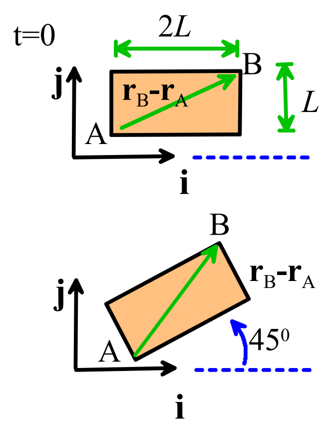
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 7: Rigid Body Kinematics, Inertial properties of rigid bodies
Due Friday April 17, 2020

1. The rigid body shown in the figure is at rest at time $t=0$, and rotates counterclockwise with constant angular acceleration of 8 rad/s^2. At the instant shown in the lower figure, find



1.1 The angular velocity vector

1.2 The spin tensor **W** (as a 2x2 matrix)

1.3 The rotation tensor (a 2x2 matrix for a 2D problem) **R** that rotates the rectangle from its initial to its position in the lower figure.

1.4 Hence, find a formula for the vector $\mathbf{r}_B - \mathbf{r}_A$ in the rotated body in (**i**,**j**) components

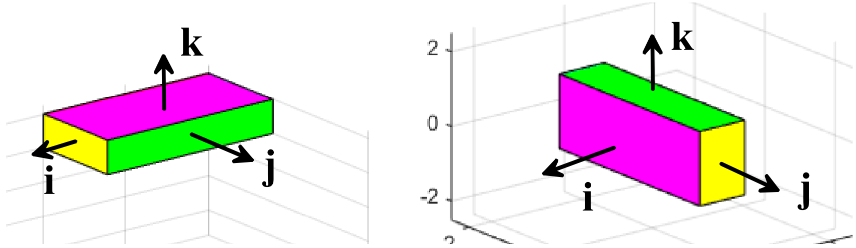
2. "Codman's paradox" is described in a 1935 medical textbook (p43). It's not really a paradox – but it does demonstrate the counter-intuitive nature of sequences of rotations. This paper has a clearer statement of the 'paradox,' than the original book, as follows: "first place your right arm hanging down along your side with your thumb pointing forward and your fingers pointing toward the ground. Next, elevate your arm horizontally so that your fingers point to the right, and then rotate your arm in the horizontal plane so that your fingers now point forward. Finally, rotate your arm downward so that your fingers eventually point toward the ground. After these three rotations, you will notice that your thumb points to the left. That is, your arm rotated by 90 degrees. The fact that you get this rotation without having performed a rotation about the longitudinal axis of your arm is known as Codman's paradox".

2.1 Take the **i,j,k** directions to be **i**: horizontal, pointing to your right; **j**: horizontal, pointing in front of you, and **k**: vertically upwards. Using this basis, write down the rotation matrices for the three rotations involved in the Codman maneuver.

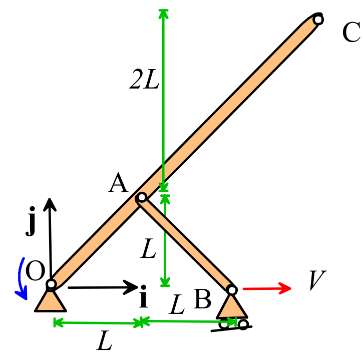
2.2 Find the rotation matrix resulting from the sequence of three rotations.

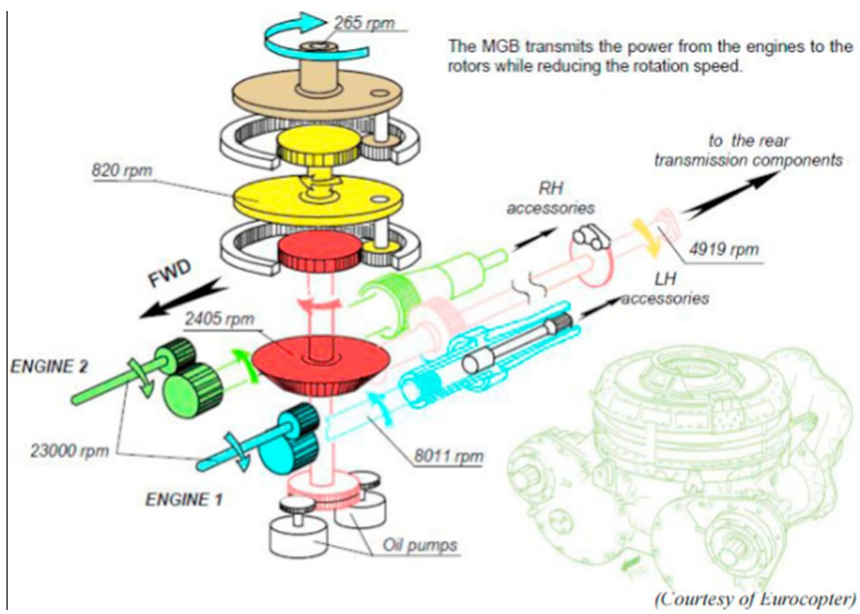
2.3 Show that 2.2 is equivalent to a 90 degree rotation about the **k** direction (hence explaining the paradox)

3. Find a (single) rotation matrix that will rotate the prism from its initial to its final configuration shown in the figure. Find the axis and angle of the rotation.



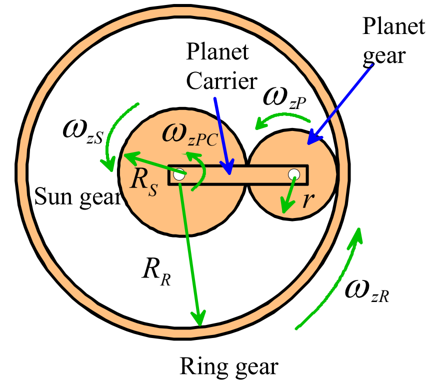
4. The figure shows a four-bar chain mechanism. Joint B moves horizontally with constant speed V. Calculate the angular velocities and angular accelerations of members OC and AB, and find the velocity and acceleration of C.



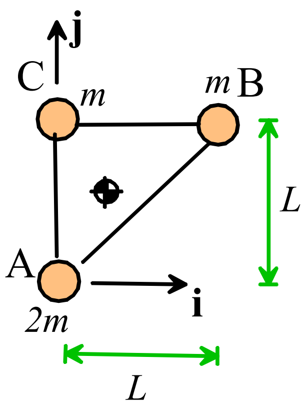


5. [This paper](#) (VPN connection to Brown required) describes a planetary gear system in a helicopter (see the figure). The ring gear in both epicyclic gears are stationary. The figure shows the angular speeds of the sun and planet carrier on the two systems. Select numbers of teeth for the ring, sun and planet gears in the two epicyclics (find the lowest possible numbers of teeth on each gear).

6 In the figure shown, the planet carrier (the bar) rotates counterclockwise with angular speed $\omega_{zPC} = \omega_0$. The planet gear has zero angular speed. Find the angular speed of the sun gear.



7. The figure shows three particles connected by rigid massless links. The particle at A has mass $2m$; those at B and C have mass m. The assembly rotates at constant angular speed ω about an axis parallel to $\mathbf{k}$ passing through the center of mass. The point of this problem is to demonstrate that the rigid body formula for the kinetic energy of the system gives the same answer as calculating the kinetic energy of each mass separately, and summing them. The rigid body formulas for angular momentum and kinetic energy are just fast ways of summing the total angular momentum and KE of a system of particles.



7.1 Calculate the position of the center of mass of the assembly

7.2 Calculate the 2D mass moment of inertia of the system about the center of mass

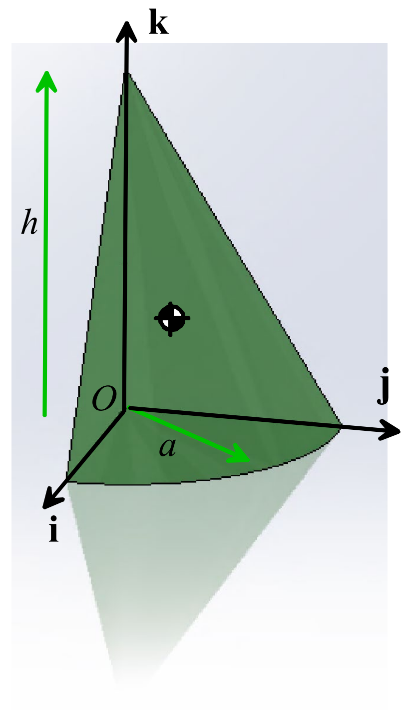
$$I_{Gzz} = \sum_i m_i \left(d_{xi}^2 + d_{yi}^2 \right)$$

where $\mathbf{d}_i = d_{xi}\mathbf{i} + d_{yi}\mathbf{j} = \mathbf{r}_i - \mathbf{r}_G$ is the position vector of the ith particle with respect to the center of mass.

7.3 Suppose that the assembly rotates about its center of mass with angular velocity $\omega\mathbf{k}$ (the center of mass is stationary). What are the speeds of the particles A,B and C?

7.4 Calculate the total kinetic energy of the system (a) using your answer to 7.2; and (b) using your answer to 7.3. (The point of this problem is to demonstrate that the rigid body formula $(1/2)I\omega^2$ is just a quick way of summing the kinetic energies of the 3 masses. For the simple 2D system here it is quite simple to prove the equivalence for any arrangement of masses. For 3D the derivation is more complicated, but the idea is the same.)

8. The figure shows a 1/4 segment of a cone with radius a, height h and uniform mass density ρ Using a Matlab 'Live Script', calculate



8.1 The total mass M (you will need to do the relevant integrals using cylindrical-polar coordinates)

8.2 The position vector of the center of mass (with respect to the origin shown in the figure)

8.3 The inertia tensor (matrix) about the center of mass, in the basis shown

8.4 Using the parallel axis theorem, calculate the mass moment of inertia about the origin O.

Please upload your 'Live script' solution to Canvas.