



School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 2: Kinematics and Dynamics of Particles Due Friday Feb 7, 2020

1. Straight Line Motion. NASA operate a [micro-gravity research facility](#) that can be used to conduct experiments in a (near) weightless environment over a short period of time. It consists of a long vertical shaft, which is pumped down to a near vacuum to minimize air drag. A capsule is dropped from rest at the top of the tube, and while the capsule is falling with gravitational acceleration, its contents experience a weightless environment. NASA specs are:

- The weightless portion of the drop lasts for 5.18s. During this portion of the drop the capsule falls vertically with acceleration of $1g$ (where $g=9.81\text{m/s}^2$ is the gravitational acceleration).
- The capsule is decelerated at $35g$



1.1 Calculate the distance traveled during the first 5.18s, and the maximum velocity of the capsule during the drop

1.2 What is the distance traveled during deceleration?

2. Calculus review (apologies for inflicting this on you but hopefully it will be helpful. We suggest doing the problems by hand rather than MATLAB; the problems are meant to help you brush up on calculus that sometimes appears on exams. The calculus in this problem appears again in later parts of the course, so try to remember it!): A particle starts at position $x = 0$ with speed V_0 at time $t=0$. For each case below, please find formulas for the speed v and position x of the particle as functions of time.

2.1 The acceleration of the particle depends on time $a(t) = A_0 \sin \omega t$ where A_0, ω are constants

2.2 The acceleration of the particle depends on its speed as $a(v) = \sqrt{1 - c^2 v^2}$, where c is a constant

2.3 The acceleration of the particle depends on its position as

$$a(x) = \begin{cases} -V_0^2(1-x) & x < 1 \\ 0 & x > 1 \end{cases}$$

(You can solve this problem by finding v as a function of x first, then find x as a function of t , and finally use your answer to find v as a function of t . There are other ways too, if you know how to solve differential equations).

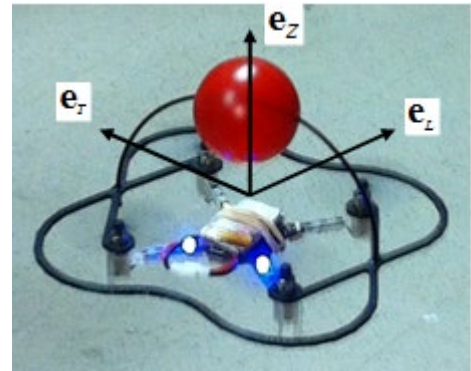
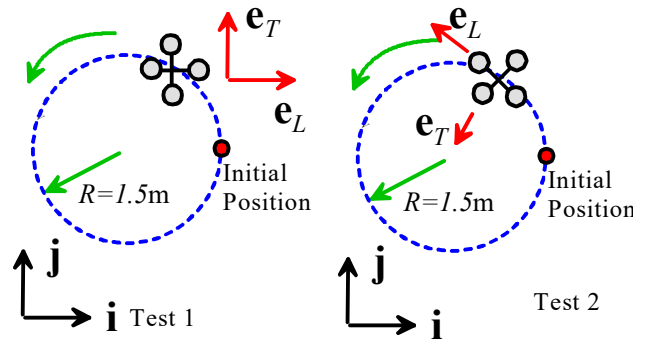
3. Using MATLAB to process position / acceleration / velocity measurements: In a class demonstration (see lecture capture to review if needed) we flew a small quadcopter around a circular path, while recording its position, velocity and acceleration.

1. We measured position using a radio system located in a fixed position in the room. The coordinate system is shown in the figure (the origin is just some random position determined by the location of the radio anchors).
2. Velocity, acceleration, and the yaw of the quadcopter were measured using on-board sensors. The sensors measure data in the $\{e_L, e_T, e_N\}$ basis that rotates with the quadcopter (to keep things simple you can assume that the quadcopter remains approximately level).

The following flight was programmed into the controller:

- (0) The quadcopter started on its flight stand, with the $\{e_L, e_T, e_N\}$ quadcopter axes parallel to $\{i, j, k\}$
- (1) Ascend to the target altitude with constant vertical speed in 5 seconds, while rotating to the correct initial orientation (with constant angular speed)
- (2) Fly two circles with 1.5m radius at constant speed, in 15 seconds, i.e. the controller was programmed to fly the path

$$\mathbf{r} = \mathbf{r}_0 + R \cos \omega(t-5)\mathbf{i} + R \sin \omega(t-5)\mathbf{j} \quad \omega = 4\pi / 15 \text{ rad} / \text{s} \quad R = 1.5\text{m}$$
- (3) Descend to a position next to the take-off stand in 5 seconds
- (4) Cut motors



Two tests were run. In the first test, the quadcopter was programmed to keep a fixed orientation, with the $\{e_L, e_T, e_N\}$ quadcopter axes parallel to $\{i, j, k\}$ throughout the flight. Data from this test are stored in the file called circle_fixedorientation.csv. In the second test, the quadcopter turned so that the e_L direction was parallel to the (desired) direction of motion around the circle. Data from this test are stored in the file called circle_with_turn.csv. In both files, the first column of data is the time (s); the second-4th columns are (x, y, z) position, in m; the 5th-7th columns are velocity (v_L, v_T, v_N) , in m/s; the 8th-10th columns are acceleration (a_L, a_T, a_N) , in m/s^2 , and the last column is the yaw (in degrees).

3.1 Before trying to process the data, sketch graphs showing:

- (a) The expected variation of (v_L, v_T) for the two tests during the time interval $5 < t < 20$ (assuming the path is a perfect circle, which of course it is not)....
- (b) The expected variation of (a_L, a_T) for the two tests during the time interval $5 < t < 20$ (again, assuming perfectly circular motion)

3.2 Write a MATLAB script that accomplishes the following tasks (Your MATLAB code should be uploaded to CANVAS as a submission to this problem): For each data file

- (a) Read the data into a matrix using the MATLAB 'csvread' command
- (b) Plot the trajectory (on a 3D plot).
- (c) Plot the measured (v_L, v_T) as a function of time
- (d) Plot the measured (a_L, a_T) as a function of time

3.3 Add MATLAB code to check the consistency of the data. For the data recorded with fixed quadcopter orientation, calculate the velocity by integrating the accelerometer readings (a_L, a_T) using the 'cumtrapz' function, and (on the same graph) plot the measured (v_L, v_T) . Explain why this procedure will not yield meaningful results for the second data set.

4. Simple circular motion problem The rotors on the fields point wind turbines have a diameter of 92m. Suppose that the turbines are spun up from rest with an angular acceleration $\alpha = 0.1 \text{ rad} / \text{s}^2$

4.1 Find formulas for the angular speed $\omega = d\theta / dt$ and the angle θ turned by the rotor as a function of time.

4.2 How long does it take for the rotor to reach an angular speed of 2 rad/s?

4.3 Calculate the speed of the blade tips on a rotor turning at 2 rad/s

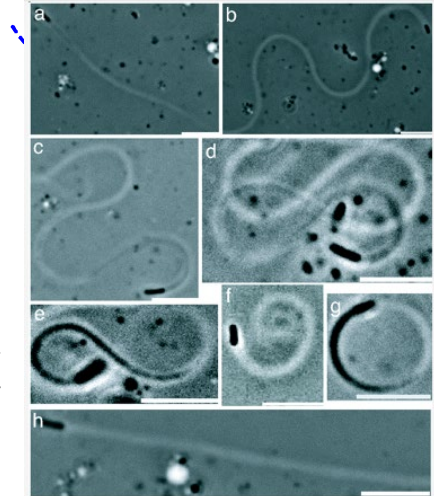
4.4 Calculate the magnitude of the acceleration of the blade tips at the instant when $\omega = 0$ and the instant when $\omega = 2 \text{ rad} / \text{s}$



5. Normal-Tangential Coordinates. [This publication](#) shows that a simple formula can predict the trajectories of the *Listeria* bacterium (some example trajectories are shown in the figure). The authors suggest that the (2D) velocity vector can be expressed as

$$\mathbf{v} = -v_0 \sin\left(\frac{\Omega}{\omega} \sin \omega t\right) \mathbf{i} + v_0 \cos\left(\frac{\Omega}{\omega} \sin \omega t\right) \mathbf{j}$$

with v_0, Ω, ω constants. The research raises many interesting questions, such as why anyone would want to predict the trajectory of a bacterium, but people were probably asking Newton and Kepler why anyone would want to predict the trajectory of a planet...



5.1 Find a formula for the speed of the bacterium, in terms of v_0

5.2 Find formulas for the acceleration vector in the $\{\mathbf{i}, \mathbf{j}\}$ basis, in terms of v_0, Ω, ω

5.3 Find formulas for the normal and tangential components of the acceleration vector, in terms of v_0, Ω, ω

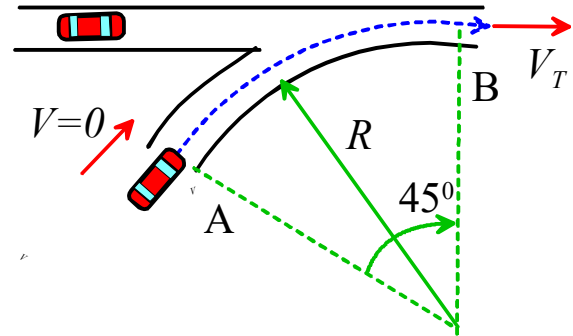
5.4 Find a formula for the radius of curvature of the path as a function of time (get your answer quickly using physics/dynamics, rather than using the long general formulas for curvature).

5.5 Write a MATLAB script to calculate the trajectory of the bacterium (include your solution in the same file that contains your solution to problem 2). You can do this in MATLAB by: (1) writing a function to calculate $d\mathbf{w}/dt = [dx/dt, dy/dt]$ given t , $\mathbf{w}=[x,y]$ and v_0, Ω, ω ; then (2) using ode45 to integrate the velocity to calculate x and y . Plot the bacterial trajectories for the following combinations of parameters

- $v_0 = 0.26 \mu\text{ms}^{-1}$ $\omega = 0.059 \text{rad/s}$ $\Omega = 0.2\omega$ $0 < t < 223.5\text{s}$
- $v_0 = 0.26 \mu\text{ms}^{-1}$ $\omega = 0.035 \text{rad/s}$ $\Omega = 1.5\omega$ $0 < t < 400\text{s}$
- $v_0 = 0.16 \mu\text{ms}^{-1}$ $\omega = 0.028 \text{rad/s}$ $\Omega = 2.39\omega$ $0 < t < 544.5\text{s}$

(these are parameters from Table 1 in the publication). You can put all the trajectories on the same plot, but offset their initial positions so the curves don't overlap.

6. A highway entry ramp is designed as $1/8$ arc of a circle with radius R . Vehicles start at rest at A and merge with traffic moving with speed V_T at B .



The magnitude of the vehicle's acceleration between A and B cannot not exceed a_{\max} , otherwise the car will skid.

6.1 Find a formula for the maximum possible tangential acceleration to travel between A and B without skidding, in terms of a_{\max} , R and the car's speed V

6.2 Hence show that if the vehicle accelerates with its max acceleration, then its speed satisfies

$$RV \frac{dV}{ds} = \sqrt{a_{\max}^2 R^2 - V^4}$$

where s is the distance traveled by the vehicle from A.

6.3 By separating variables and evaluating the integral with respect to V using MATLAB, show that the car's speed at B is

$$V = \sqrt{Ra_{\max}}$$

6.4 What is the radius of curve required to merge with traffic travelling at 55mph, if $a_{\max} = 0.2g$?