School of Engineering
Brown University

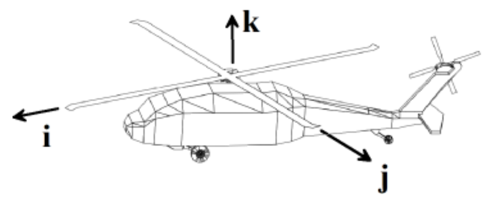
EN40: Dynamics and Vibrations

Homework 8: Rigid Body Dynamics
Due Friday April 26, 2018

1. The Sikorsky UH-60 Black Hawk helicopter has a 4 bladded rotor with the following specifications

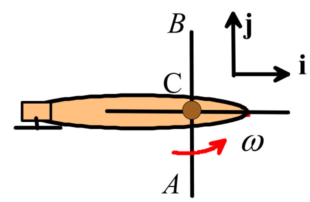
- Blade mass 113kg (for one blade)
- Rotor diameter: 16.36m
- Rotor speed 258 rpm

1.1 Calculate the mass moment of inertia matrix of the rotor. Take the basis vectors to be aligned with the blades as shown in the figure. (find the full 3x3 matrix for I, not just the I_{Gzz})

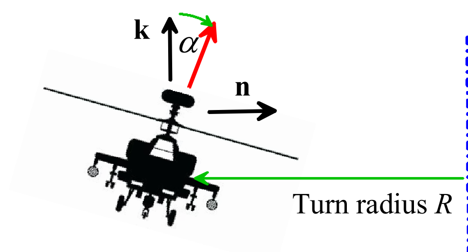


1.2 What (constant) torque (moment) must be applied to the rotor to spin it up to operating speed in 60sec (neglect drag on the rotors)?

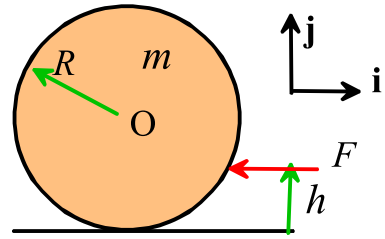
1.3 If the helicopter is in cruise flight at its maximum airspeed of 294 km/hr, what are the maximum and minimum tip speeds of the rotors through the air? (The fig shows a plan view of the rotor; hopefully you can see that A is moving faster through the air than B).



1.4 If the helicopter is in a standard rate turn at 60 knots (30.86m/s), (with the direction of turn in the same direction as the direction of rotation of the rotor, and the aircraft traveling out of the plane of the figure), what is the angular velocity vector of the rotor, in the $\{\mathbf{n},\mathbf{t},\mathbf{k}\}$ basis shown in the figure? (Recall that aircraft in a standard rate turn was analyzed in Lecture 3. Neglect drag and the **t** component of lift for simplicity.)



2. The figure shows a spherical billiard ball with mass m and radius R at rest on a pool table. It is subjected to a horizontal force F by a cue at a height h above the table.



2.1 Draw a free body diagram showing all the forces acting on the ball (include gravity and assume no slip at the contact)

2.2 Write down the equations of motion (Newton's law, and the angular momentum equation).

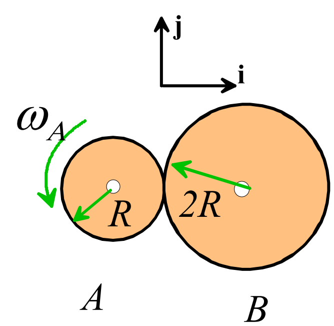
2.3 Write down the kinematics equation relating the acceleration of the center at O and the angular acceleration of the sphere

2.4 Hence, calculate the acceleration of the sphere

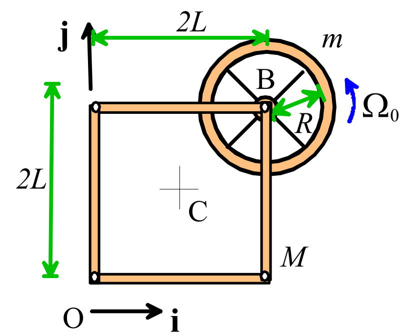
2.5 Calculate the reaction forces at the contact. If the coefficient of friction at the contact is μ , find a formula for the critical value of F that will cause slip at the contact, in terms of m, g, h, R.

2.6 Where should the ball be struck to guarantee no slip for any value of F?

3. The two gears A and B in the figure have radii R and $2R$, and mass moments of inertia $mR^2/2$ and $2mR^2$, respectively. Their centers are stationary. If gear A rotates counterclockwise at angular speed ω_A , find a formula for the total angular momentum of the system (including both gears)



4. Reaction/Momentum wheels are used to control the attitude of spacecraft



The figure shows a 2D idealization of a cube-sat (idealized as a frame with four members with length 2*L* and combined mass *M*, with a ring-shaped momentum wheel with mas *m* and radius *R* at one corner.

At time *t=0* the frame is at rest (no translation or rotation), and the wheel spins counterclockwise with angular speed Ω_0 . The motor is then spun up rapidly to increase the speed of the motor to a speed Ω_1 The goal of this problem is to calculate the resulting angular speed of the frame.

4.1 Find formulas for the out-of-plane components of the mass moments of inertia (I_{Gzz})of the frame (about its center of mass at C) and the wheel (about the center of the wheel), in terms of *m*, *M*, *R* and *L*.

4.2 Find a formula for the total angular momentum of the system about the corner at O at time *t=0*, in terms of Ω_0 and other relevant variables.

4.3 What is the total linear momentum of the system at time *t=0*?

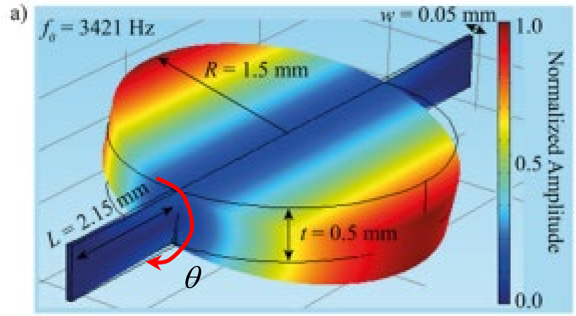
4.4 Suppose that after the motor is spun up the center of the frame (at C) has a velocity $\mathbf{v}_C = v_x\mathbf{i} + v_y\mathbf{j}$ and the frame rotates with an angular velocity $\omega_f\mathbf{k}$. Find a formula for the velocity $\mathbf{v}_B$ of the corner at B, in terms of $\omega_f, \mathbf{v}_C, L$

4.5 Why is the total linear momentum of the system conserved? Use linear momentum conservation to find formulas for $\mathbf{v}_C$ and $\mathbf{v}_B$ in terms of ω_f, M, m, L

4.6 Hence, write down the total angular momentum of the frame and the motor about O after the change in motor speed. Note that the motor speed Ω_1 specifies the rate of rotation of the motor, not the angular speed of the ring.

4.7 Finally, find a formula for the angular velocity of the frame after the change in motor speed in terms of Ω_1, M, m, L, R

5. The figure (from this publication) shows a MEMS mirror (used in laser scanning applications). It consists of a disk with radius R and thickness t, made from Silicon with mass density ρ with dimensions shown in the figure. The disk is mounted on two slender beams that operate as torsional springs with stiffness (Nm/radian) κ (for one beam). For laser scanning the mirror is driven at resonance.



5.1 Write down the total kinetic and potential energy of the system, in terms of the angle of rotation of the mirror θ and its time derivative $d\theta / dt$

5.2 Use the energy method to derive an equation of motion for θ

5.3 Find a formula for the natural frequency of vibration

5.4 The authors report a resonant frequency of 3421Hz. Calculate the torsional stiffness κ. (Si has a mass density of 2329kg/m^3 – you will need this to find the mass moment of inertia of the cylinder)