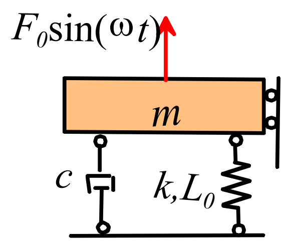
School of Engineering
Brown University

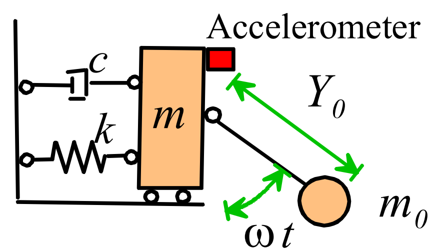
EN40: Dynamics and Vibrations

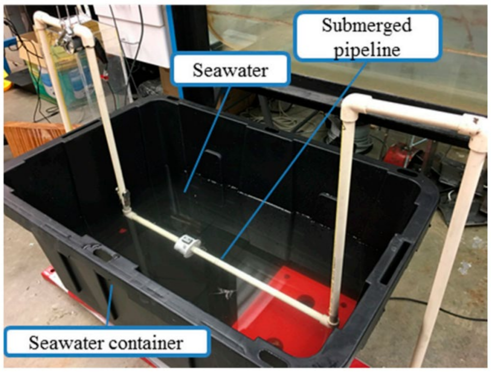
Homework 6: Vibrations
Due Friday April 8 2022

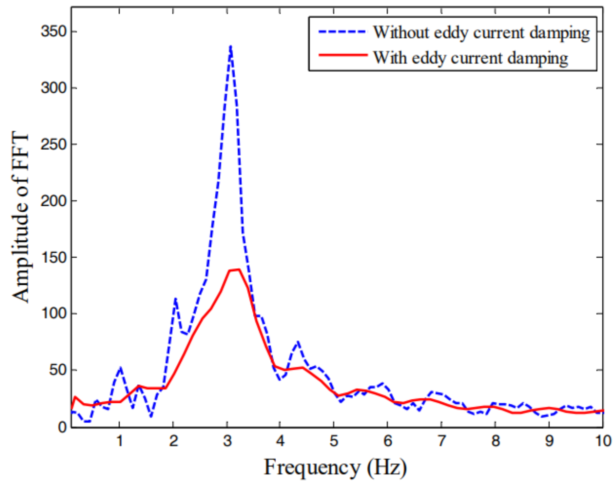
1. The spring-mass system shown is a simple idealization of a building subjected to wind loading. The mass is $250\text{x}10^6$ kg, spring stiffness is 2.5 GN/m and the dashpot coefficient is 160MNs/m. The building is observed to vibrate at a frequency of 1Hz with an amplitude of 3mm. Find the amplitude of the force.



2. An unbalanced rotating fan is mounted on isolation pads giving a spring mass system with natural frequency $\omega_n = 10$ rad/s, damping factor $\zeta = 0.2$, total mass $m + m_0 = 10kg$, and mass unbalance $Y_0 m_0 = 0.1kgm$. The system is at rest for time $t<0$. The fan is then turned on and spins with angular speed $20rad/s$. Plot a graph showing the acceleration of the mass as a function of time, for a time interval 0<t<4s. (Be careful when calculating the phase of the steady state response, the phase must be in the range $0 < \phi < -\pi$. When $\omega > \omega_n$ the phase should be less than $-\pi/2$). It is best to do the calculation and plot in MATLAB. You need not submit your MATLAB code, however – just explain how you did the calculation and submit your plot.







3. The figure (from this publication) shows the measured vibration amplitude of a submerged pipeline with, and without, and eddy current damper installed, as a function of frequency.

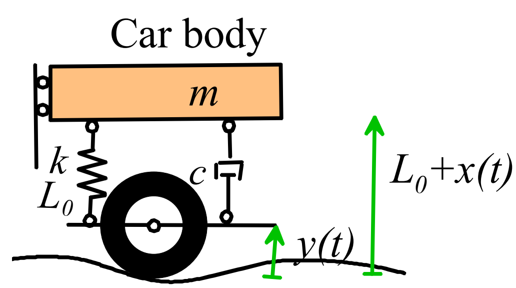
3.1 What (approximately) is the undamped natural frequency of vibration of the system ω_n ?

3.2 What is the damping factor ζ , before and after the damper is added?

4. The re-designed vehicle suspension in Section 5.6.9 of the lecture notes had the following specification:

- Natural frequency $\omega_n = 5\pi / 2.1$ rad/s
- Damping factor $\zeta = 0.38$

Suppose that the car drives over a road with roughness amplitude 5cm and wavelength 8m.



4.1 Show that if the mass of the wheel can be neglected, the normal force exerted by the wheel on the ground is given by

$$N = kY_0\left[\frac{g}{Y_0\omega_n^2} - M\frac{\omega^2}{\omega_n^2}\sin(\omega t + \phi)\right]$$

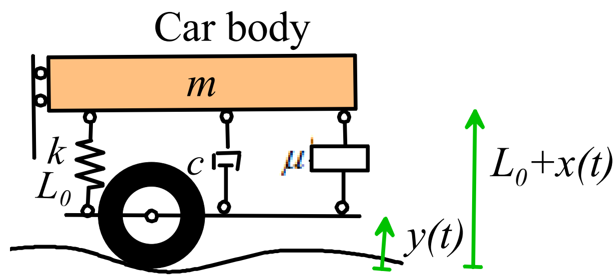
where Y_0 is the amplitude of the roughness, $\omega = 2\pi V / L$ is the excitation frequency, ω_n is the natural frequency of the suspension system and M is the magnification.

4.2 Hence, find the car speed where the wheel will lose contact with the ground. You might find it helpful to plot a graph of $(\omega / \omega_n)^2 M$ as a function of ω / ω_n

5. Formula 1 racecars have been using ‘inerters’ in their suspensions for over 10 years. An ‘inerter’ is a mechanical element that, like a spring or dashpot, can be stretched by a force. In an inerter, the force is proportional to the relative acceleration of its ends (the second time derivative of its length)

$$F_I = \mu \frac{d^2L}{dt^2}$$

The goal of this problem is to investigate the effects of adding an inerter to a suspension.



Before attempting this problem you might find it helpful to review the analysis of a conventional spring-mass-damper suspension system discussed in the lecture videos (see Example 5.6.9 in the Detailed Syllabus).

5.1 Draw a free body diagram showing the forces acting on the mass (the car body). Neglect gravity.

5.2 Hence, show that the displacement of the wheel y, and the displacement of the mass are related by an equation of the form

$$\frac{1}{\omega_n^2}\frac{d^2x}{dt^2} + \frac{2\zeta}{\omega_n}\frac{dx}{dt} + x = \frac{\lambda^2}{\omega_n^2}\frac{d^2y}{dt^2} + \frac{2\zeta}{\omega_n}\frac{dy}{dt} + y$$

and give formulas for ω_n, λ, ζ in terms of c, k, μ

5.3 The solution to the EOM in part 4.2 can be found in the tabulated solutions to differential equation for common vibration problems. Note that (like the conventional suspension system) the amplitude of vibration of the car body is given by

$$X_0 = KM\left(\omega / \omega_n, \zeta, \lambda\right) Y_0$$

where K=1. To understand the effects of the inerter, plot (on the same axes) a graph of the magnification M as a function of ω / ω_n for $0 < \omega / \omega_n < 3$, $\zeta = 0.1$, and $\lambda = 0$ (a conventional suspension), $\lambda = 0.5$, and $\lambda = 0.7$ (representing the addition of a weak and a strong inerter). You will find that the inerter gives the suspension a so-called ‘anti-resonance’ – a magic frequency where M has a minimum value.

5.4 What is the frequency corresponding to the anti-resonance (the minimum value of M), in terms of λ, ω_n (give an approximate solution for $\zeta << 1$) ? What is (approximately) the smallest vibration amplitude (in terms of λ, ζ)?

5.5 For what range of frequency (in terms of λ, ω_n) does the antiresonant system give better performance than the simpler spring-mass-damper system with an identical value of ζ, ω_n but $\lambda = 0$?

5.6 Re-visit the suspension system that was re-designed in Section 5.6.9 of the lecture videos. Re-design the system (with an inerter) to meet the following specifications:

(1) The vehicle is to be designed to drive over a roadway with roughness wavelength 10m, and amplitude is 20cm.
(2) The suspension should give the minimum vibration amplitude of the car body at 50 mph
(3) The suspension should be effective (i.e. the car should vibrate with amplitude less than that of the roadway) for all vehicle speeds exceeding 20 mph
(4) The amplitude of vibration of the car's body should not exceed 35cm at any speed
(5) The static deflection of the suspension spring should be as small as possible, while still satisfying (1-4)

For your design, recommend values of ω_n, ζ, λ , and k, c, μ . You can use small ζ approximations to simplify calculations (it's possible to get a solution without this approximation in MATLAB but it's quite tricky to set up – try it if you would like a challenge. It changes the numbers you get a bit, but doesn't have an appreciable effect on the performance of the design at the end).

5.7 Compare your new design to the one in the notes by plotting graphs (on the same figure) of the predicted vibration amplitude for the two designs (use the system designed at the end of Section 5.6.9 as the 'original' design, and plot your new design on the same graph) as a function of car speed in the range $0<V<80$ mph.