School of Engineering
  Brown University

- Your solution to this homework should consist of two files:
    1. A commented MATLAB Live Script stored as a .mlx file
    2. A commented MATLAB function stored as a .m file
- Please submit the assignment electronically on the EN40 canvas website. You can log into canvas at http://brown.edu/it/canvas/ (the login link is near the top right of the page). You can find some instructions on Canvas use (from CIT) at https://ithelp.brown.edu/kb/articles/canvas-for-students
- (You might find the solutions to homework 1, 2009-2021 helpful, if you get stuck – you could use the 2021 HW as a template, if you like.)

Part 1: Write a Matlab 'Live Script' to solve the following problems.

1. Find all the solutions to the simultaneous equations

$$\frac{1}{x} + \frac{1}{(y+z)} = -2 \qquad \frac{1}{y} + \frac{1}{x+z} = -4 \qquad \frac{1}{z} + \frac{1}{x} = -5$$

2. Plot the function

$$P(x,\mu,\beta) = \frac{1}{\beta}\exp\left(-\left[\frac{x-\mu}{\beta} + \exp\left\{-\frac{(x-\mu)}{\beta}\right\}\right]\right)$$

in the range $-5 < x < 15$, for $(\mu = 0.5, \beta = 1)$; $(\mu = 2, \beta = 1)$; $(\mu = 2, \beta = 4)$; (on the same figure). (Here, 'exp(x)' denotes the exponential function, i.e. $\exp(x) \equiv e^x$, and P is the 'Gumbel' distribution, which is used to predict the likelihood of extreme events – earthquakes, floods, etc). A few points of credit are awarded for making the plot look nice.

3. Evaluate the integral

$$S(x,\mu,\beta) = \int_{-\infty}^{x} P(\xi,\mu,\beta)d\xi$$

Hence, plot $S(x,\mu,\beta)$ in the range $0 < x < 100$, for $(\mu = 0.5, \beta = 1)$; $(\mu = 2, \beta = 1)$; $(\mu = 2, \beta = 4)$; (on the same figure). A few points of credit are awarded for making the plot look nice.

4. Find a formula for the value of x that maximizes $P(x,\mu,\beta)$ (you can find potential maxima by differentiating P with respect to x; then solving $dP/dx = 0$).

5. Find a formula (i.e. an equation) for the maximum value of P

6. The differential equation

$$\frac{dN}{dt} = \mu \exp(-kt)N - \lambda N$$

predicts the growth patterns of the mammary gland of a cow during pregnancy and lactation (after parturition). The time dependent variable N represents the mass of mammary tissue; μ is a constant quantifying the specific rate of cell proliferation (per day) at parturition, k is a decay constant, and λ is a constant that represents the specific rate of cell death. You can find more details in this reference.

6.1 Use the 'dsolve' function to solve the differential equation for N as a function of time. Use the initial condition $N(t) = N_0 \quad t = 0$.

6.2 Plot a graph showing $N(t)$ for an initial cell mass $N_0 = 13.2mg$, and

(a) $\mu = 0.33 \ day^{-1}$, $k = 0.4 \ day^{-1} \ \lambda = 0.04 \ day^{-1}$

(b) $\mu = 0.15 \ day^{-1}$, $k = 0.4 \ day^{-1} \ \lambda = 0.04 \ day^{-1}$

(c) $\mu = 0.33 \ day^{-1}$, $k = 0.2 \ day^{-1} \ \lambda = 0.04 \ day^{-1}$

(d) $\mu = 0.33 \ day^{-1}$, $k = 0.4 \ day^{-1} \ \lambda = 0.02 \ day^{-1}$

for $0 < t < 30$ days.

Part 2: Please solve the remaining problems using MATLAB (write your code in a matlab .m file). You should make your MATLAB (.m) file a function, so that when the file is executed, it will solve all the homework problems. For example:

```
function EN40_Homework1_2020
        code that will solve problems 7-11
end
function [time_vals,sol_vals] = mynewode45(f,time_int,initial_w)
    Code for function you write in problem 10…
end
Code for function you write for problem 11….
```

7. Using a loop, or dot notation, or the 'linspace' function, create a vector of 501 equally spaced points between 0 and 2π

8. Using the solution to problem 7, along with a loop and conditional statement, create vectors x and y that contain values of the function

$$x = \begin{cases} 4\cos(t)\sin^3(t) & 0 < t < 2\pi \\ 0.75\sin(t) & 2\pi < t < 4\pi \end{cases}$$

$$y = \begin{cases} 4\cos^2(t)\sin^2(t) & 0 < t < 2\pi \\ 0.75(\cos(t)-1) & 2\pi < t < 4\pi \end{cases}$$

for $0 < t < 4\pi$ Hence, plot a graph of y (on the vertical axis of the plot) against x (on the horizontal axis).

A few points of credit are awarded for making the plot look nice.

9. The 'Collatz conjecture' asserts that the sequence of integers generated by the following recursive sequence

$$I_{n+1} = \begin{cases} I_n / 2 & I_n \text{ even} \\ 3I_n + 1 & I_n \text{ odd} \end{cases}$$

will always reach $I_n = 1$ (and thereafter cycles through a sequence of $(1,4,2,1....)$. It has never been proved. The sequence of numbers is known as the 'hailstone sequence'

Write a function that takes as input an integer I_1, and outputs a vector containing the hailstone sequence starting at I_1 and ending at the first occurrence of $I_n = 1$.

You can do the calculation with a loop, but use a 'while' loop instead of a 'for loop.' Use the search keyword 'while' in the MATLAB documentation to see instructions and examples for writing a 'while' loop.

You can use the matlab functions 'mod' or 'rem' to check whether I_n is odd or even. Again, use the MATLAB documentation to learn how to use these functions.

9.1 Test your code by calculating the hailstone sequence starting at $I_1 = 7$.

9.2 Plot a graph (on the same figure) of the value of the hailstone sequence I_n on the vertical axis, and iteration number n on the horizontal axis, for $I_1 = 54$ and $I_1 = 97$

10. The 'Ross-Macdonald' equations (which exist in many different forms) describe the spread of malaria between mosquitos and humans. One version of the equations (which assume that the total populations of humans and mosquitos are constant) states that the fraction of infected humans h and the fraction of infected mosquitos m are related by

$$\frac{dh}{dt} = n_m am(1-h) - rh$$

$$\frac{dm}{dt} = ach(1-m) - \mu m$$

Here, n_m is the number of female mosquitoes per human host; a is the number of infected bites by a single mosquito that produce infection per day; c is the transmission efficiency from human to mosquito; r is the average recovery rate of humans, and μ is the mortality rate of mosquitoes.

- Write a function that computes the vector of time derivatives $d\mathbf{w}/dt = [dh/dt; dm/dt]$ given a value of t and the current values of $\mathbf{w} = [h; m]$ and the values of n_m, a, c, r, μ.
- Hence, use the ode45 function in MATLAB to integrate (approximately) the system of equations with respect to time, and plot a graph of h, m as a function of time (put the curves on the same plot)
  Run the simulation for 200 days with initial conditions $h=0, m=0.01$. Plot separate figures for the following values of parameters:

(i) $n_m = 2$, $a = 0.2 / day$, $c = 0.5$, $r = 0.05 / day$, $\mu = 0.4 / day$

(ii) $n_m = 1$, $a = 0.2 / day$, $c = 0.5$, $r = 0.05 / day$, $\mu = 0.4 / day$