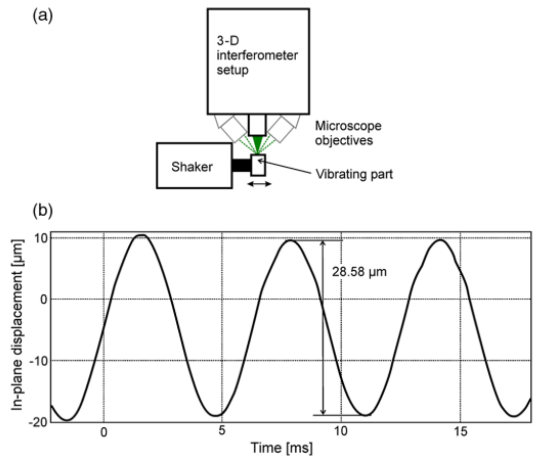
EN40: Dynamics and Vibrations

Homework 5: Vibrations
Due Friday March 20, 2020

School of Engineering
Brown University

1. The figure (from this publication) shows a vibration measurement from vibrometer microscope. Use the figure to estimate



1.1 The amplitude of the displacement

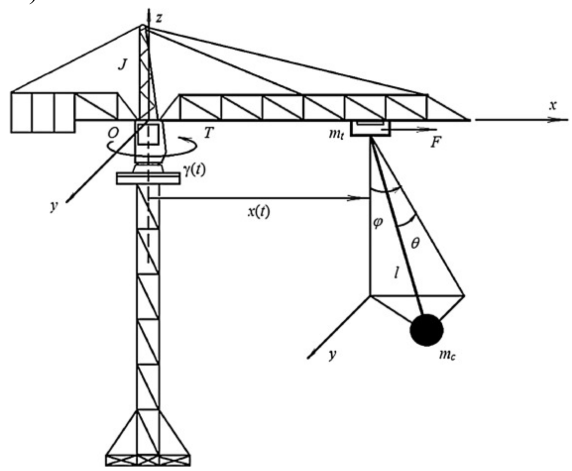
1.2 The period of the vibration

1.3 The frequency (in Hertz) and angular frequency (in rad/s)

1.4 The amplitude of the velocity

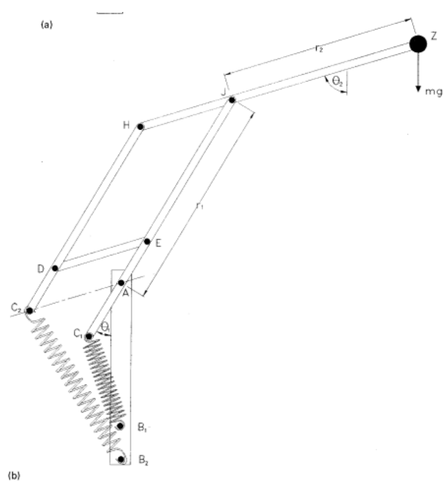
1.5 The amplitude of the acceleration

2. Find the number of degrees of freedom and vibration modes for each of the systems shown in the figures (you may need to consult the publications to understand the system)

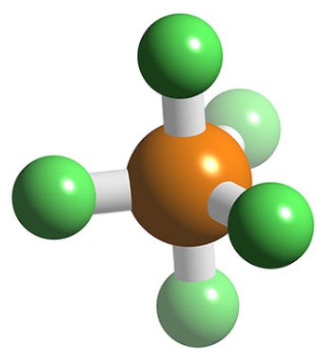


(a) [Theoretical model of a tower crane] (you may need to check the publication for a clearer description of the system than the figure

(b) [Universal Stage]



(c) [AnglePoise Lamp] (treat the system as 2D)



(d) Phosphorus pentafluoride molecule

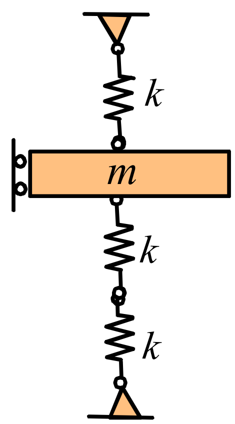
3. Solve the following differential equations (please solve them by hand, using the tabulated solutions to differential equation – you can check the answers with matlab if you like)

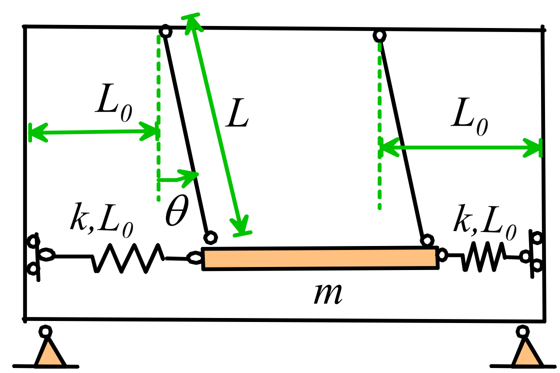
3.1 $\frac{1}{5}\frac{d^2y}{dt^2}+5y=0 \qquad y=1 \qquad \frac{dy}{dt}=0 \qquad t=0$

3.2 $\frac{d^2y}{dt^2}-4y=-4 \qquad y=1 \qquad \frac{dy}{dt}=-1 \qquad t=0$

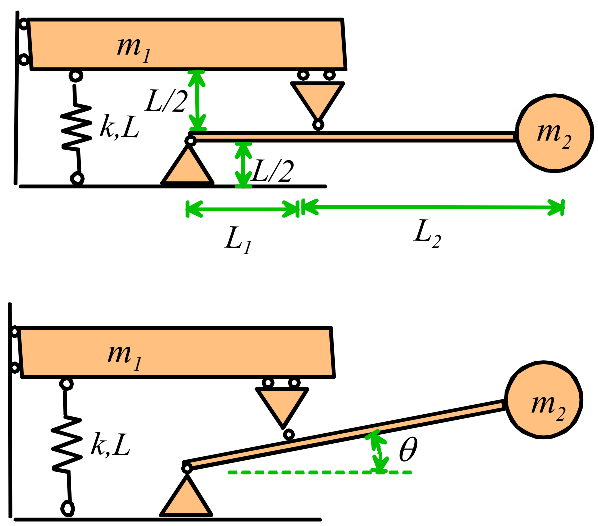
3.3 $\frac{1}{4}\frac{d^2y}{dt^2}+2\frac{dy}{dt}+4y=32\sin(t)+16\cos(t) \qquad y=0 \qquad \frac{dy}{dt}=0 \qquad t=0$

4. Find formulas for the natural frequency of vibration for the systems shown in the figure





5. The figure shows an 'anti-resonant' vibration isolation system (we'll analyze its behavior in more detail in next week's HW).

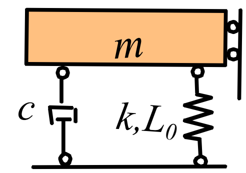


5.1 Find formulas for the kinetic and potential energy of the system in terms of the angle θ and its time derivatives. **Neglect gravity**.

5.2 Hence, find the equation of motion for θ

5.3 Linearize the equation for small θ, and hence find the natural frequency of the system. (Your equation of motion will include a term that is multiplied by $\sin\theta(d\theta/dt)^2$ - for small harmonic θ , the angular velocity $d\theta/dt$ is also small, so when the equation of motion is linearized this term is neglected)

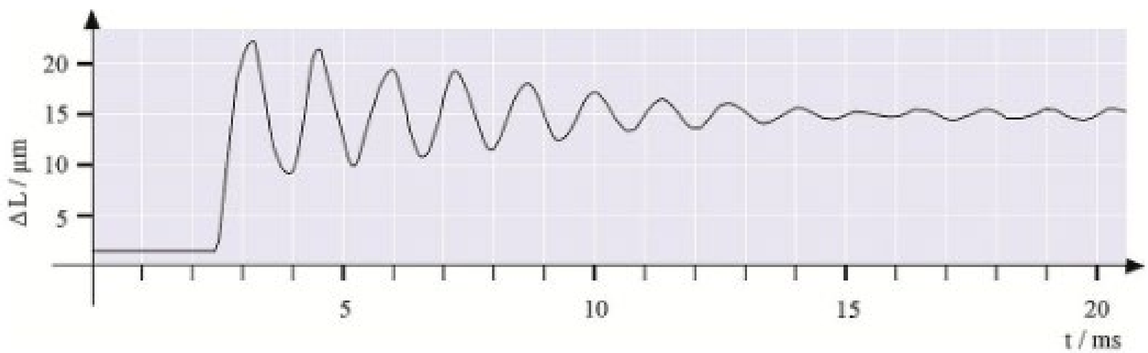
6. The spring-mass-dashpot system shown in the figure has an undamped natural frequency $\omega_n = 100 rad/s$ and damping factor $\zeta = 0.1$. The spring has stiffness $k = 10^3 N/m$.



6.1 Calculate the mass *m* and dashpot coefficient *c*.

6.2 The system is released from rest in the vertical configuration shown. At time *t=0* the spring is free of force. Plot a graph of the subsequent vertical displacement of the mass from its initial position (i.e. the change in length of the spring, in mm) as a function of time (for 0<t<0.5s). Include the effects of gravity. You only need to submit relevant calculations and the plot, a MATLAB upload is not required.

6.3What value of the dashpot coefficient *c* is required to make the system critically damped?



7. The figure ([from this product spec]) shows the measured damped vibration response of a piezoelectric actuator.

7.1 Calculate the period and log decrement for the signal

7.2 Hence determine the undamped natural frequency ω_n and damping coefficient ζ for the system