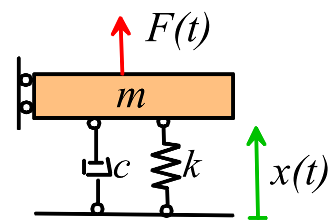
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 6: Forced Vibrations
Due Friday April 10, 2020



1. The spring-mass system shown in the figure has

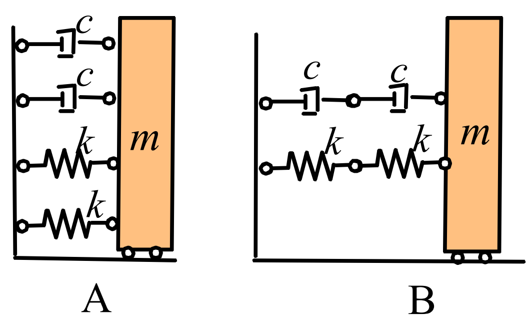
- m=40kg,
- $k = 38.6 kN/m$
- $c = 0.2518 kNs/m$

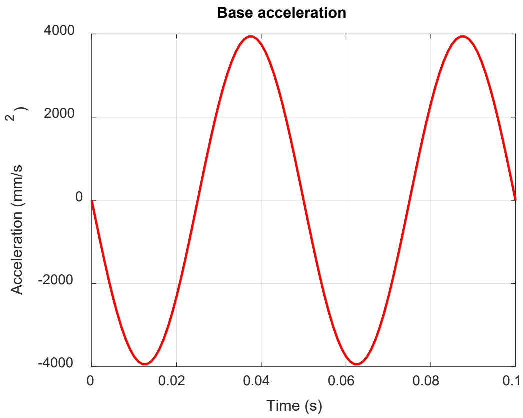
The mass vibrates harmonically with an amplitude of 1mm and angular frequency $\omega = 100 rad/s$.

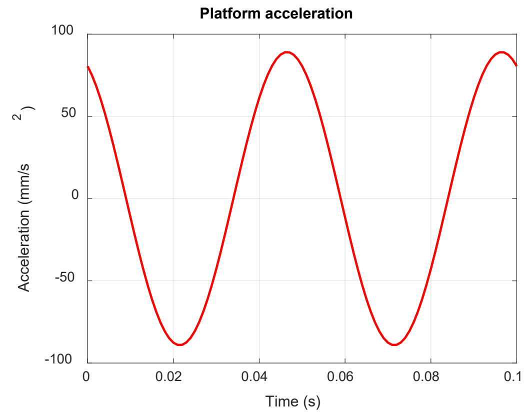
1.1 What is the amplitude of the force?

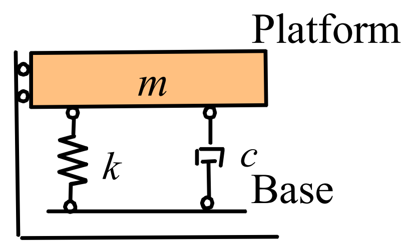
1.2 What is the frequency of the force that yields the maximum vibration amplitude? If the force amplitude is the same as the value computed in 1.1, what is the resulting vibration amplitude?

2. System A has natural frequency and damping coefficient ω_n, ζ . Find the natural frequency and damping coefficient of system B









3 The figure shows readings from accelerometers attached to the base and platform of a vibration isolation table.

3.1 Find

(1) The period of the vibration;

(2) The frequency of the vibration (in both Hz and radians per second);

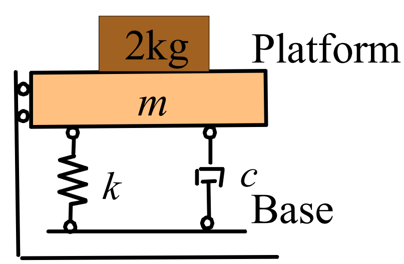
(3) The amplitude of the displacement (in mm) of the base

(4) The amplitude of the displacement (in mm) of the platform

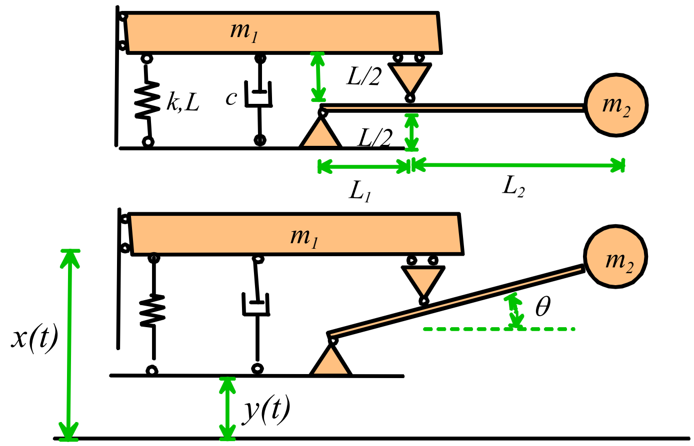
(5) The magnification.

3..2 In a separate experiment, the natural frequency of vibration of the table is measured to be 2Hz. Use this information and your solution to 21.1 to calculate the damping coefficient ζ for the platform

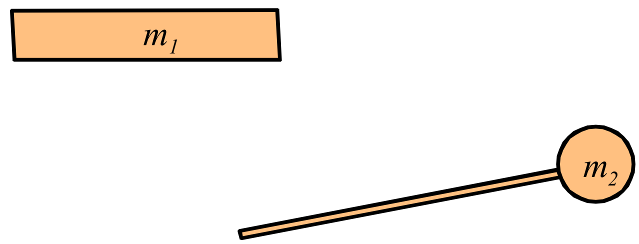
3.3 A 2kg mass is placed on the table. It is found that the natural frequency of the table with the mass on its surface decreases to 1Hz. Calculate the value of the table mass m, the spring constant k and the dashpot coefficient c.



4. In this problem we will analyze the behavior of the 'anti-resonant' vibration isolation system introduced in Homework 5. The system is illustrated in the figure. Assume that the base vibrates vertically with a displacement $y(t) = Y_0 \sin \omega t$. Our goal is to calculate a formula for the steady-state vertical motion $x(t)$ of the platform, and to compare the behavior of this system with the standard base excited spring-mass-damper design for an isolation system.



4.1 Draw free body diagrams showing the forces acting on the mass m_1 and the pendulum assembly (see the figure).



4.2 Using geometry, find an expression for the acceleration of mass m_2 in terms of θ and y (and their time derivatives as well as relevant geometric constants) (You could write down a formula for the position vector relative to a fixed origin and differentiate it). Show that if θ and its time derivatives are small the result can be approximated by

$$\mathbf{a} \approx \left[\frac{d^2 y}{dt^2} + (L_1 + L_2)\frac{d^2\theta}{dt^2} \right] \mathbf{j}$$

Show also (using geometry) that

$$\frac{d^2 x}{dt^2} - \frac{d^2 y}{dt^2} \approx L_1 \frac{d^2\theta}{dt^2}$$

4.3 For the pendulum, write down $\mathbf{F} = m\mathbf{a}$ and $\mathbf{M} = 0$ about the center of mass, in terms of reaction forces shown in your FBD. Use the approximation in 4.2 for the acceleration.

4.4 Write down $\mathbf{F} = m\mathbf{a}$ for mass m_1 , and hence use 4.3 and the second of 4.2 to show that (if θ and its time derivatives are small) then

$$\left(m_1 + m_2 \frac{(L_1 + L_2)^2}{L_1^2} \right) \frac{d^2 x}{dt^2} + c\frac{dx}{dt} + kx = kL + m_2 \frac{(L_1 + L_2)L_2}{L_1^2} \frac{d^2 y}{dt^2} + c\frac{dy}{dt} + ky$$

4.5 Show that the equation can be re-arranged into the form

$$\frac{1}{\omega_n^2}\frac{d^2x}{dt^2}+\frac{2\zeta}{\omega_n}\frac{dx}{dt}+x=\frac{\lambda^2}{\omega_n^2}\frac{d^2y}{dt^2}+\frac{2\zeta}{\omega_n}\frac{dy}{dt}+y+C$$

and show that

$$\omega_n=\sqrt{\left(\frac{kL_1^2}{L_1^2 m_1+m_2(L_1+L_2)^2}\right)} \qquad \lambda=\sqrt{\left(\frac{m_2(L_1+L_2)L_2}{L_1^2 m_1+m_2(L_1+L_2)^2}\right)}$$

$$\zeta=\frac{c}{2\sqrt{k(m_1L_1^2+m_2(L_1+L_2)^2)/L_1^2}}$$

4.6 The solution to the EOM in part 4.5 can be found in the pdf solution to equations of motion for vibration systems. What is the frequency corresponding to the anti-resonance (the minimum value of M), in terms of λ,ω_n (give an approximate solution for $\zeta << 1$; it is fine to write down the answer without doing any calculus) ? What is (approximately) the smallest vibration amplitude (in terms of λ,ζ)?

4.7 For what range of frequency (in terms of λ , ω_n) does the antiresonant system give better performance than the simpler spring-mass-damper system?

4.8 This paper analyzes a design for an anti-resonant isolator to be used in a submarine (you will need a VPN connection to Brown for the link to work). The isolator has the following specifications:

- $m_1 = 345.37kg \qquad m_2 = 23.65kg$
- $(L_1+L_2)/L_1 = 9$
- $k = 160$ MN/m

The authors do not state the value they used for the dashpot coefficient c. Select a value for c that will give $M<2$ for all frequencies, and gives the best vibration isolation at the anti-resonance (you can use any method you like to do this – trial and error, or if you want to be super-precise use more fancy math. For example you can find the frequency that maximizes M and then solve $\max(M)=2$ for c). For this value of c plot a graph showing the predicted magnification for the isolator.