School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 4: Conservation Laws for Particles
Due Friday Feb 28, 2020

Lecture Schedule – please note that this week's HW is rather closely synchronized with lectures/Sections. We will work through an example similar to Problem 2 in Section on Monday Feb 24; Material for problems 5 and 6 will be discussed in lecture on Tues Feb 25, and we may not get to the material for problem 7 until Thurs Feb 27.

1. LAMMPS is an open-source molecular dynamics code maintained by Sandia National Labs. It is used to model material behavior, chemical reactions, and biochemical processes at molecular scales. It includes a large library of interatomic potentials: the 'Beck' potential is one example. It gives the potential energy of a pair of He atoms that are separated by a distance r as

$$U = E_0 \left[\exp\left\{ -\alpha r - (\beta r)^6 \right\} - \frac{b^6}{\left(r^2+a^2\right)^3} \left\{ 1 + \frac{c^2+3a^2}{r^2+a^2} \right\} \right]$$

Values for the constants are listed in the table

E_0	6.3878x10^{-17} J (398.7eV)
a	*0.675* Angstroms
α	4.39 Angstroms^{-1}
β	0.1829 Angstroms^{-1}
b	0.3616 Angstroms
c	1.6459 Angstroms

1.1 Plot the energy as a function of r (use units of eV for the energy, and Angstroms for the distance r. 2.6<r<5 Angstroms gives a clear plot) (Just hand in your plot; there is no need to submit MATLAB code)

1.2 Plot a graph showing the force as a function of separation r. (use units of picoNewtons for the force, and angstroms for the distance r. *2.9*<r<5 Angstroms gives a clear plot) (Just hand in your plot; there is no need to submit MATLAB code)

1.3 Find the (static) force required to break the bond between the He atoms). If you are using a MATLAB live script to do the calculation you will need to use the 'vpasolve' function to find the value of r that maximizes the force of attachment. See Sect 6.1 of the MATLAB tutorial for an example.

1.4 Find the equilibrium separation between two He atoms (the separation when $F=0$) (in Angstroms)

1.5 Find the stiffness of the bond (at the equilibrium separation) in N/m

2. Eviation Alice is a prototype 9+2 seat electric commuter aircraft.

The aircraft has the following specifications

- Max take-off weight 6350kg
- Max power output from each of three electric motors 260 kW (kiloWatts)
- Cruise speed 260 knots
- Battery capacity 900 kWh (the total energy in the battery , in kilo-Watt hours)
- Range, (at cruise speed, with 45 min reserve): 650 Nautical miles
- Approach speed 100 knots

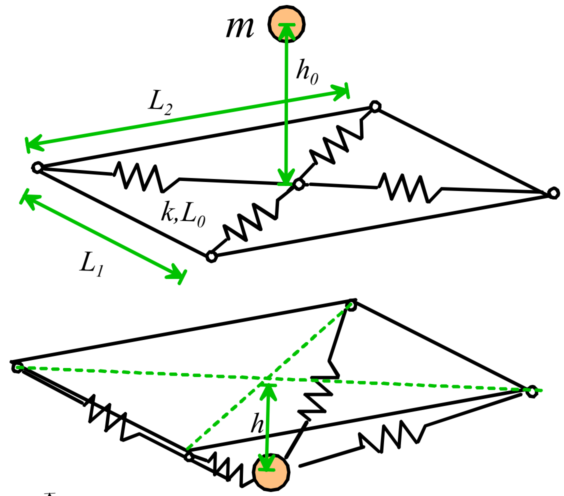
2.1 Use the given cruise speed, battery capacity, and range – accounting for the reserve – to estimate the power expenditure during level cruise.

2.2 Hence, estimate the drag force on the aircraft during level cruise

2.3 If the aircraft descends from 10000ft to sea level with zero engine power at the same speed and lift:drag ratio as cruise, how far will the aircraft travel during the descent?

2.4 Estimate the maximum (constant speed) climb rate of the aircraft (at max takeoff weight; assume the same power loss to drag as in level cruise)

2.5 Assuming that the engine produces a constant power, find a formula for the acceleration of the aircraft during take-off (use the power-KE relation for a particle – if you take a time derivative you get the acceleration – see class notes for an example. You can neglect drag to keep things simple). Hence, estimate the minimum length of runway required to take off (assume takeoff speed is the same as the approach speed; this is probably an overestimate.).



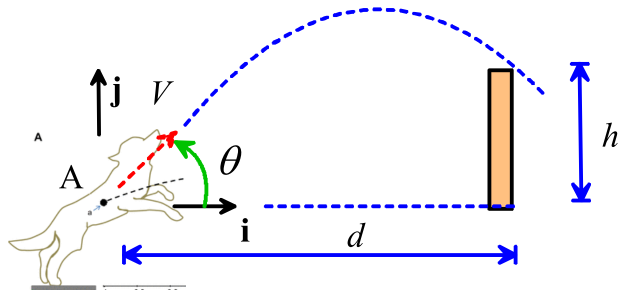
3. In this report a 7m x 5m fall protection net is tested by dropping a 106 kg sphere onto the net. The subsequent deflection of the sphere inside the net is measured with a high-speed camera. In a test with an initial drop height of 3m above the flat net, a deflection of h=2.6m was measured.

The net can be idealized as four springs, as shown in the sketch.

3.1 Use energy conservation and the known deflection and drop height to estimate the stiffness k of the springs (assume they are free of force before the sphere impacts the net)

3.2 Hence, calculate the maximum acceleration of the sphere.

4 In this publication it is shown that when dogs jump over an obstacle, they select a jump trajectory that minimizes the energy required to clear the obstacle. The goal of this problem is to find formulas for the angle and energy of the optimal trajectory, as a function of the distance and height of the obstacle (presumably dogs do this calculation in their heads).



Suppose that the dog jumps so that its COM follows the trajectory shown: at time $t=0$ it has speed V and an angle θ, and just clears the obstacle at a distance d and height h an unknown time t later.

4.1 Use the trajectory equations to find a formula for the minimum jump speed V necessary to clear the obstacle, in terms of g,d,h and the jump angle θ (one way to do this is to use the **i** component of the trajectory equation to find the time required to reach the obstacle, and substitute this into the **j** component and then solve for V)

4.2 Hence, show that the energy (i.e. the KE of the dog just after jumping) required to clear the obstacle can be expressed as

$$\frac{E}{mgh} = \frac{(d/h)^2}{4\cos\theta((d/h)\sin\theta - \cos\theta)}$$

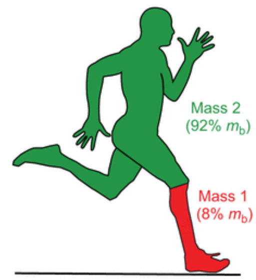
4.3 Plot a graph of E/mgh as a function of θ for $d/h = 2$, with $0.7 < \theta < 1.2$. Note that there is a critical angle that minimizes the energy. (There is no need to submit MATLAB code, just submit the plot)

4.4 Find a formula for the optimal angle in terms of d/h. (If you use MATLAB for this you will likely get a very weird looking answer. But you can get a simple expression if you do the calculation by hand).

4.5 Hence, plot a graph of the optimal value of E/mgh as a function of d/h, for $0.7<d/h<2$. Be careful with the range of θ: the formula in 4.4 has solutions with both positive and negative values of θ, and if you solved 4.4 with MATLAB you will get at least 4 roots, maybe more, depending on your version of MATLAB. But of course only the positive values less than $\pi/2$ are relevant. Substituting some numbers for d/h into MATLAB's solution will show you which root to use. If you solve 4.4 by hand it's easier to do 4.5). There is no need to submit matlab code.

4.6 (optional) If you have a dog, please submit a picture (pictures of other pets may be submitted if you don't have a dog)!

5. A csv file on the engn40 website homework page contains the results of a force-plate measurement of the force acting on a runner's foot while it is in contact with the ground (estimated from this publication). The first column in the file is time (in seconds); the second column is the measured force (in Newtons). The runners mass is 70kg.



5.1 Write a MATLAB script to read the file and plot the force as a function of time, and also calculate the total impulse exerted by the contact force.

5.2 Hence, use the impulse-momentum equations to estimate the vertical velocity of the runner just before and just after the impact with the ground (assume steady running, and neglect energy loss during the airborne phase of the stride)

5.5 Calculate the number of foot strikes per second (assume each stride is identical). There is more than one way to solve this problem – you could use the trajectory equations or the impulse-momentum equations…

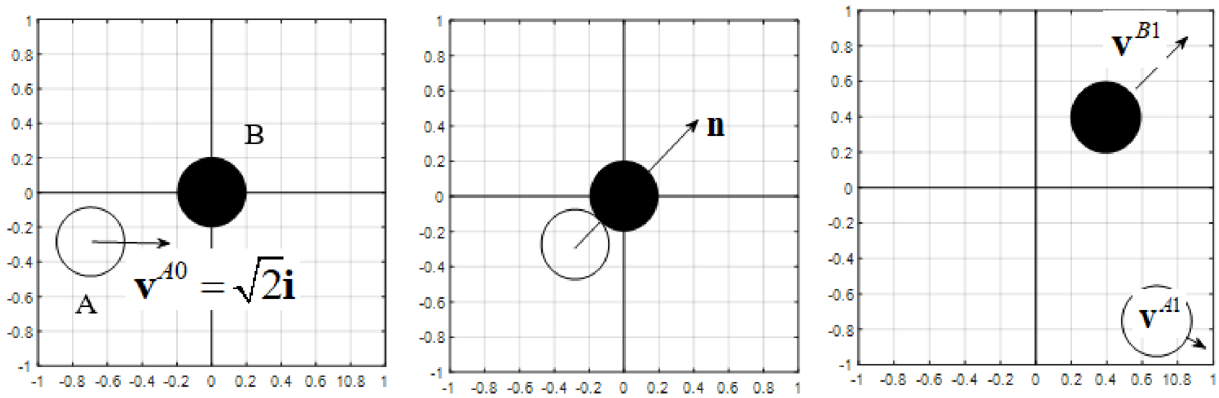
6. This website suggests that increasing the restitution coefficient between a golf club and ball from e=0.78 to e=0.822 will add 21 yards to a drive with a V=100mph club tip speed (just before impact). Check this estimate by calculating the flight distance. Use the following approach and data:

- Golf ball mass m_B=*45.93* grams
- Club head mass m_C = 200grams
- Idealize the collision between the club and head as a straight line collision; neglect any force exerted on the head of the club by the driver shaft during the collision
- Use the trajectory equations without air resistance to estimate the flight distance (this is not very accurate – and it is not hard to incorporate lift and drag forces with MATLAB calculations - but it's not worth the trouble….). Assume that the trajectory starts at a 45 degree angle.

Show that the additional distance resulting from changing the restitution coefficient from e_1 to e_2 is

$$\Delta x = \frac{\left[(1+e_2)^2 - (1+e_1)^2\right]}{\left(1+(m_B/m_C)\right)^2}\frac{V^2}{g}$$

Hence, calculate Δx with the numbers given.



Before impact At impact After impact

7. The figure shows a frictionless oblique impact between two spheres. The spheres both have mass 1 kg. At time $t=0$ sphere B is at rest, and sphere A has velocity vector $\mathbf{v}^{A0} = \sqrt{2}\mathbf{i}$ m/s. The collision direction $\mathbf{n}$ is at 45 degrees to the $\mathbf{i}$ and $\mathbf{j}$ directions.

7.1 The figure shows the magnitude of the force acting on the particles at the point of contact during the collision. Calculate the magnitude of the impulse.

7.2 Explain why the direction of the impulse on each individual particle during the collision must be parallel to $(\mathbf{i}+\mathbf{j})$

7.3 Hence, calculate the velocity vectors for particles A and B after the collision

7.4 Calculate the restitution coefficient for the collision.