# Goal for Sections (Jan 25, Jan 30, Feb 1)

## Worked examples on the following topics:

- Vectors/matrices ✓
- Loops ✓
- Conditional statements ✓
- Functions
- Solving ODEs using Mupad
- Using ode45 to solve ODEs

# Example (Homework 2012)

Write a MATLAB function that will count the number of entries in a vector whose value exceeds a critical value

Use the function to count the number of the first 17 digits of pi that exceed 7

You can use the following method to create a vector containing the first 17 digits of pi

```
for i=1:17
pivec(i) = floor(pi*10^(i-1))-10*floor(pi*10^(i-2));
end
```

See MATLAB script online for solution

# Solving Ordinary Differential Equations (ODEs)

- What is an ODE?
- Solving an ODE with Mupad
- Solving an ODE with Matlab
- How the ODE solver works (Euler method)
- Solving two simultaneous ODEs with MATLAB

# Simple example of an ODE

$$\frac{dw(t)}{dt} = w(t)$$

‘Ordinary Differential Equation’

$$w = 0.1 \quad \text{at} \quad t = 0$$

‘Initial Condition’

In words: find a function w(t) such that

(1) w(t) is equal to its slope dw/dt for all values of t

(2) w(t)=0.1 at t=0

- Solve the equation by hand (separate variables)
- Solve the equation with Mupad (See code for solution)
- Solve the equation with Matlab (See code for solution)

# Simple example of an ODE

## Hand Solution

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Solve $\frac{dw}{dt} = w$ for $w(t)$ given $w = 0.1$ @ $t = 0$

$$\int_{0.1}^{w} \frac{dw}{w} = \int_0^t dt$$

$$\left[\log w\right]_{0.1}^{w} = \left[t\right]_0^t$$

$$\log \frac{w}{0.1} = t \Rightarrow w = 0.1 \exp(t)$$

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# Example

6. The differential equations

$$\frac{dm}{dt} = m(\alpha - \beta c) \qquad \frac{dc}{dt} = -c(\gamma - \delta m)$$

are the famous 'predator-prey' equations used by mathematical biologists to predict the populations of two interacting species. In these equations

- $m$ (mice) represents the population density of the prey
- $c$ (cats) represents the population density of the predators
- $\alpha$ is the rate at which mice reproduce – for large $\alpha$, the mice reproduce very fast. The model assumes that mice have lots of food, so their population will be infinite if they don't get eaten.
- $\beta$ specifies how fast the cats eat the mice – for large $\beta$, each cat eats lots of the available mice.
- $\gamma$ is the rate at which cats perish if they can't catch any mice
- $\delta$ is the rate at which cats reproduce if they have a supply of mice. Note that the cats reproduce faster if there are lots of mice.

Following the example in the tutorial, write a function that will calculate $dm/dt$ and $dc/dt$ given values of $t$ and $[m, c]$. Then add commands to your MATLAB file that will integrate the equations and plot the solution $m(t)$, $c(t)$ as functions of time. Find and plot the solution up to time $t=15$ days, for the following parameter values:

Initial conditions: $m = 20, c = 1$ at time $t=0$

Parameters $\alpha = 4$ $\beta = 3$ $\gamma = 1$ $\delta = 0.1$ (in days$^{-1}$)