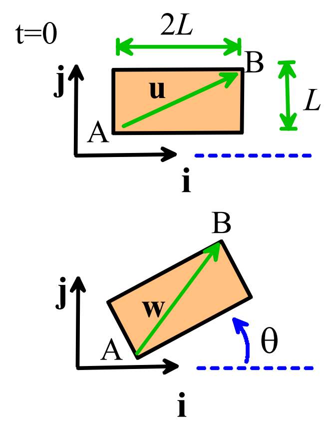
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 7: Rigid Body Kinematics, Inertial properties of rigid bodies
Due Friday April 22 2022

1 The rectangle shown in the figure starts with $\theta = 0$ at time $t=0$ and rotates with an angular velocity vector $\boldsymbol{\omega} = 4t^3\mathbf{k}$. At a time $t=2$sec



1.1 Find the angular acceleration vector

1.2 Find the rotation matrix

1.3 Find the spin tensor

1.4 Find the time derivative of the vector **w**.

2. Let $\mathbf{R}_x(\theta_x), \mathbf{R}_y(\theta_y), \mathbf{R}_z(\theta_z)$ denote rotations through angles $(\theta_x, \theta_y, \theta_z)$ about the {**i,j,k**} axes (with RH screw convention), respectively. This publication shows that a sequence of 6 rotations of the form

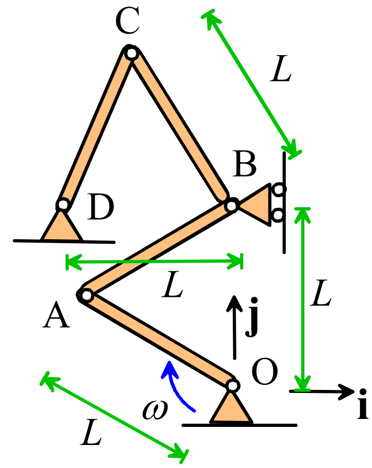
$$\mathbf{R} = \mathbf{R}_x(\theta_x)\mathbf{R}_y(\theta_y)\mathbf{R}_z(\theta_z)\mathbf{R}_x(-\theta_x)\mathbf{R}_z(-\theta_z)\mathbf{R}_y(-\theta_y)$$

can rotate an object to any desired orientation, without any net rotation about the {**i,j,k**} axes (this is helpful in robots whose actuators can only turn through a finite angle).

2.1 Find the rotation matrix caused by this sequence of rotations with $\theta_x = \pi/2$, $\theta_y = -\pi/2$, $\theta_z = -\pi/2$. (you can use MATLAB to do the matrix multiplications, if you wish)

2.2 Find the axis and angle of the rotation matrix you found in 3.1.

3. The figure (from this publication) shows a candidate design for the 6 bar chain mechanism in an electrically actuated clamp. Member OA rotates clockwise with constant angular speed ω For the configuration shown in the figure (note that OAB and BCD are equilateral triangles):

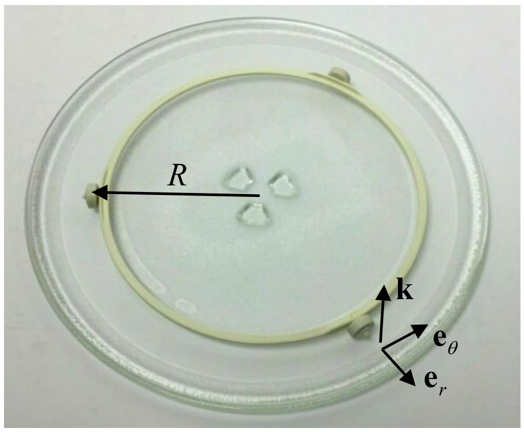


3.1 calculate the velocity of point B, along with the angular speeds of members AB, BC and BD.

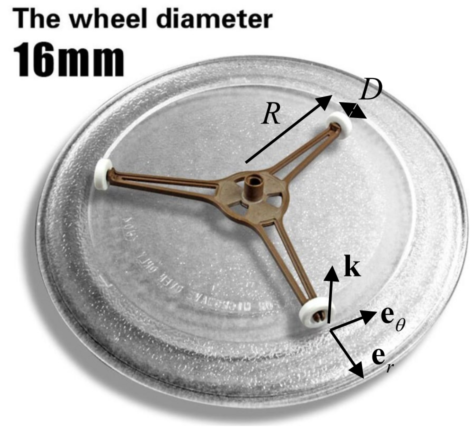
3.2 calculate the acceleration of point B, along with the angular accelerations of members AB, BC and BD.

4. The figure shows a portal lift gear. Find the ratio of the gearbox (count the teeth on the gears). What is the purpose of the two smaller 'idler' gears? Why are there two of them?

5. This video shows an epicyclic gear clock in which the sun gear is connected to the minute hand, the planet carrier is connected to the hour hand and the ring gear is stationary. Calculate the number of teeth on the sun, ring, and planet gear (of course the solution is not unique – recommend something sensible!)



(a)

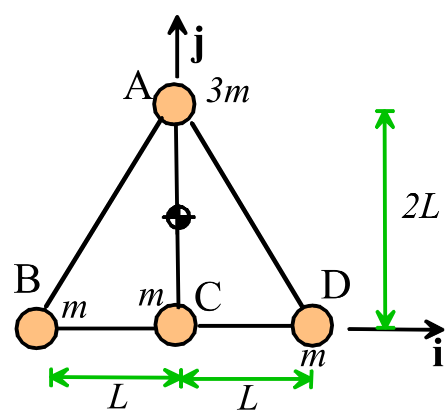


(b)

6. The figure shows two designs for a microwave turntable. In (a) the motor drives the glass plate directly, while in (b) it drives the plastic support.

If the motor turns with angular velocity $\omega\mathbf{k}$, find formulas for the angular velocity vectors of the support, the turntable, and the white rollers for each design. Express your answer in a basis that rotates with the support, as shown in the pictures.

7. The figure shows four particles connected by rigid massless links. The particle at A has mass $3m$; those at B, C and D have mass m. The assembly rotates at constant angular speed ω about an axis parallel to **k** passing through the center of mass. The point of this problem is to demonstrate that the rigid body formula for the kinetic energy of the system gives the same answer as calculating the kinetic energy of each mass separately, and summing them. The rigid body formulas for angular momentum and kinetic energy are just fast ways of summing the total angular momentum and KE of a system of particles.



7.1 Calculate the position of the center of mass of the assembly

7.2 Calculate the 2D mass moment of inertia of the system about the center of mass

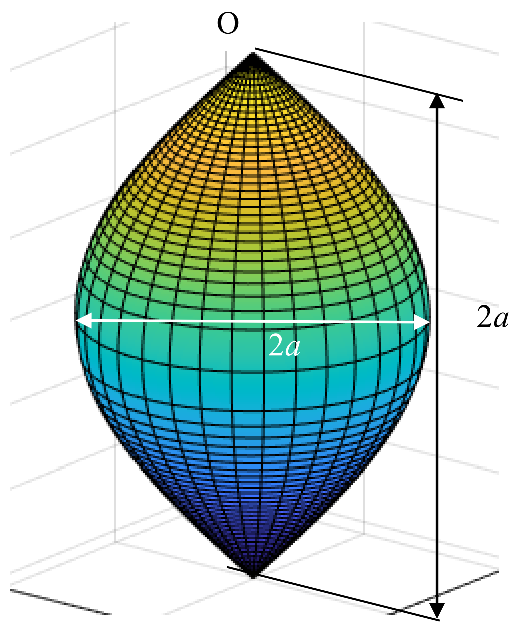
$$I_{Gzz} = \sum_i m_i \left(d_{xi}^2 + d_{yi}^2 \right)$$

where $\mathbf{d}_i = d_{xi}\mathbf{i} + d_{yi}\mathbf{j} = \mathbf{r}_i - \mathbf{r}_G$ is the position vector of the ith particle with respect to the center of mass.

7.3 Suppose that the assembly rotates about its center of mass with angular velocity $\omega\mathbf{k}$ (the center of mass is stationary). What are the speeds of the particles A,B and C?

7.4 Calculate the total kinetic energy of the system (a) using your answer to 7.2; and (b) using your answer to 7.3. (The point of this problem is to demonstrate that the rigid body formula $(1/2)I\omega^2$ is just a quick way of summing the kinetic energies of the 4 masses. For the simple 2D system here it is quite simple to prove the equivalence for any arrangement of masses. For 3D the derivation is more complicated, but the idea is the same.)

8 The figure shows a solid of revolution with base radius *a*, height 2*a*, and profile $r = a\cos(\pi z/(2a))$. It has uniform mass density ρ. Using a Matlab 'Live Script', calculate



8.1 The total mass M (you will need to do the relevant integrals using cylindrical-polar coordinates)

8.2 The inertia tensor (matrix) about the center of mass (which must be at the origin, by symmetry, but you could verify this by doing the integrals if you wish), in the basis shown

8.3 Using the parallel axis theorem, calculate the mass moment of inertia about the tip O.