School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 8: Rigid Body Dynamics
Due Friday April 24, 2020

1 A two bladed wind-turbine with total mass 120kg (60kg per blade) and rotor diameter 20m is spun up from rest to an angular speed of 10 radians per second in 100 sec. Calculate the torque acting on the turbine.

2. In the planetary gear system shown,

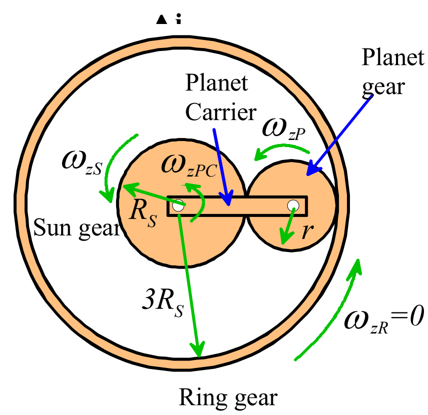
(i) the sun gear has radius R_S and mass m ,

(ii) the ring gear has radius $3R_S$,

(iii) the planet gear has mass m and

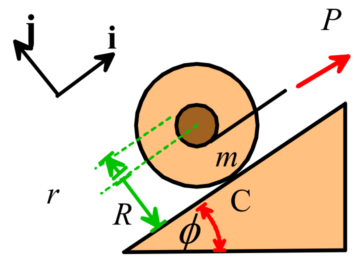
(iv) the planet carrier has mass $m/2$.

The sun gear rotates with angular speed ω_{zS} and the ring gear is stationary.



Find a formula for the total angular momentum of the assembly about the center of the sun gear, in terms of ω_{zS} , R_S and m. Treat the gears as disks, the planet carrier as a 1D rod and assume there's only one planet gear as shown to keep things simple; this would be a rather unusual gear system but adding more gears just makes the problem tedious without illustrating any new concepts...

3. The point of this problem is to illustrate the choices you can make when you apply the moment-angular momentum equation to calculate the acceleration and angular acceleration of a rigid body subjected to forces. The figure shows a spool with radius R and mass moment of inertia $mR^2/2$ that is being pulled up a ramp by a cable subjected to a (known) force P. Assume that the spool rolls without slip.



3.1 Draw a free body diagram showing the forces acting on the spool

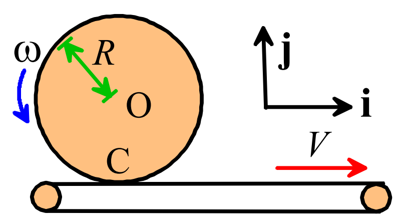
3.2 Write down the rigid body kinematics equation that relates the angular acceleration α_z and linear acceleration a_{Gx} of the center of mass of the spool

3.3 Write down the equation relating forces and the time derivative of linear momentum of the spool (i.e. Newton's law, but the version for rigid bodies)

3.4 Write down the equation that relates the total moment acting on the spool to the time derivative of its angular momentum. Take moments (and angular momentum) about the center of the spool. Hence, solve the equations in 3.3. and 3.4 to calculate the angular acceleration of the spool and the acceleration of its COM a_{Gx}

3.5 Repeat 3.4, but this time apply the moment – d**h**/dt relation by taking moments about the contact point C. Notice that (just like when you do statics) you can simplify the algebra by choosing to take moments about a convenient point – it doesn't change the answer.

4. A solid cylinder with mass m and radius R rests on a conveyor belt. At time $t=0$ the belt moves with speed V to the right, and the cylinder has zero angular velocity (so its center is also moving with speed V. The belt is then suddenly reversed, so it moves with speed V to the left. The goal of this problem is to analyze the subsequent motion of the cylinder.

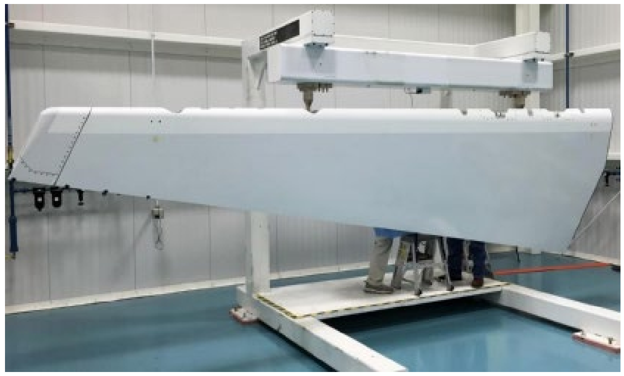


4.1 Draw a free body diagram showing the forces acting on the cylinder just after the belt is reversed

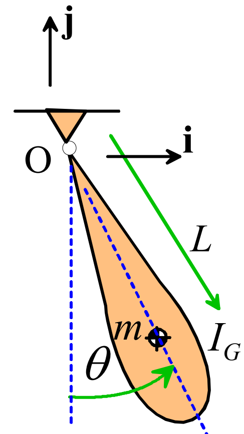
4.2 Write down the equations of translational and rotational motion (ie **F**=m**a** and the moment-angular momentum relation) for the cylinder (use the 2D equations)

4.3 Use the friction law and the solution to 4.2 to calculate the linear and angular acceleration of the cylinder during the period of slip

4.4 Hence, calculate how long it takes before the cylinder begins to roll without slip on the belt. Find the linear and angular velocity at this time. Does the cylinder end up moving to the left or to the right?



5. This product specification describes a device that is designed to measure the position of the center of mass and mass moment of inertia of large aircraft parts. Their approach is to suspend the part of interest as a pendulum, and then to measure the amplitude and period of free oscillation of the part. They also measure the reaction forces at the pivots. This information is sufficient to determine the mass moment of inertia, as well as the distance of the center of mass of the part from the pivot (the remaining two coordinates of the COM can be found from static measurements).



The goal of this problem is to find the formulas that can be used to determine the COM position and inertia from measurements of the period and reaction forces on a suspended rigid body.

5.1 Write down a formula for the mass moment of inertia of the pendulum about O, in terms of I_G, m, L .

5.2 Use the energy method discussed in class to find the equation of motion for the pendulum, and hence find the formula for its natural frequency of vibration, in terms of I_G, m, L, g

5.3 Draw a free body diagram showing the forces acting on pendulum

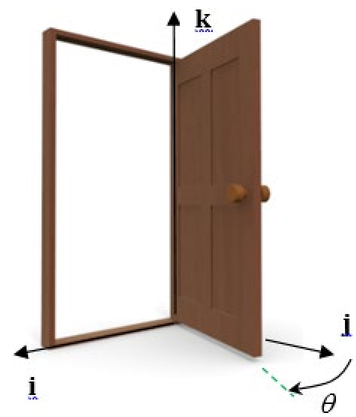
5.4 Find a formula for the acceleration of the center of mass in terms of θ and its time derivatives, and use Newton's laws to find formulas for the reactions at the pivot.

5.5 Suppose that the pendulum is set swinging with a very small amplitude θ_0 . The amplitude θ_0; the natural frequency ω_n and the amplitude of vibration of the horizontal force H_0 are measured. Show that (provided $\theta_0 << 1$) $H_0 \approx mL\theta_0\omega_n^2$, and hence that the mass moment of inertia I_G and the distance of the COM from the pivot can be calculated from these measurements as

$$L = \frac{H_0}{m\omega_n^2\theta_0} \qquad I_G = \frac{H_0 g}{\omega_n^4\theta_0}\left\{1 - \frac{H_0}{mg\theta_0}\right\}$$

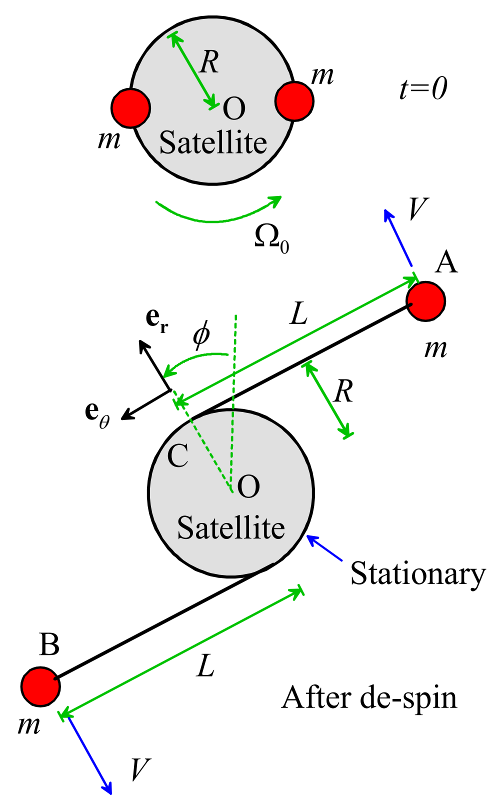
6. A wooden door with dimensions 7'x4'x1.5" is hung on hinges with a helical thread. The thread has pitch p – i.e. if the door is rotated through 360 degrees its height will decrease by a distance p

Design the pitch p so that if the door is released from rest and swings through a 90 degree angle, it will close in 10 seconds. (Start by finding the potential energy and kinetic energy of the door in terms of θ and its time derivative, then differentiating the energy conservation equation $T+U=C$ to find the angular acceleration of the door in terms of p. You can use the constant acceleration formula to find p. You should end up with a number for p, in inches or cm)



7 The figure shows a so called 'yoyo de-spin' of a satellite. The satellite is cylindrical, with radius R and mass moment of inertia about its center I_{Gzz} . Two masses m (with negligible mass moment of inertia about their COM) are attached to the cylinder by inextensible, massless tethers.

At time $t=0$ the tethers are wound tightly around the cylinder and the assembly spins with angular speed Ω_0 . To de-spin the satellite, the masses are released, and the rotational motion of the assembly causes the tethers to unwind from the cylinder. This slows the rotation of the cylinder. When the rotation of the cylinder stops, the tethers are cut. The goal of this problem is to determine the length of the tethers necessary to stop the rotation of the satellite.



7.1 Consider the system at time $t=0$. Assume that the center of the satellite at O is stationary and the satellite (with masses attached) spins at angular speed Ω_0 . Find the total kinetic energy of the system (small masses + cylinder together) at time $t=0,$ in terms of Ω_0, m, R, I_{Gzz} . Treat the small masses m as particles.

7.2 Find a formula the total angular momentum of the system (masses + cylinder) about the center of the cylinder at time $t=0$ in terms of Ω_0, m, R, I_{Gzz}.

7.3 Consider the assembly at the instant that the cylinder just comes to rest (after de-spin). Write down the position vector of the mass at A at this instant, (taking the origin to be at O) in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis in terms of R, L.

7.4 Suppose that at the instant the satellite stops rotating, the tether CA has angular speed $\omega = d\phi / dt$ and length L. Note that the cylinder (i.e. the satellite) and tether must have the same velocity where they touch at point C. Use the rigid body kinematics formula to find the velocity vector of the mass at A in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis in terms of ω, L .

7.5 Hence, find formulas for the total kinetic energy and angular momentum about O of the system (i.e. the satellite and both masses combined) at the instant that the cylindrical satellite comes to rest, in terms of ω, L, m.

7.6 Finally, by considering the energy and angular momentum of the system show that the cylinder comes to rest when the tether length reaches

$$L = \sqrt{\frac{\left(I_{Gzz} + 2mR^2\right)}{2m}}$$