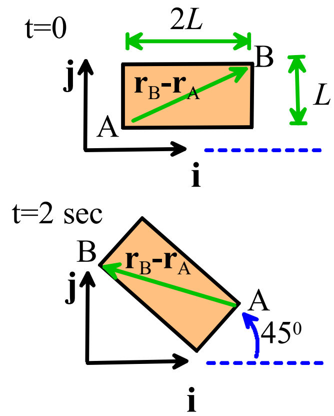
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 7: Rigid Body Kinematics, Inertial properties of rigid bodies
Due Friday April 19, 2019

1. The rigid body shown in the figure is at rest at time t=0, and rotates counterclockwise with constant angular acceleration. After 2 sec it has the orientation shown. Find



1.1 The angular acceleration vector

1.2 The angular velocity vector as a function of time

1.3 The spin tensor **W** (as a 2x2 matrix, also a function of time)

1.4 The rotation tensor (a 2x2 matrix for a 2D problem) **R** that rotates the rectangle from its initial to its position at 2 sec.

1.5 Hence, find a formula for the vector $\mathbf{r}_B - \mathbf{r}_A$ in (**i**,**j**) components at time t=2sec

2. A rigid body is subjected to a sequence of two rotations.

$$\mathbf{R}^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \qquad \mathbf{R}^{(2)} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.1 Describe each rotation matrix in a sentence (eg a 30 degree rotation about the **i** axis)

2.2 Find the rotation matrix that produces the sequence of rotations $\mathbf{R} = \mathbf{R}^{(2)}\mathbf{R}^{(1)}$

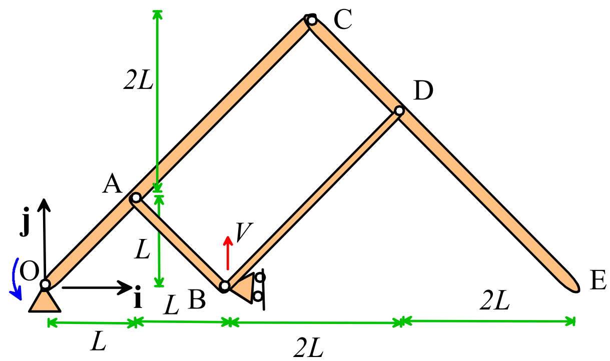
2.3 Find the axis **n** and rotation angle θ that will complete the rotation $\mathbf{R}$ directly

Optional: You can check your answers by downloading a matlab script from the homework page of the course website that will animate a rigid rotation through an angle θ about an axis parallel to a unit vector **n**. You can use the code by navigating to the directory storing the file in the Matlab command window, and then typing

Animate_rotation(*angle*,[n_x, n_y, n_z])

Where *angle* is your solution for the rotation angle θ (in radians), and n_x, n_y, n_z are the components of your solution for the unit vector **n**.

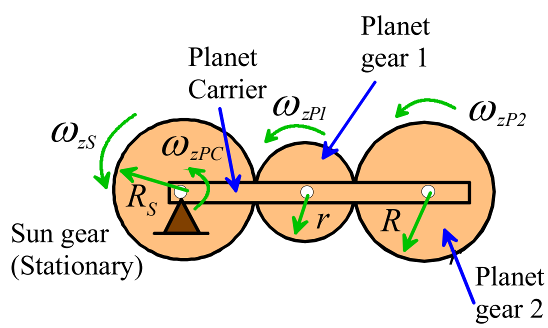
3. The figure shows a four-bar chain mechanism. Joint B moves vertically with constant speed V. Calculate the angular velocities and angular accelerations of members OC and CE, and find the velocity and acceleration of E.



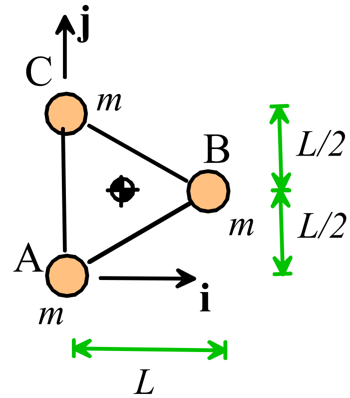
4. Pratt and Whitney started production of the first geared turbofan engine in 2016. The engine allows the fan and compressor stages (which normally run on the same shaft) to spin at different speeds, which improves the efficiency of the engine (but the additional complexity of the engine has led to some reliability issues). Rolls Royce are currently rolling out their 'Ultrafan' engine, which will keep patent lawyers in business for years. The figure shows the P&W gearbox. It operates with the planet carrier held fixed, the fan connected to the ring gear, and the compressor to the sun.

Use the figure to calculate the ratio of the angular speeds of the compressor and fan (you will need to count the teeth on the gears).

5. 'Ferguson's paradox' is a special arrangement of epicyclic gears used by its inventor to make an orrery. Ferguson's version made three non-standard co-axial sun gears mesh with the same planet gear, but the basic principle can be illustrated by the simple design shown in the figure (see also this animation). The sun gear is stationary, and the planet carrier rotates about O. Find a formula for the angular speed ω_{zP2} of planet gear 2, in terms of the angular speed of the planet carrier ω_{zPC} and the radii R and R_S .



6. The figure shows three particles with equal mass m connected by rigid massless links.



6.1 Calculate the position of the center of mass of the assembly

6.2 Calculate the 2D mass moment of inertia of the system about the center of mass

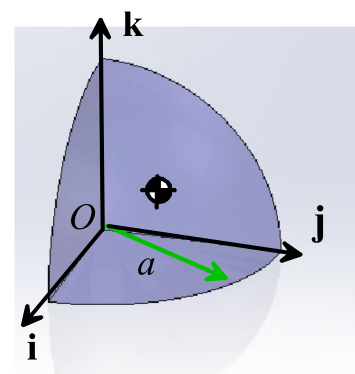
$$I_{Gzz} = \sum_i m_i \left(d_{xi}^2 + d_{yi}^2 \right)$$

where $\mathbf{d}_i = d_{xi}\mathbf{i} + d_{yi}\mathbf{j} = \mathbf{r}_i - \mathbf{r}_G$ is the position vector of the ith particle with respect to the center of mass.

6.3 Suppose that the assembly rotates about its center of mass with angular velocity $\omega\mathbf{k}$ (the center of mass is stationary). What are the speeds of the particles A,B and C?

6.4 Calculate the total kinetic energy of the system (a) using your answer to 6.2; and (b) using your answer to 6.3. (The point of this problem is to demonstrate that the rigid body formula $(1/2)I\omega^2$ is just a quick way of summing the kinetic energies of the 3 masses. For the simple 2D system here it is quite simple to prove the equivalence for any arrangement of masses. For 3D the derivation is more complicated, but the idea is the same.)

7. The figure shows a 1/8 segment of a sphere with radius a and uniform mass density ρ Using a Matlab 'Live Script', calculate



7.1 The total mass M (you will need to do the relevant integrals using spherical-polar coordinates)

7.2 The position vector of the center of mass (with respect to the origin shown in the figure)

7.3 The inertia tensor (matrix) about the center of mass, in the basis shown

7.4 Using the parallel axis theorem, calculate the mass moment of inertia about the tip O.

Please upload your 'Live script' solution to Canvas.