1. This publication (Kulakova *et al*, 2017) compares two interatomic potentials for liquid Argon:

(i) The classic 'Lennard-Jones' potential $U_{LJ} = 4E_{LJ}\left\{ \left(\dfrac{a_{LJ}}{r} \right)^{12} - \left(\dfrac{a_{LJ}}{r} \right)^{6} \right\}$

(ii) The (6-p) potential $U_{(6,p)} = 4E_{(6,p)}\left\{ \left(\dfrac{a_{(6,p)}}{r} \right)^{p} - \left(\dfrac{a_{(6,p)}}{r} \right)^{6} \right\}$

Values for the constants in the two potentials were estimated by fitting to experimentally measured 'radial distribution functions' (the probability distribution of distances between neighboring atoms in the fluid). The table shows values determined from a dataset taken at temperature of 84.4K and pressure of 0.8atm.

E_{LJ}	0.0123 eV
a_{LJ}	3.305 Angstroms
$E_{(6,p)}$	0.0983 eV
$a_{(6,p)}$	3.431 Angstroms
p	6.644

1.1 Plot the energy as a function of r for both potentials, on the same graph (use units of eV for the energy, and Angstroms (1 Angstrom is 10^{-10}m) for the distance r. 3<r<8Angstroms gives a clear plot).

1.2 Plot a graph showing the force as a function of separation r, for both potentials, on the same graph. (use units of eV/Ansgstrom[1] for the force, and angstroms for the distance r. *3.5<r<8* Angstroms gives a clear plot) (Just hand in your plot; there is no need to submit MATLAB code)

1.3 Find the equilibrium separation between atoms for the two potentials (in Angstroms). You can use the MATLAB 'vpasolve' function to solve the equation for the separation – see 2021 HW4 solutions for an example. MATLAB will give you several solutions for the Lennard-Jones potential but only one of them is physically meaningful.

1.4 Find the stiffness of the bond for the two potentials (at the equilibrium separation) in N/m

1.5 Find the (static) force required to break the bond, in eV/A. If you are using a MATLAB live script to do the calculation you will need to use the 'vpasolve' function to find the value of r that maximizes the force of attachment. See Sect 6.1 of the MATLAB tutorial for an example.

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[1] This seems a weird force unit to those of us who don't usually think about individual atoms, but it is quite common in the nanoscience field.

2. A rider on an upright bicycle is subjected to a drag force with magnitude $F_D = (\rho C_D A V^2)/2$, where $\rho = 1.22 kg/m^3$ is the air density, $C_D = 0.9$ is the drag coefficient, $A = 0.5 m^2$ is the projected frontal area of the rider and bicycle together, and V is the bicycles speed.

2.1 Calculate the power expenditure of a rider traveling at constant speed of 7m/s (about 16mph) in still air on level ground.

2.2 Suppose that the rider makes a round trip along a straight road with length 23km (46km for the round trip. This is about the length of the Providence – Bristol East Bay bike path, excluding the distance you have to travel from campus to reach the start of the path….). If the rider travels with constant power expenditure with the value you found in 2.1, how long does the trip take (a) in still air; and (b) with a 4 m/s (about 9 mph) wind blowing parallel to the road (a headwind in one direction, and a tailwind in the other). You will need to solve a cubic equation for the bike's speed with a head/tailwind – you can use the MATLAB 'vpasolve' function to solve the equation numerically.

2.3 Suppose that while accelerating from rest to cruise speed the rider (temporarily) expends power at a rate[2]

$$P_{rider} = 2V(14 - V) \qquad Watts$$

where V is the bike speed. Find a formula for the acceleration of the bike during this time, and hence calculate (a) how long it takes to reach cruise speed of 7 m/s and (b) the distance travelled during this time (the energy equation will give you a differential equation for the speed, which you can solve by separating variables. You will need to use MATLAB to do the integrals for you. Use the MATLAB 'double()' function to convert the incomprehensible answer to a floating point number) Use your mass + 10kg for the mass of rider + bicycle.

On the East Bay bike path the rider would probably need to come to a stop at about 10 cross-streets and will also probably come to a stop at the end, so they stop a total of 21 times on the round trip. Calculate how long it would take the rider to make the round trip with no wind.

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[2] The formula is an estimate for the rider's power curve, which is assumed to be quadratic in V with a peak of about 98W at $V=7$m/s. It assumes that the rider does not change gears…

3. The figure (from [this publication](#)) shows force-v-stretch factor curves for latex rubber tubes used in a slingshot. The different curves show force curves for various different maximum values of stretch factor. The red curve shows the force during loading; the black one shows the force during (quasi-static) unloading.
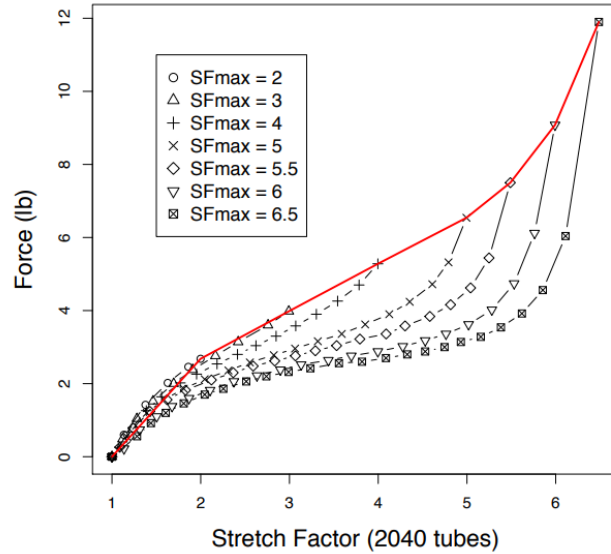
The 'stretch factor' is defined as

$$\lambda = L / L_0$$

where L is the stretched length and L_0 is the unstretched length of the tube.

Stretch Factor (2040 tubes)

The curves for a maximum stretch factor of 6 can be approximated by

$$F_L = 0.1277\lambda^3 - 1.3123\lambda^2 + 5.4929\lambda - 4.3084$$

$$F_U = 0.003834\lambda^5 - 0.01917\lambda^4 + 0.0383\lambda^3 - 0.6348\lambda^2 + 3.5979\lambda - 2.9861$$

where F_L is the (red) loading curve and F_U is the unloading curve is the unloading curve, and the forces are in lb.

3.1 Suppose that a slingshot is made with two parallel tubes with 20cm unstretched length. Plot the force-draw curves (both loading and unloading) for the slingshot, for a draw distance $0 < x < 1$m

3.2 Calculate the work done to draw the slingshot to 1m (calculate the work in Joules)

3.3 Calculate the work done by slingshot during unloading from 1m

3.4 Suppose that the projectile has a mass of 3.6 grams, and the bands and pouch have a combined effective mass of 5.6 grams (i.e. the kinetic energy of the bands and pouch at the instant of launch is $m_{eff}V_p^2 / 2$, where V_p is the projectile speed and m_{eff} is the effective mass of 5.6g). Estimate the launch speed of the projectile.

3.5 Hence, estimate the energy efficiency of the device.
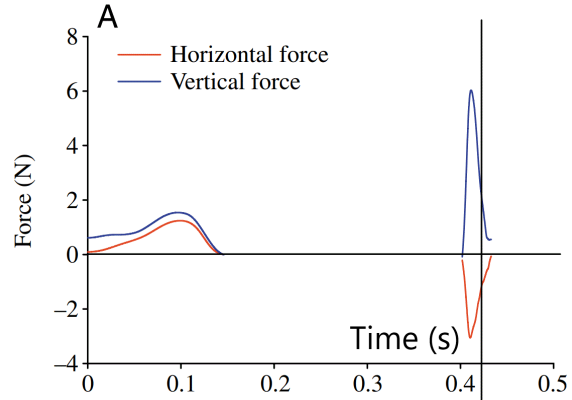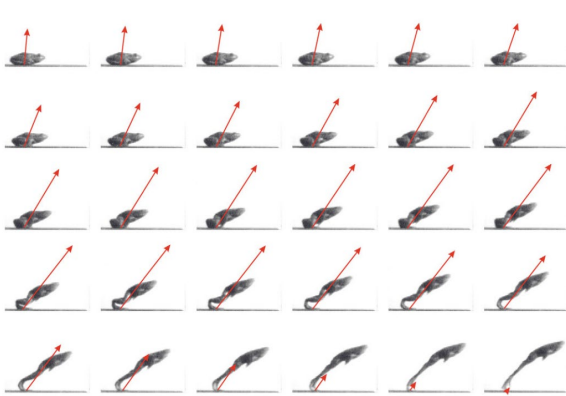
**4.** [This publication](#) reports measurements of the forces exerted by a frog on a force plate during a jump

4.1 Download the image file frog_jump.jpg from the homework page of the course website, along with the MATLAB file called digitize.m   Save both files to the same directory on your PC.   Run the digitize.m file (just follow the instructions in the message boxes) to generate two files called, eg, frog_force_vertical.csv and frog_force.horizontal.csv, which store columns of time and force during the jump (the portion of the curve between $0<t<0.2s$). There is no need to submit a solution to this problem.

4.2 Read the .csv files generated in part 4.1 into MATLAB and (a) plot the data, to check it; and (b) calculate the horizontal and vertical impulses exerted on the frog during its jump. Include your calculations plots and your values for the impulse in your uploaded pdf.

4.3 Use the results of 4.2 to calculate the horizontal and vertical components of the frog's velocity just after it leaves the ground. You can find the frog's mass from the force reading at time $t=0$.

4.4 Check your answer to 4.3 by using Newton's law to calculate the frog's acceleration while it is touching the ground, and using MATLAB to integrate the acceleration to find its velocity. Also, integrate the velocity to calculate the displacement of the frog's COM during the period that it is in contact with the ground

4.5 Use the results of 4.4 and 4.3 to find the time that the frog is airborne, as well as the height h_{max} and length L of its jump (include the displacement while it is touching the ground)
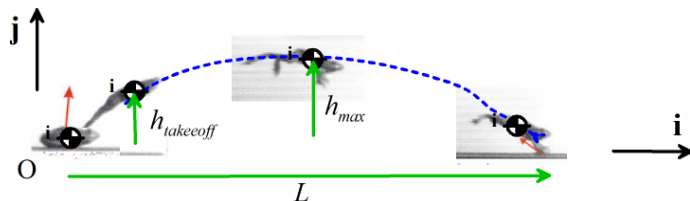
4.6 What is the energy cost (i.e. the work done by the frog) of the jump?

5. describes an experimental method for measuring the restitution coefficient using a double-pendulum apparatus. In a representative experiment pendulum A is released from rest from a known height h_{A1}. At the lowest point of its swing it strikes a second pendulum B that hangs vertically. The height h_{B2} of pendulum B at the instant that it comes to rest is measured.
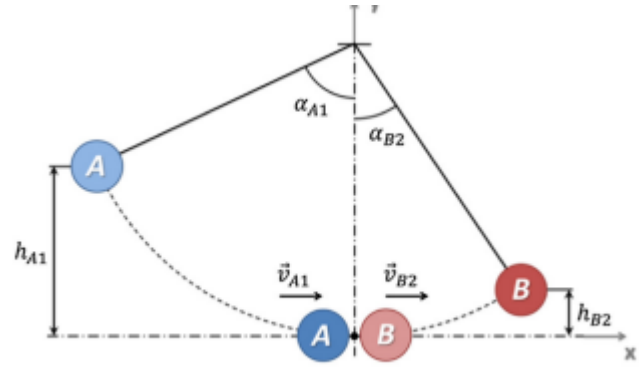
5.1 Find a formula for the restitution coefficient of the collision, in terms of h_{A1} and h_{B2} and the particle masses m_A, m_B. (use energy conservation, linear momentum conservation, and the definition of the restitution coefficient)

5.2 The figure below shows experimental measurements for the collision between two corn kernels with identical mass. Find the restitution coefficient for one of the experiments (be sure to state which one you choose)
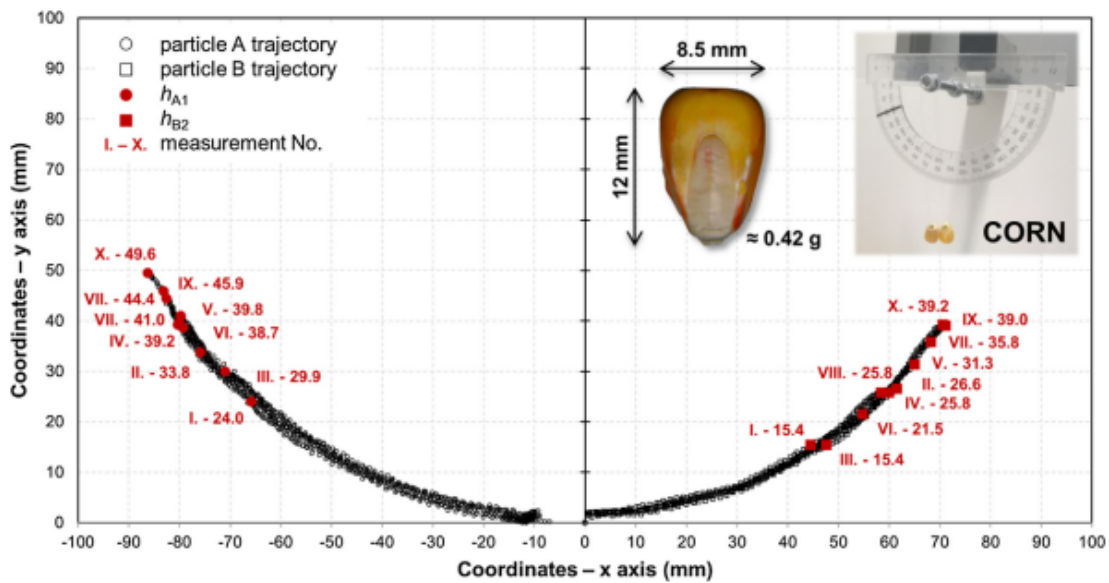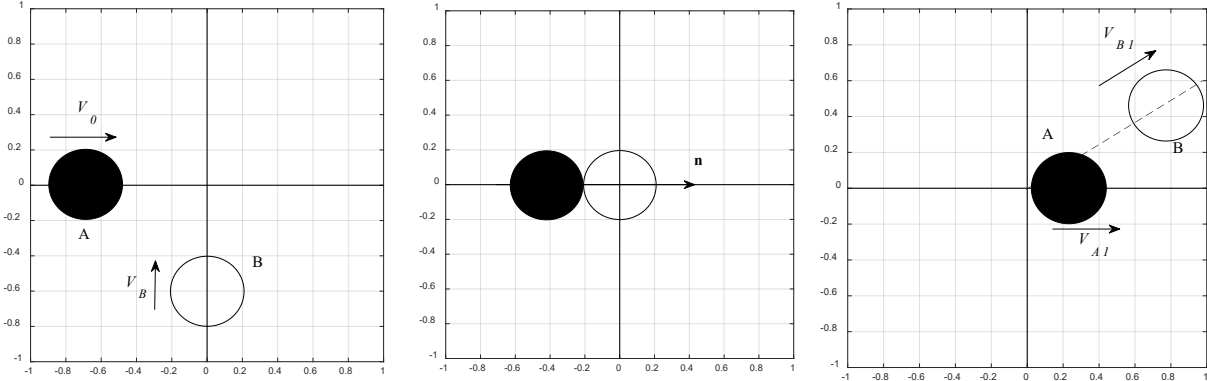

Fig. 7 – Evaluation of ten measurements of coefficient of restitution e_h for pair of corn grains using h_{A1} and h_{B2}, the left part of the diagram shows motion trajectories of motion and the height of the release of particle A (h_{A1}), whereas the right part depicts motion trajectory and rebound height of particle B (h_{B2}).

6. The figure shows an oblique impact between two frictionless spheres with identical mass. Before the impact sphere A has a velocity $\mathbf{v}^{A0} = V_0\mathbf{i}$ and sphere B moves in the $\mathbf{j}$ direction with a speed to be determined. The collision direction $\mathbf{n}$ is parallel to $\mathbf{i}$. After impact, sphere B moves with a speed (to be determined) along a line parallel to $2\mathbf{i} + \mathbf{j}$

6.1 Explain briefly why sphere A must continue to move parallel to the $\mathbf{i}$ direction after the collision.

6.2 Explain briefly why the $\mathbf{j}$ component of sphere B's velocity cannot change during the collision.

6.3 Hence, or otherwise, find formulas (in terms of V_0 and the restitution coefficient e) for the velocity vectors of spheres A and B after the collision, and the speed of sphere B before the collision.