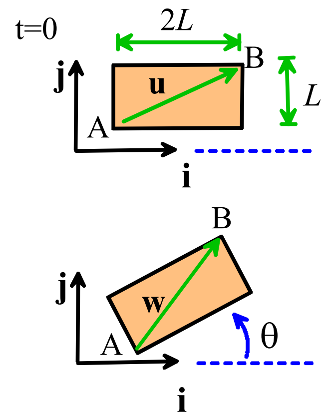
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 6: Rigid Body Kinematics, Inertial properties of rigid bodies
Due Friday July 30, 2021

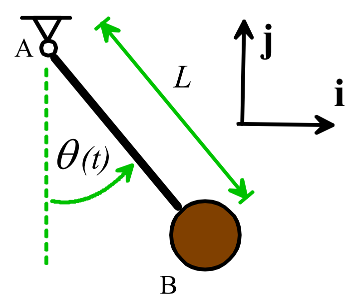
1. A vector **u** connecting two points A and B in a rigid body changes to a new vector **w** after the body is rotated through an angle θ about the **k** axis. The matrix describing the rotation is

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Show that **w** and **u** have the same length (this verifies that **R** is a rigid rotation)

2. A pendulum with length L swings in the $\{\mathbf{i},\mathbf{j}\}$ plane. It is at rest at time $t=0$ and its shaft subtends a small angle θ_0 to the vertical. Write down formulas for the angular velocity vector and angular acceleration vector of the shaft of the pendulum, as a function of L, θ_0, t and g. (You don't need to analyze the motion of the pendulum – just use the solution derived in lectures)



3. A time dependent 3D rotation is described by the matrix

$$\mathbf{R} = \begin{bmatrix} \cos^2(\Omega t) & \cos(\Omega t)\sin(\Omega t) & \sin(\Omega t) \\ \cos(\Omega t)\sin(\Omega t) & \sin^2(\Omega t) & -\cos(\Omega t) \\ -\sin(\Omega t) & \cos(\Omega t) & 0 \end{bmatrix}$$

where Ω is a constant.

3.1 Find a formula for the spin tensor (matrix) (MATLAB will make the calculus/matrix product painless)

3.2 Find a formula for the angular velocity vector

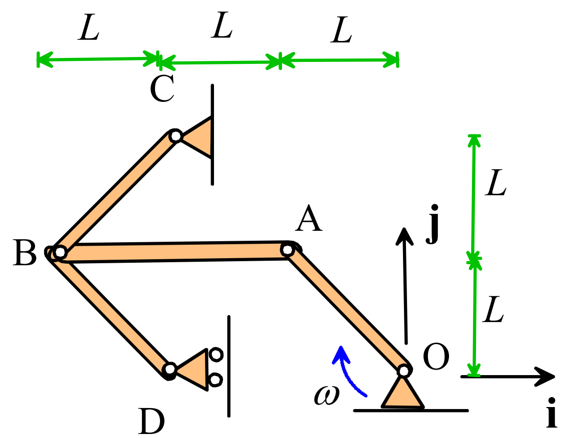
3.3 Find a formula for the angular acceleration vector

4. Let {**i**,**j**,**k**} be a Cartesian basis. A rigid body is subjected to a sequence of two rotations:
(a) A 90 degree clockwise rotation about the **i** axis
(b) A 90 degree clockwise rotation about the **j** axis.

An intelligent life-form without knowledge of linear algebra or rigid body dynamics (eg a humanities major) suggests that it should be possible to return the body to its original orientation by:
(c) A 90 degree counterclockwise rotation about the **i** axis, followed by
(d) A 90 degree counterclockwise rotation about the **j** axis.

Does this work? If not, find the axis and angle of the (single) rotation that will return the body to its initial orientation after step (d).

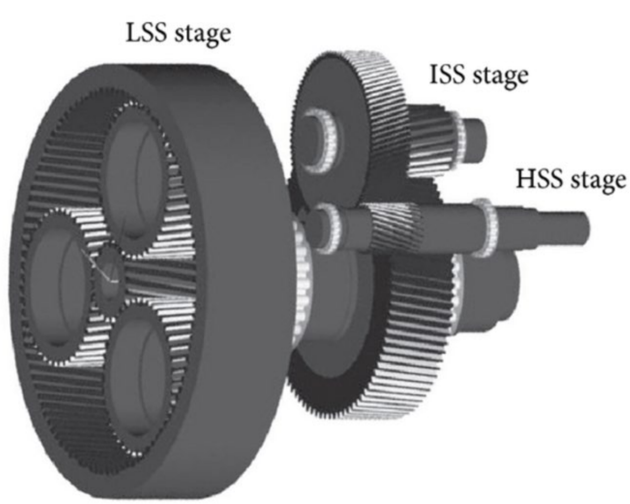
5. The figure shows a mechanism used in a hand-operated press. Member OA rotates clockwise with constant angular speed ω For the configuration shown in the figure:



5.1 calculate the velocity of point D, along with the angular speeds of members AB, BC and BD.

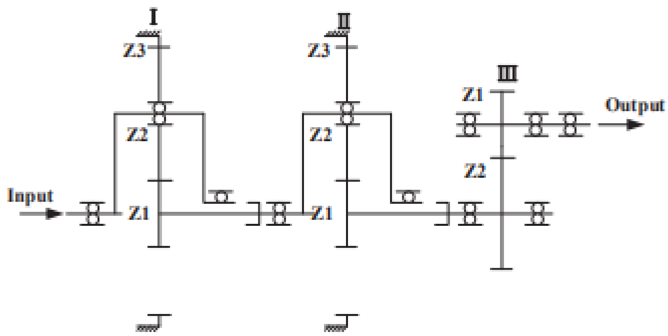
5.2 calculate the acceleration of point D, along with the angular accelerations of members AB, BC and BD.

6. The gearbox in a wind turbine is intended to allow the turbine (which rotates at a slow angular speed) and the generator (which runs at a fast angular speed – about 100 times faster than the turbine) to rotate at their most efficient speeds. The gearbox typically has two or 3 stages of gearing. The first stage is nearly always an epicyclic gearbox. The input to the epicyclic is connected to the turbine; the output must rotate as fast as possible.



There are 3 possible ways to connect the epicyclic: (i) run with the sun stationary and the planet carrier or ring connected to the turbine; (ii) the planet carrier stationary, and the sun or ring connected to the turbine; and (iii) the ring stationary, and the planet carrier or sun connected to the turbine.

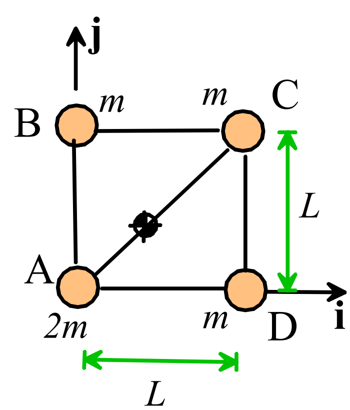
Which option will give the biggest ratio between the speed of the output shaft and the speed of the turbine?



Stage	Number of planet gears	Number of teeth		
		z_1	z_2	z_3
I	3	22	36	95
II	3	22	39	101
III	\	29	98	\

7. This publication analyzes vibrations in a 3 stage wind-turbine gearbox. A schematic diagram of the transmission is shown in the figure. The first two stages are epicyclics, operating with the ring gear fixed, input to the planet carrier, and output from the sun. The final stage is a regular gear pair. The table lists the number of teeth on the various gears in the system (for example, z1 is the number of teeth on the sun, z3 is the number of teeth on the ring in the two epicyclic gears. Note that the gears are helical so don't quite satisfy the usual relation $N_R = N_S + 2N_P$). Calculate the gear ratio of the gearbox (i.e. the ratio of the output speed to the input speed).

8. The figure shows four particles connected by rigid massless links. The particle at A has mass $2m$; those at B, C and D have mass m. The assembly rotates at constant angular speed ω about an axis parallel to $\mathbf{k}$ passing through the center of mass. The point of this problem is to demonstrate that the rigid body formula for the kinetic energy of the system gives the same answer as calculating the kinetic energy of each mass separately, and summing them. The rigid body formulas for angular momentum and kinetic energy are just fast ways of summing the total angular momentum and KE of a system of particles.



8.1 Calculate the position of the center of mass of the assembly

8.2 Calculate the 2D mass moment of inertia of the system about the center of mass

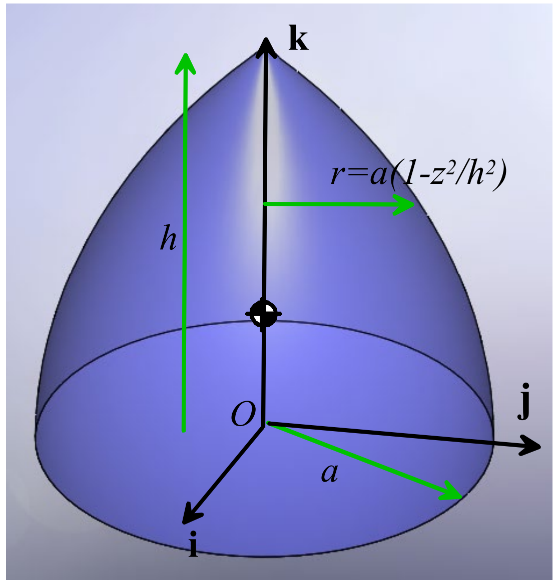
$$I_{Gzz} = \sum_i m_i \left(d_{xi}^2 + d_{yi}^2 \right)$$

where $\mathbf{d}_i = d_{xi}\mathbf{i} + d_{yi}\mathbf{j} = \mathbf{r}_i - \mathbf{r}_G$ is the position vector of the ith particle with respect to the center of mass.

8.3 Suppose that the assembly rotates about its center of mass with angular velocity $\omega\mathbf{k}$ (the center of mass is stationary). What are the speeds of the particles A,B and C?

8.4 Calculate the total kinetic energy of the system (a) using your answer to 8.2; and (b) using your answer to 8.3. (The point of this problem is to demonstrate that the rigid body formula $(1/2)I\omega^2$ is just a quick way of summing the kinetic energies of the 7 masses. For the simple 2D system here it is quite simple to prove the equivalence for any arrangement of masses. For 3D the derivation is more complicated, but the idea is the same.)

9 The figure shows a proposed design for a rocket nose-cone (or if you prefer, a Hershey's Kiss). It is a solid of revolution with base radius *a,* height *h*, and profile $r = a(1 - z^2 / h^2)$. It has uniform mass density ρ . Using a Matlab 'Live Script', calculate



9.1 The total mass *M* (you will need to do the relevant integrals using cylindrical-polar coordinates)

9.2 The position vector of the center of mass (with respect to the origin shown in the figure)

9.3 The inertia tensor (matrix) about the center of mass, in the basis shown

9.4 Using the parallel axis theorem, calculate the mass moment of inertia about the origin O.