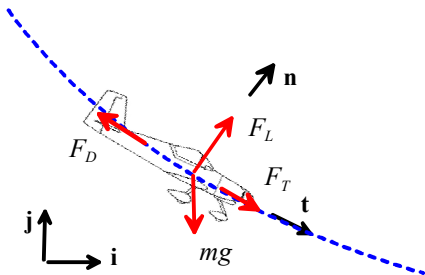
The figure shows a FBD. We can write Newton's law as

$$(F_T - F_D)\mathbf{t} + F_L\mathbf{n} - mg\mathbf{j} = m\mathbf{a}$$

Take the dot product with \mathbf{n}

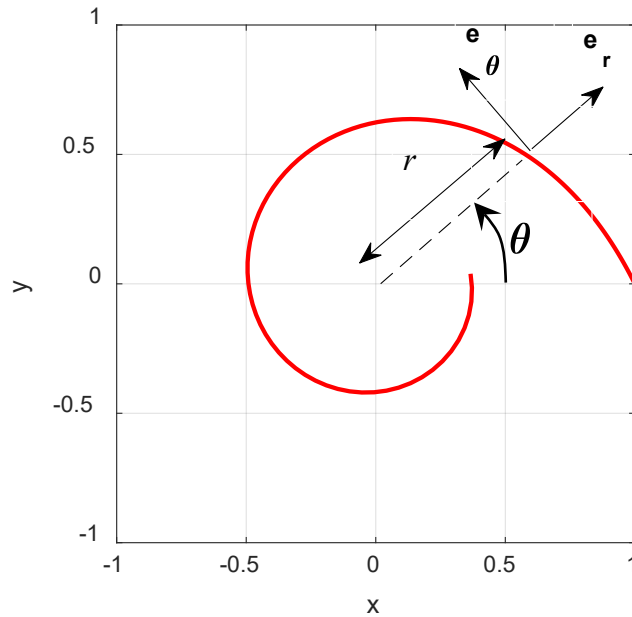
$$F_L - mg\mathbf{j} \cdot \mathbf{n} = m\mathbf{a} \cdot \mathbf{n} \Rightarrow \frac{F_L}{mg} = \frac{\mathbf{a} \cdot \mathbf{n}}{g} + \mathbf{j} \cdot \mathbf{n} = \frac{a_n}{g} + \frac{1}{V} \frac{dx}{dt}$$

Where a_n is from 5.3.



[3 POINTS]

GRADERS: Live scripts were not required for this problem; please grade the plots and any brief description of the method that has been provided with the plots.



6. Polar Coordinates: The figure shows the trajectory of a particle with mass m in an electric field. The polar coordinates of the particle vary with time t according to the formulas

$$r = e^{-t} \quad \theta = e^{2t} - 1$$

6.1 Find a formula for the velocity vector of the particle as a function of time, as components in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis

$$\mathbf{v} = \frac{dr}{dt} \mathbf{e}_r + r \frac{d\theta}{dt} \mathbf{e}_\theta = -e^{-t} \mathbf{e}_r + 2e^t \mathbf{e}_\theta$$

[1 POINT]

6.2 Find a formula for the acceleration vector of the particle as a function of time, as components in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis

$$\begin{aligned} \mathbf{a} &= \left(\frac{d^2r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 \right) \mathbf{e}_r + \left(r \frac{d^2\theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right) \mathbf{e}_\theta \\ &= \left(e^{-t} - e^{-t} (2e^{2t})^2 \right) \mathbf{e}_r + \left(e^{-t} 4e^{2t} - 2e^{-t} 2e^{2t} \right) \mathbf{e}_\theta \\ &= \left(e^{-t} - 4e^{3t} \right) \mathbf{e}_r \end{aligned}$$

[2 POINTS]

6.3 Show that the force on the particle acts in the \mathbf{e}_r direction, and show that the magnitude of the force as a function of r and m is

$$|\mathbf{F}| = m \left(r - \frac{4}{r^3} \right)$$

$\mathbf{F} = m\mathbf{a}$ shows

$$\mathbf{F}(t) = m(e^{-t} - 4e^{3t})\mathbf{e}_r = m \left(r - \frac{4}{r^3} \right) \mathbf{e}_r$$

This is in the \mathbf{e}_r because there is no \mathbf{e}_θ component.

[2 POINTS]