School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 5: Vibrations
Due Friday March 22, 2019

1. The figure (from this publication) shows a vibration measurement from a test on an accelerometer. Use the figure to estimate

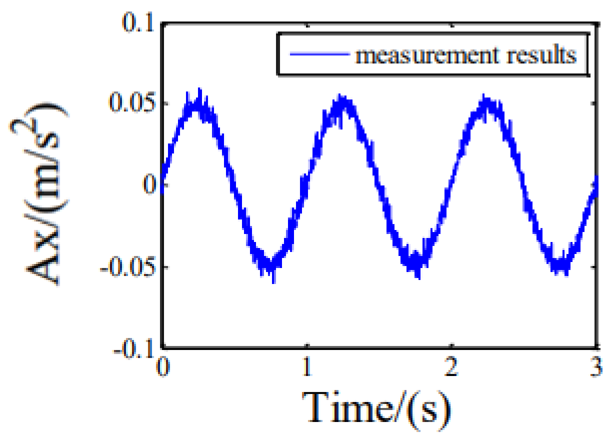
1.1 The amplitude of the acceleration

1.2 The period of the vibration

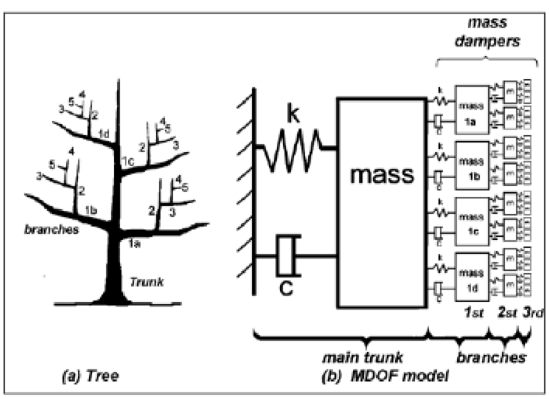
1.3 The frequency (in Hertz) and angular frequency (in rad/s)

1.4 The amplitude of the velocity

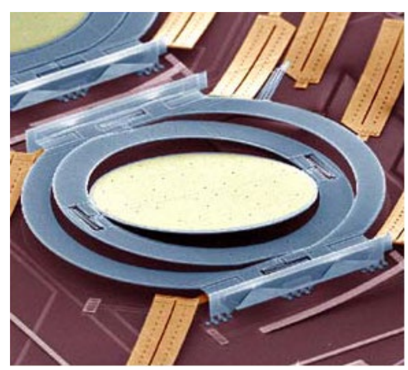
1.5 The amplitude of the displacement



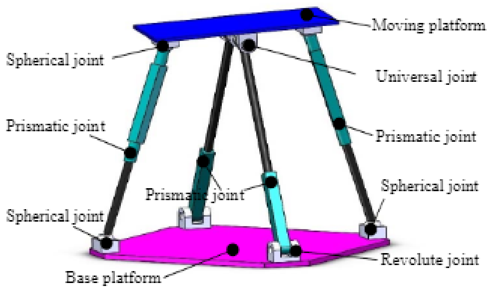
2. Find the number of degrees of freedom and vibration modes for each of the systems shown in the figures (you may need to consult the publications to understand the system)



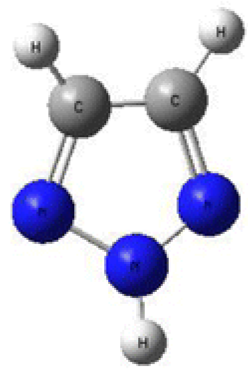
(a) Theoretical model of vibrations in a tree
(use the figure on the right)



(b) Mems Mirror



(c) Model of an articulated platform



(d) Thiazole molecule (the publication shows some of the vibration modes)

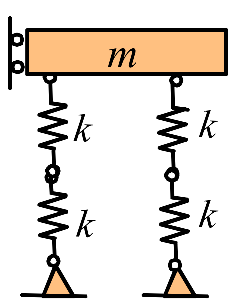
3. Solve the following differential equations (please solve them by hand, using the tabulated solutions to differential equation – you can check the answers with matlab if you like)

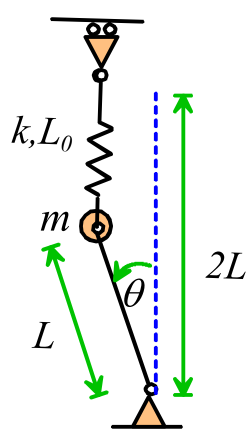
3.1 $\dfrac{d^2y}{dt^2}+9y=9 \qquad y=1 \qquad \dfrac{dy}{dt}=-3 \qquad t=0$

3.2 $\dfrac{d^2y}{dt^2}-9y=-9 \qquad y=1 \qquad \dfrac{dy}{dt}=-1 \qquad t=0$

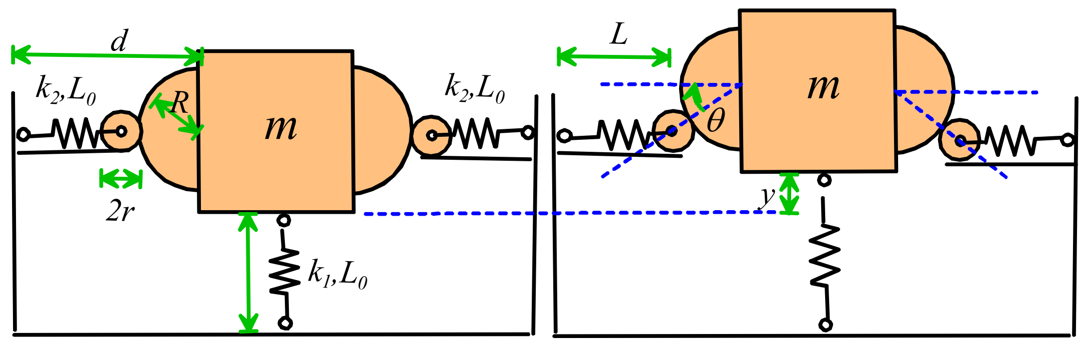
3.3 $\dfrac{d^2y}{dt^2}+100\dfrac{dy}{dt}+25y=50\sin(t) \qquad y=0 \qquad \dfrac{dy}{dt}=0 \qquad t=0$

4. Find formulas for the natural frequency of vibration for the systems shown in the figure





5. This publication shows an alternative design for the 'minus-k' vibration isolation system discussed in class. Like the minus-k design, the platform of mass m is supported by a vertical spring, and loaded from the side by two horizontal springs. The horizontal springs are connected to the mass through a system of rollers, instead of being directly attached.



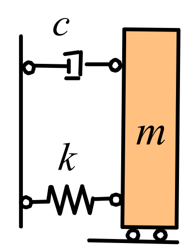
5.1 Using geometry, find a formula for the angle θ in terms of the vertical deflection y of the mass. Hence, find a formula for the stretched length L of the horizontal springs, in terms of y.

5.2 Write down the total kinetic and potential energy of the system, in terms of y (and its time derivatives), d, k_1, k_2, L_0, r, R, m . **Neglect gravity**.

5.3 Hence, find the equation of motion for y

5.4 Linearize the equation for small y, and hence find the natural frequency of the system.

6. The spring-mass-dashpot system shown in the figure has $k = 10^3 \, N/m$, $c = 100 \, Ns/m$, $m=100$grams.



6.1 Calculate the natural frequency and damping factor for the system (just use the standard formulas)

6.2 If the system is released from rest with a 1cm deflection from its equilibrium position, how long will it take to reach 0.1cm deflection? (there is no need to derive and solve the EOM; just use the tabulated standard solutions)

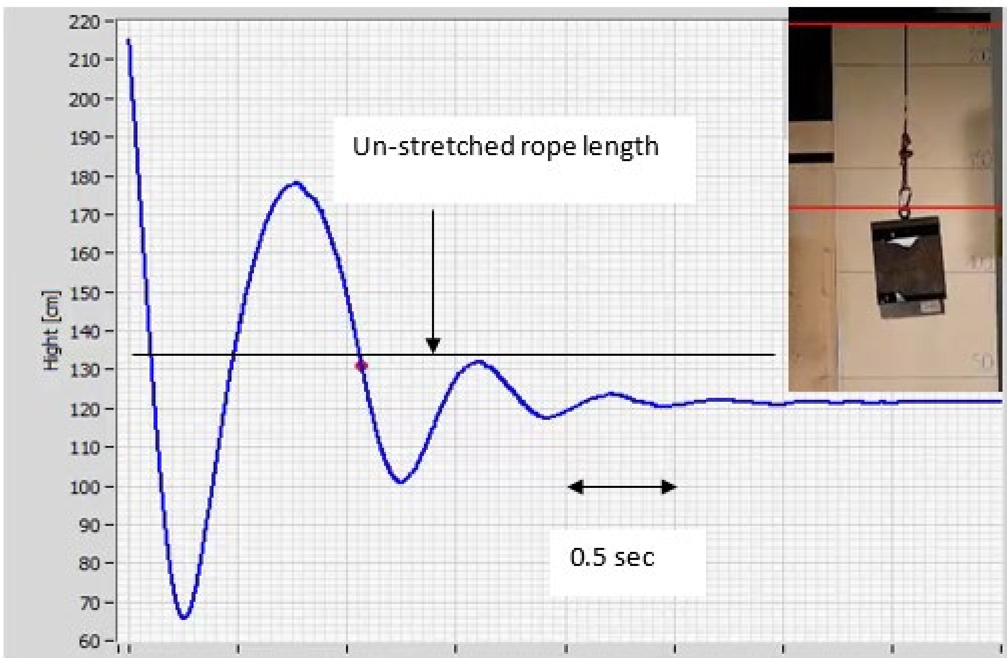
6.3 How much mass needs to be added to the system (with no change in c or k) to make it critically damped? What is the new natural frequency?

6.4 How long will the modified system (in 6.3) take to reach 0.1 cm deflection? (there is no need to derive and solve the EOM; just use the tabulated standard solutions)

7. This website describes a series of tests to measure the dynamic properties of a climbing rope. The figure shows the results of an experiment with the following parameters:

- Rope length 294.7cm,
- Mass 80 kg
- Fall factor 0.96

Before you do this problem watch a movie of the test here. Note that the rope goes slack during part of the experiment – the standard spring-mass vibration formulas don't apply during this phase of the experiment, so the data has to be interpreted carefully.



7.1 Suppose that the rope and weight can be idealized as a spring-mass-damper system. Use the data in the figure to estimate

(a) The period of oscillation (use only peaks after the rope remains stretched)

(b) The log decrement

(c) The values of ζ and ω_n

7.2 Hence, determine values for

(a) the stiffness of the rope

(b) The dashpot coefficient c that best approximates the damping in the system.

7.2 If the length of the rope is doubled, what will be its new stiffness and dashpot coefficient?

7.3 What length of rope would give a critically damped response in the drop test (with an 80kg mass)?